高技术通讯 2013 年 第23 卷 第3 期:325 ~329

doi:10.3772/j.issn.1002-0470.2013.03.017

非均质材料瞬态热传导的时域自适应等效分析①

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摘　要　为实现非均质材料瞬态热传导的时域自适应等效分析,提出了一种分析线性非均质材料瞬态热传导的等效模型,用来预测等效性能并评估宏观温度场的等效行为;基于时域自适应算法,将瞬态热传导方程在时段上展开,结合渐进展开均匀化技术,得到递推格式的等效热传导方程,再利用有限元方法得到等效瞬态温度场求解模型。在实现等效分析的计算中,可通过自适应计算避免时段大小变化时可能出现的计算误差。通过数值算例,将等效分析的计算结果与基于 ANSYS 的有限元非均质解进行了比较,结果是令人满意的。

关键词　瞬态热传导, 时域, 自适应, 均匀化, 非均质材料

0　引　言

非均质材料热传导问题与核工业[1,2]、航空航天[3,4]、土木工程[5-10]等诸多工程领域密切相关,在工程上需要进行非均质材料的与时间相关的热传导分析。为避免计算量过大,常将非均质材料作为一种宏观物质材料进行等效求解,其中一个重要问题是如何考虑其计算模型。本文提出了一种分析线性非均质材料瞬态热传导的数值模型,用来预测等效性能并评估宏观温度场的等效行为。为此,基于时域自适应算法,将瞬态热传导方程在时段上展开,结合渐进展开均匀化技术,得到了递推格式的等效热传导方程,再利用有限元方法得到等效瞬态温度场求解模型。在计算中,可通过自适应计算避免时段大小变化时可能出现的计算误差。本文通过数值算例,将等效分析的计算结果与基于 ANSYS 的有限元非均质解进行了比较。

1　非均质材料瞬态热传导等效模型

1.1　基本问题

二维瞬态温度场 $\varphi(x,y,t)$ 在宏观坐标系 x_1-x_2 中应满足的微分方程为[11,12]

$$\rho^\varepsilon c^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial t^\varepsilon} = \frac{\partial}{\partial x_1}\left(k_{11}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_1}\right) + \frac{\partial}{\partial x_1}\left(k_{12}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_2}\right) + \frac{\partial}{\partial x_2}\left(k_{21}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(k_{22}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_2}\right) + Q^\varepsilon \tag{1}$$

边界条件如下

$$\Phi^\varepsilon = \overline{\Phi^\varepsilon} \quad (\text{在 } \Gamma_1 \text{ 边界上}) \tag{2}$$

$$k_{11}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_1} n_{x_1} + k_{22}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_2} n_{x_2} = \bar{q}^\varepsilon (\text{在 } \Gamma_2 \text{ 边界上}) \tag{3}$$

$$k_{11}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_1} n_{x_1} + k_{22}^\varepsilon \frac{\partial \Phi^\varepsilon}{\partial x_2} n_{x_2} = h^\varepsilon (\Phi_\alpha^\varepsilon - \Phi^\varepsilon) \quad (\text{在 } \Gamma_3 \text{ 边界上}) \tag{4}$$

其中,Φ^ε 和 t 分别表示温度和时间,ρ^ε、c^ε、Q^ε 分别表示材料的密度、比热容和热源密度;$k_{ij}^\varepsilon(i, j = 1, 2)$ 是热传导系数,$\overline{\Phi}^\varepsilon$ 和 $\bar{q}^\varepsilon$ 分别是给定温度和热流密度;n_x, n_y 是边界外法向的方向余弦;h^ε 是对流换热系数,对于 Γ_3 边界,在自然对流条件下,t 是外界环境温度;在强迫对流条件下,Φ_α^ε 是边界层的绝热壁温度。$\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3$ 是整个边界。在此令 $\Gamma_3 = 0$。

1.2　等效模型

根据时域自适应算法,在离散的时段内,Φ^ε、

① 国家自然科学基金(10772035,10721062,10472019)和973 计划(2005CB321704)资助项目。

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(收稿日期:2012-03-30)

$\overline{\Phi}^{\varepsilon}$、$\bar{q}^{\varepsilon}$、$Q^{\varepsilon}$ 通过 s 展开为

$$\Phi^{\varepsilon} = \sum_{m=0}\Phi_m^{\varepsilon}s^m \tag{5}$$

$$\overline{\Phi}^{\varepsilon} = \sum_{m=0}\overline{\Phi}_m^{\varepsilon}s^m \tag{6}$$

$$\bar{q}^{\varepsilon} = \sum_{m=0}\bar{q}_m^{\varepsilon}s^m \tag{7}$$

$$Q^{\varepsilon} = \sum_{m=0}Q_m^{\varepsilon}s^m \tag{8}$$

其中

$$s = \frac{t - t_{k-1}}{T_k},\ s \in [0,1],\ k = 1,2,3,\cdots \tag{9}$$

Φ_m^{ε},$\overline{\Phi}_m^{\varepsilon}$,$\bar{q}_m^{\varepsilon}$ 和 Q_m^{ε} 是 Φ^{ε},$\overline{\Phi}^{\varepsilon}$,$\bar{q}^{\varepsilon}$ 和 Q^{ε} 的展开系数,T_k 是时段长度,m 是时域上的展开阶数。将式(6)带入式(1)-(3),得到展开格式的瞬态热传导方程为

$$\frac{\partial}{\partial x_1}(k_{11}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_1}) + \frac{\partial}{\partial x_1}(k_{12}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_2}) + \frac{\partial}{\partial x_2}(k_{21}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_1}) + \frac{\partial}{\partial x_2}(k_{22}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_2}) = (m+1)\frac{\rho^{\varepsilon}c^{\varepsilon}\Phi_{m+1}^{\varepsilon}}{T_k} - Q_m^{\varepsilon} \quad (m = 0,1,2,\cdots) \tag{10}$$

边界条件如下:

$$\Phi_m^{\varepsilon} = \overline{\Phi}_m^{\varepsilon},\ m = 0,1,2,\cdots\ (\text{在}\Gamma_1\text{边界上}) \tag{11}$$

$$k_{11}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x}n_x + k_{22}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial y}n_y = \bar{q}_m^{\varepsilon},\ m = 0,1,2,\cdots\ (\text{在}\Gamma_2\text{边界上}) \tag{12}$$

根据泛函驻值原理,式(10)-(12)可写作[14]

$$\int_{\Omega}(k_{11}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_1}\frac{\partial u}{\partial x_1} + k_{12}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_1}\frac{\partial u}{\partial x_2} + k_{21}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_1}\frac{\partial u}{\partial x_2} + k_{22}^{\varepsilon}\frac{\partial \Phi_m^{\varepsilon}}{\partial x_1}\frac{\partial u}{\partial x_2} - (Q_m^{\varepsilon} - \frac{(m+1)\rho^{\varepsilon}c^{\varepsilon}\Phi_{m+1}^{\varepsilon}}{T_k})u)\mathrm{d}\Omega + \int_{\Gamma_2}\overline{q_m}\,u\mathrm{d}\Gamma = 0 \tag{13}$$

其中 u 是任意函数。

根据渐进展开均匀化技术,$\Phi_m^{\varepsilon}(x_1, x_2)$ 和 $\Phi_{m+1}^{\varepsilon}(x_1, x_2)$ 按照 ε 展为

$$\Phi_m^{\varepsilon}(x_1, x_2) = \Phi_m^0(x_1, x_2) + \Phi_m^1(x_1, x_2, y_1, y_2)\varepsilon^1 + \Phi_m^2(x_1, x_2, y_1, y_2)\varepsilon^2 + \cdots \tag{14}$$

$$\Phi_{m+1}^{\varepsilon}(x_1,x_2) = \Phi_{m+1}^0(x_1,x_2) + \Phi_{m+1}^1(x_1,x_2,y_1,y_2)\varepsilon^1 + \Phi_{m+1}^2(x_1, x_2, y_1, y_2)\varepsilon^2 + \cdots \tag{15}$$

其中 y_1 和 y_2 是微观坐标系 $y_2 - y_1$ 中的坐标。考虑到对任意函数 $F^{\varepsilon}(x_1, x_2)$ 有

$$\frac{\partial F^{\varepsilon}(x_1, x_2)}{\partial x_i} = \frac{\partial F(x_1, x_2)}{\partial x_i} + \frac{1}{\varepsilon}\frac{\partial F(x_1, x_2, y_1, y_2)}{\partial y_i} \quad (i = 1,2) \tag{16}$$

将式(14)带入到式(13)并比较 ε 同次幂系数可得

$$\frac{1}{\varepsilon}\int_{\Omega}\sum_{i=1}^{2}\sum_{j=1}^{2}k_{ij}^{\varepsilon}(\frac{\partial \Phi_m^0}{\partial x_i} + \frac{\partial \Phi_m^1}{\partial y_i})\frac{\partial u}{\partial y_i}\mathrm{d}\Omega = 0 \tag{17}$$

$$\int_{\Omega}\{\sum_{i=1}^{2}\sum_{j=1}^{2}k_{ij}^{\varepsilon}[(\frac{\partial \Phi_m^0}{\partial x_i} + \frac{\partial \Phi_m^1}{\partial y_i})\frac{\partial u}{\partial x_j} + (\frac{\partial \Phi_m^1}{\partial x_i} + \frac{\partial \Phi_m^2}{\partial y_i})\frac{\partial u}{\partial x_j}]\} - [Q^{\varepsilon} - \frac{(m+1)\rho^{\varepsilon}c^{\varepsilon}\Phi_{m+1}^0}{T_k}]u\mathrm{d}\Omega + \int_{\Gamma}\bar{q}_m\,u\mathrm{d}\Gamma = 0 \tag{18}$$

式(17)两边同乘 ε 后取极限 $\varepsilon \to 0^+$,并考虑对任意函数 $F(x_1, x_2, y_1, y_2)$ 有关系式

$$\lim_{\varepsilon\to 0^+}\int_{\Omega}F(x_1, x_2, y_1, y_2)\mathrm{d}\Omega = \int_{\Omega}(\frac{1}{|Y|}F(x_1, x_2, y_1, y_2)\mathrm{d}Y)\mathrm{d}\Omega \tag{19}$$

得

$$\int_{\Omega}\frac{1}{|Y|}(\int_{Y}\sum_{i=1}^{2}\sum_{j=1}^{2}k_{ij}^{\varepsilon}(\frac{\partial \Phi_m^0}{\partial x_i} + \frac{\partial \Phi_m^1}{\partial y_i})\frac{\partial u}{\partial y_i}\mathrm{d}Y)\mathrm{d}\Omega = 0 \tag{20}$$

其中,$\int_{\Omega}$ 和 $\int_{Y}$ 分别表示在整个域和单胞上积分,$|Y|$ 表示单胞的面积。由于 u 的任意性,令

$$u(x_1, x_2, y_1, y_2) = g(x_1, x_2)p(x_1, x_2) \tag{21}$$

其中

$$g(x_1, x_2) \in \Phi_{\Omega} = \{\Phi(x_1, x_2) \mid x_1, x_2 \in \Omega\} \tag{22}$$

$$p(y_1, y_2) \in \Phi_{Y} = \{\Phi(y_1, y_2) \mid y_1, y_2 \in Y\} \tag{23}$$

考虑 $g(x_1, x_2)$ 和 $p(y_1, y_2)$ 均为任意,由式(20)得

$$\int_{Y}\sum_{i=1}^{2}\sum_{j=1}^{2}k_{ij}^{\varepsilon}(\frac{\partial \Phi_m^0}{\partial x_i} + \frac{\partial \Phi_m^1}{\partial y_i})\frac{\partial p}{\partial y_i}\mathrm{d}Y = 0 \tag{24}$$

式(24)为定义在单胞上的热平衡方程。Φ_m^0 可以认为是非均匀荷载项[13]。由于该方程和荷载项 Φ_m^1 和 $\frac{\partial \Phi_m^0}{\partial x_i}$ 都为线性,$i = 1,2$ 存在线性关系

$$\Phi_m^1(x_1, x_2, y_1, y_2) = -\Psi_1(y_1,y_2)\frac{\partial \Phi_m^0}{\partial x_1} - \Psi_2(y_1,y_2)\frac{\partial \Phi_m^0}{\partial x_2} \tag{25}$$

将式(25)带入式(24)得

$$\int_{Y}\sum_{i=1}^{2}\sum_{j=1}^{2}k_{ij}^{\varepsilon}(\frac{\partial \Phi_m^0}{\partial x_i} - \frac{\partial \psi_1}{\partial y_i}\frac{\partial \Phi_m^0}{\partial y_i} - \frac{\partial \psi_2}{\partial y_i}\frac{\partial \Phi_m^0}{\partial y_i})\frac{\partial p}{\partial y_i}\mathrm{d}Y = 0 \tag{26}$$

若 ψ_1, ψ_2 满足

$$\int_Y((k_{11}^{\varepsilon}-k_{11}^{\varepsilon}\frac{\partial\psi_1}{\partial y_i}-k_{21}^{\varepsilon}\frac{\partial\psi_1}{\partial y_2})\frac{\partial u}{\partial y_i}+(k_{11}^{\varepsilon}-k_{11}^{\varepsilon}\frac{\partial\psi_1}{\partial y_i})\frac{\partial u}{\partial y_i})\mathrm{d}Y=0 \tag{27}$$

$$\int_Y((k_{21}^{\varepsilon}-k_{11}^{\varepsilon}\frac{\partial\psi_2}{\partial y_i}-k_{21}^{\varepsilon}\frac{\partial\psi_2}{\partial y_2})\frac{\partial u}{\partial y_i}+(k_{22}^{\varepsilon}-k_{12}^{\varepsilon}\frac{\partial\psi_2}{\partial y_i}-k_{22}^{\varepsilon}\frac{\partial\psi_2}{\partial y_2})\frac{\partial u}{\partial y_2})\mathrm{d}Y=0 \tag{28}$$

则式(25)亦满足式(26)。将式(25)带入式(18)中,由于 u 为任意函数,不妨令 $u=u(x_1,x_2)\in\Phi_\Omega$,并取极限 $\varepsilon\to0^+$ 得

$$\int_\Omega\sum_{i=1}^{2}\sum_{j=1}^{2}(k_{ij}^{\varepsilon}\frac{\partial\Phi_m^0}{\partial x_i}-(Q_m^H-\frac{\rho^Hc^H}{T_k}\Phi_{m+1}^0)u)\mathrm{d}\Omega+\int_{\Gamma_2}\bar{q}_m\mathrm{d}\Gamma=0 \tag{29}$$

其中

$$k_{11}^H=\frac{1}{|Y|}\int_Y(k_{11}^{\varepsilon}-k_{11}^{\varepsilon}\frac{\partial\psi_1}{\partial y_1}-k_{21}^{\varepsilon}\frac{\partial\psi_1}{\partial y_2})\mathrm{d}Y \tag{30}$$

$$k_{12}^H=\frac{1}{|Y|}\int_Y(k_{12}^{\varepsilon}-k_{12}^{\varepsilon}\frac{\partial\psi_1}{\partial y_1}-k_{22}^{\varepsilon}\frac{\partial\psi_1}{\partial y_2})\mathrm{d}Y \tag{31}$$

$$k_{21}^H=\frac{1}{|Y|}\int_Y(k_{21}^{\varepsilon}-k_{11}^{\varepsilon}\frac{\partial\psi_2}{\partial y_1}-k_{21}^{\varepsilon}\frac{\partial\psi_2}{\partial y_2})\mathrm{d}Y \tag{32}$$

$$k_{22}^H=\frac{1}{|Y|}\int_Y(k_{22}^{\varepsilon}-k_{12}^{\varepsilon}\frac{\partial\psi_2}{\partial y_1}-k_{22}^{\varepsilon}\frac{\partial\psi_2}{\partial y_2})\mathrm{d}Y \tag{33}$$

$$Q_m^H=\frac{1}{|Y|}\int_YQ_m^{\varepsilon}\mathrm{d}Y \tag{34}$$

$$\rho^Hc^H=\frac{1}{|Y|}\int_Y\rho^{\varepsilon}c^{\varepsilon}\mathrm{d}Y \tag{35}$$

式(29)是宏观尺度的热传导方程的积分形式。k_{ij}^H $(i,j=1,2)$ 表示宏观等效热传导系数,Q_m^H、ρ^H 和 c^H 分别表示宏观的热源密度、材料密度和比热容。Φ_m^0、Φ_{m+1}^0 分别是时域算法中第 m 和 $m+1$ 阶的宏观温度场函数。目标是得到宏观热传导系数和求解宏观温度场。求解步骤如下:

(a) 首先在单胞上求解式(27)和(28),求得 ψ_1 和 ψ_2。

(b) 将 ψ_1 和 ψ_2 带入式(30)－(33)求得 k_{ij}^H。

(c) 再将 k_{ij}^H 带入到式(29)求解温度场。

2　等效瞬态温度场数值模拟

2.1　概述

本节给出两个瞬态热传导的数值模型,对等效解与使用 ANSYS 计算的非均质解在计算精度和消耗上做了比较,同时对不同步长尺寸下的等效解做了比较。材料参数如表 1 所示,单位均为国际标准单位,每种材料的 $k_{11}^H=k_{22}^H$,$k_{12}^H=k_{21}^H=0$。两种模型之间的相对误差定义为

$$Error(t)_i=\frac{|\Phi(t)_i^{Equ}-\Phi(t)_i^{Het}|}{\Phi(t)_i^{Het}}\times100\% \tag{36}$$

其中 $\Phi(t)_i$ 表示样本点 i 的温度,上标 Equ 和 Het 分别表示等效解和非均质解。定义等效模型和非均质模型的 CPU 时间分别是 CPU^{Equ} 和 CPU^{Het}

表 1　材料参数

材料	热传导系数 (k_{11})	密度 (ρ)	比热容 (c)
基体	1	1	1
纤维	5	2	2

2.2　算例:空心材料瞬态热传导等效分析

计算结构如图 1 所示,在边界 $X=0$ 和 $X=100$ 处温度为 0;在 $Y=0$ 和 $Y=100$ 处,热流密度为 0;非均质模型和等效模型的有限元网格分别如图 2 和图 3 所示,自由度分别是 7741 和 225。边界条件为

$$\Phi(t)=0,\ X=0\cap X=100 \tag{37}$$

$$\frac{\partial\Phi}{\partial t}=0,\ Y=0\cap Y=100 \tag{38}$$

初始条件为 $\Phi(0)=10$。

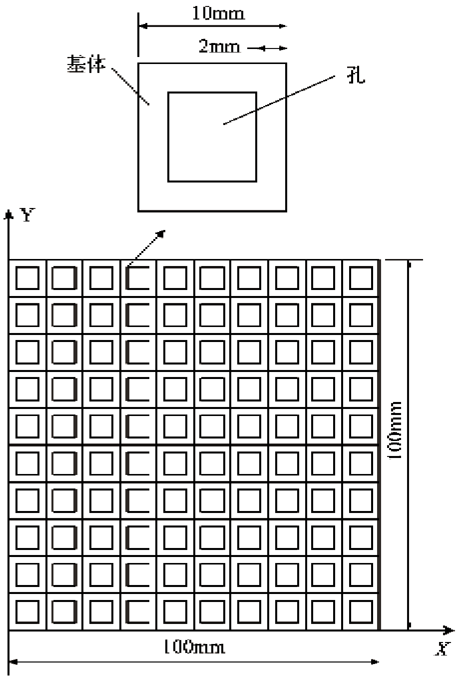

图 1　结构和单胞示意图(算例 1)

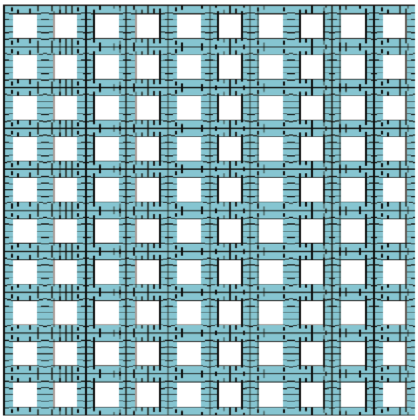

图 2　非均质模型有限元网格(算例 1)

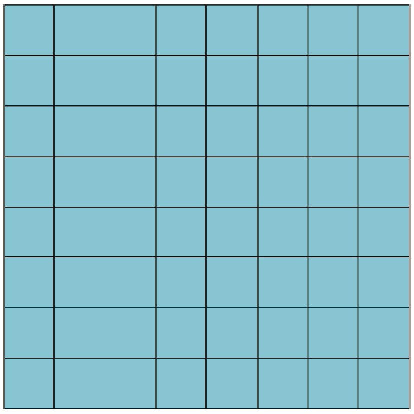

图 3　等效模型有限元网格(算例 1)

对两种模型的 $\Phi(t)_i$ 进行了比较。CPU^{Equ} = 132.344s, CPU^{Hex} = 1904.047s,两者相差 14 倍,误差如表 2 所示。等效解与非均质解的比较见图 4。

表 2　误差(算例 1)

样本点编号	坐标	$Error(0)_i$ (%)	$Error(150)_i$ (%)
1	(1,9)	1.31	5.01
2	(5,5)	1.80	5.43
5	(1,1)	1.28	4.95

2.3　结果分析

(1) 等效分析与基于 ANSYS 的非均质有限元分析的比较表明,本文提出的非均质线性瞬态热传导等效模型可提供快速且计算精度合理的数值结果。

(2) 当步长尺寸变化时,由于自适应计算保证了各时段内误差标准不变,因此避免了在步长尺寸变化时可能造成的计算误差。

3　结　论

本文将时域自适应算法与渐进展开均匀化方法相结合,导出了递推格式线性非均质瞬态温度场的等效求解格式及等效热传导物性参数的表达式。所提出的线性非均质瞬态热传导等效模型可通过自适应计算,避免步长变化时可能出现的计算误差,综合考虑计算精度和效率,其进行等效计算的结果是令人满意的。

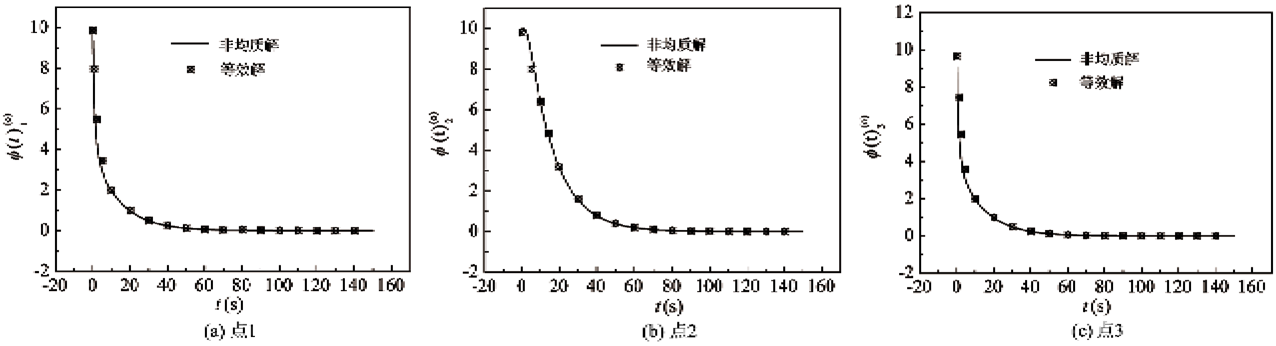

图 4　等效解与非均质解的比较(算例 1)

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Equivalent analysis of heterogeneous material's transient heat conduction based on adaptive computation in time domain

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Abstract

An equivalent model for analysis of the transient heat conduction of linear heterogeneous materials was presented to predict and evaluate the equivalent properties and the equiralent behavior of the macroscopic temperature field. Based on adaptive algorithm in time domain, the transient heat conduction equation was expanded in time domain, and by combining with the asymptotic expansion homogenization technique, the recursive equivalent heat conduction equation was obtained. Finally, the model for calculation of the equivalent transient temperature field was acquired by the finite element method. During the calculation based on the model, calculation errors were avoided through the adaptive computation when the size of time step changed. In numerical examples the calculation results of the equivalent analysis and the heterogeneous solutions based on ANSYS were compared, and the satisfactory results were achieved.

Key words: transient heat conduction, time domain, adaptive, homogenization, heterogeneous material