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Maneuvering target tracking algorithm based on CDKF in observation bootstrapping strategy①

Hu Zhentao (胡振涛)*, Zhang Jin*, Fu Chunling②**, Li Xian*
(* Instituteof Image Processing and Pattern Recognition, Henan University, Kaifeng 475004, P. R. China)
(** School of Physics and Electronics, Henan University, Kaifeng 475004, P. R. China)

Abstract

The selection and optimization of model filters affect the precision of motion pattern identification and state estimation in maneuvering target tracking directly. Aiming at improving performance of model filters, a novel maneuvering target tracking algorithm based on central difference Kalman filter in observation bootstrapping strategy is proposed. The framework of interactive multiple model (IMM) is used to realize identification of motion pattern, and a central difference Kalman filter (CDKF) is selected as the model filter of IMM. Considering the advantage of multi-sensor fusion method in improving the stability and reliability of observation information, the hardware cost of the observation system for multiple sensors is adopted, meanwhile, according to the data assimilation technique in Ensemble Kalman filter(EnKF), a bootstrapping observation set is constructed by integrating the latest observation and the prior information of observation noise. On that basis, these bootstrapping observations are reasonably used to optimize the filtering performance of CDKF by means of weight fusion way. The object of new algorithm is to improve the tracking precision of observed target by the multi-sensor fusion method without increasing the number of physical sensors. The theoretical analysis and experimental results show the feasibility and efficiency of the proposed algorithm.

Key words: maneuvering target tracking, interacting multiple model (IMM), central difference Kalman filter (CDKF), bootstrapping observation

0 Introduction

The key of target tracking is to estimate its motion state by using the priori pattern information of target motion and the latest observation. The availability and reliability of an algorithm depend on two aspects including the matching level between motion model and real motion pattern, and the performance of model filter. According to the pattern and strength of target motion, it is usually divided into non-maneuvering target tracking and maneuvering target tracking[1]. When an observed target moves in the non-maneuvering pattern, it can be described by single model. At this point, it is not related to the model matching problem, and the precision of state estimation mainly relies on the performance of the used filter. When the observed target moves in maneuvering pattern, the structure of multiple models needs to be generally adopted because of the uncertainty of the motion model. For such problem, a group of models usually needs to be designed to describe the different motion behavior. In the model set, each model matches a specific behavior pattern, and the estimation results of more than one parallel filters are organically integrated to constitute state estimation. According to the differences of model switching principle, the structure of multiple models is divided into static multiple models estimation[2] and dynamic multiple models estimation[3]. The hard decision mechanism of binary decision is adopted in the static multiple models estimation, and target motion model is identified by the accumulative result of estimation error. Its weakness is that the threshold value of model decision relies heavily on expert knowledge. Besides, the accumulative process of estimation error results in the delay of model switching time. The typical realization of dynamic multiple models estimation is IMM[4]. A kind of soft decision mechanism of model selection is used in IMM,

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② To whom correspondence should be addressed. E-mail: fuchunling@henu.edu.cn
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which adopts the balance strategy between the precision of model identification and the precision of state estimation. So it avoids the dependence for expert knowledge. At present, IMM is considered as the mainstream approach to solve the maneuvering target tracking problem.

When state model and observation model are linear or weak nonlinear, in order to obtain better performance in the process of model identification and state estimation, Kalman filter (KF)[5] or extended Kalman filter (EKF)[6] are used as a model filter in IMM. However, when they are strong nonlinear, KF and EKF are no longer applicable. Considering that it is much easier to approximate probability density distribution of nonlinear function than nonlinear function itself, meanwhile, accompanied by rapid development of computer performance, the filter design according to sampling strategy becomes the most active research hotspot in nonlinear estimation[7,8]. Recently, some domestic and foreign scholars put forward a series of solutions for the design and optimization of nonlinear filters. Those realizing principles can roughly be divided into deterministic sampling and random sampling. The typical method of deterministic sampling is unscented Kalman filter (UKF)[9]. Its basic idea is that a set of carefully chosen sigma points are used to deliver the statistic characteristics of random variables by UT transform, and then the mean and the covariance can be estimated by the weighted statistical linear regression way. Its advantages are that UKF is insensitive to system nonlinear degree, meanwhile, it avoids the calculation process of Jacobian matrix appearing in EKF. However, the filtering precision of UKF is limited by parameter selection of sigma point and weight, and the non-positive definite problem of estimation error covariance appears easily in filtering iteration. Similar to the implementation of UKF, there are some solutions such as Gauss-Hermite filter (GHF)[10] adopting the numerical integration principle of Gaussian-Hermite, cubature Kalman filtering (CKF)[11] adopting the third-order volume integral principle and so on. The typical methods of random sampling are particle filter (PF)[12] and Ensemble Kalman filter (EnKF)[13], and their common disadvantages are that the filtering precision and computation complexity are limited by the dimension of estimated state and the number of samples. Considering the parallel filtering way used in IMM, many PFs or EnKFs need to be run at the same time. Therefore, the calculation amount will be increased sharply along with the number of target motion models, and real-time is damaged. Aiming at solving the problem, combining with the Stirling interpolation principle, the central difference Kalman filter (CDKF) gives a novel realizing structure of deterministic sampling[14], and it will deals with the contradiction between the estimation precision of nonlinear state and the computational complexity. According to above analysis, through the dynamic combination of IMM and CDKF in observation bootstrap strategy, a novel maneuvering target tracking algorithm is designed in the paper, and the feasibility and efficiency of the algorithm are verified by emulation experiment.

1 The central difference Kalman filter in observation bootstrapping strategy

1.1 Central difference Kalman filter

CDKF is considered as a classic nonlinear filter based on the Stirling interpolation principle. In realization of CDKF, sigma points are sampled according to state prior distribution of observed system, and its posterior distribution is expressed by sigma points using linear regression transformation[15]. Let L denote the state dimension of the observed system, the number of sigma points is $2L+1$. In order to make sigma points have the same mean value, variance and higher-order center distance with real state, sigma points and their corresponding weight are expressed as

$$\begin{cases} \boldsymbol{\xi}^l_{k-1|k-1} = \hat{\boldsymbol{x}}_{k-1|k-1} & l = 0 \\ \boldsymbol{\xi}^l_{k-1|k-1} = \hat{\boldsymbol{x}}_{k-1|k-1} + \lambda(\sqrt{\boldsymbol{P}_{k-1|k-1}})_l & l = 1,2,\cdots,L \\ \boldsymbol{\xi}^l_{k-1|k-1} = \hat{\boldsymbol{x}}_{k-1|k-1} - \lambda(\sqrt{\boldsymbol{P}_{k-1|k-1}})_l & l = L+1, L+2,\cdots,2L \end{cases} \quad (1)$$

$$\begin{cases} \omega^l = (\lambda^2 - L)/\lambda^2 & l = 0 \\ \omega^l = 1/2\lambda^2 & l = 1,2,\cdots,2L \end{cases} \quad (2)$$

$\hat{\boldsymbol{x}}_{k-1|k-1}$ denotes the state estimation at time $k-1$, and $\boldsymbol{P}_{k-1|k-1}$ denotes error covariance matrix of $\hat{\boldsymbol{x}}_{k-1|k-1}$. λ denotes the half-step length in central difference principle, its optimum value is $\sqrt{3}$ in Gaussian distribution. $(\sqrt{\boldsymbol{P}_{k-1|k-1}})_l$ deontes the lth column of square-rooting matrix of $\boldsymbol{P}_{k-1|k-1}$. The concrete realization of CDKF is as follows.

1) Initialization

$$\hat{\boldsymbol{x}}_0 = E[\boldsymbol{x}_0] \quad (3)$$

$$\boldsymbol{P}_0 = E[(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)^{\mathrm{T}}] \quad (4)$$

$\boldsymbol{x}_0$ and $\hat{\boldsymbol{x}}_0$ denote the real state and the state estimation in initial time, and $\boldsymbol{P}_0$ denotes the error covariance matrix of $\hat{\boldsymbol{x}}_0$

2) Time update

According to Eq. (1), sigma point $\boldsymbol{\xi}^l_{k-1|k-1}$ is sampled, and then $\boldsymbol{\xi}^l_{k|k-1}$ (the spread value of $\boldsymbol{\xi}^l_{k-1|k-1}$) is calculated in line with the state transition function

$f(\cdot)$.

$$\boldsymbol{\xi}_{k|k-1}^{l} = f(\boldsymbol{\xi}_{k-1|k-1}^{l}) \quad l = 0,1,2,\cdots,2L \tag{5}$$

$$\hat{\boldsymbol{x}}_{k|k-1} = \sum_{l=0}^{2L} \omega^{l} \boldsymbol{\xi}_{k|k-1}^{l} \tag{6}$$

$$\boldsymbol{P}_{k|k-1} = \sum_{l=0}^{2L} \omega^{l} (\boldsymbol{\xi}_{k|k-1}^{l} - \hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{\xi}_{k|k-1}^{l} - \hat{\boldsymbol{x}}_{k|k-1})^{\mathrm{T}} + \boldsymbol{\Gamma}_{k-1} \boldsymbol{\sigma}_{\boldsymbol{u}_k}^{2} (\boldsymbol{\Gamma}_{k-1})^{\mathrm{T}} \tag{7}$$

$\hat{\boldsymbol{x}}_{k|k-1}$ denotes the one-step state prediction, and $\boldsymbol{P}_{k|k-1}$ denotes the error covariance matrix of $\hat{\boldsymbol{x}}_{k|k-1}$. $\boldsymbol{\Gamma}_{k-1}$ and $\boldsymbol{\sigma}_{\boldsymbol{u}_k}^{2}$ denote the process noise matrix and the process noise variance, respectively.

3) Observation update

Combining with the construction of observation prediction $\boldsymbol{\zeta}_{k|k-1}^{l}$, one-step observation prediction $\hat{z}_{k|k-1}$, state estimation $\hat{\boldsymbol{x}}_{k|k}$ and estimation error covariance matrix $\boldsymbol{P}_{k|k}$ can be obtained.

$$\boldsymbol{\zeta}_{k|k-1}^{l} = h(\boldsymbol{\xi}_{k|k-1}^{l}) \tag{8}$$

$$\hat{z}_{k|k-1} = \sum_{l=0}^{2L} \omega^{l} \boldsymbol{\zeta}_{k|k-1}^{l} \tag{9}$$

$$\boldsymbol{P}_{xz} = \sum_{l=0}^{2L} \omega^{l} (\boldsymbol{\xi}_{k|k-1}^{l} - \hat{z}_{k|k-1})(\boldsymbol{\zeta}_{k|k-1}^{l} - \hat{z}_{k|k-1})^{\mathrm{T}} \tag{10}$$

$$\boldsymbol{P}_{zz} = \sum_{l=0}^{2L} \omega^{l} (\boldsymbol{\zeta}_{k|k-1}^{l} - \hat{z}_{k|k-1})(\boldsymbol{\zeta}_{k|k-1}^{l} - \hat{z}_{k|k-1})^{\mathrm{T}} + \boldsymbol{\sigma}_{\boldsymbol{v}_k}^{2} \tag{11}$$

$$\boldsymbol{K}_{k} = \boldsymbol{P}_{xz} (\boldsymbol{P}_{zz})^{-1} \tag{12}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_{k} (z_{k} - \hat{z}_{k|k-1}) \tag{13}$$

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_{k} \boldsymbol{P}_{zz} (\boldsymbol{K}_{k})^{\mathrm{T}} \tag{14}$$

where $h(\cdot)$ and $\boldsymbol{\sigma}_{\boldsymbol{v}_k}^{2}$ denote the observation function and the observation noise variance, respectively. $\boldsymbol{P}_{zz}$ and $\boldsymbol{P}_{xz}$ denote the observation error covariance matrix and the interactive error covariance matrix between state and observation, respectively. $\boldsymbol{K}_{k}$ denotes the gain matrix.

1.2 CDKF in observation bootstrapping strategy

According to the multi-source information fusion theory, the uncertainty of observation information can be weakened by the rational utilization of multi-sensor observation information. However, if multiple sensors are adopted, hardware cost of observation system will be increased inevitably. In addition, the selection of sensor accuracy, the position configuration and the random fault of sensor need to also be taken into account. Referencing the data assimilation method in EnKF, the bootstrapping observation set is structured by using the latest observation and the statistical information of observation noise. On this basis, combining with the weighted fusion approach, the central difference Kalman filter based on observation bootstrapping strategy (CDKF-OBS) is proposed. Let $\tilde{z}_{k}^{n}$ is the nth bootstrapping observation, the sampling mechanism of $\tilde{z}_{k}^{n}$ is expressed by

$$\tilde{z}_{k}^{n} = z_{k} + \boldsymbol{v}_{k}^{n} = h(\boldsymbol{x}_{k}) + \boldsymbol{v}_{k} + \boldsymbol{v}_{k}^{n} \quad n = 1,2,\cdots,N \tag{15}$$

It is known that $\tilde{z}_{k}^{n}$ only relies on the existing information including z_{k} and $\boldsymbol{\sigma}_{\boldsymbol{v}_k}^{2}$. $\boldsymbol{v}_{k}^{n}$ and $\boldsymbol{v}_{k}$ denote zero-mean Gaussian noise, and $Cov[\boldsymbol{v}_{k}^{n}, \boldsymbol{v}_{k}] = 0$, $Cov[\boldsymbol{v}_{k}^{n}, \boldsymbol{v}_{k}^{\eta}] = \begin{cases} \boldsymbol{\sigma}_{\boldsymbol{v}_k}^{2} & n = \eta \\ 0 & n \neq \eta \end{cases}$, $\eta = 1,2,\cdots,N$. N denotes the number of bootstrapping observations. According to the properties of the Gaussian distribution, $\boldsymbol{v}_{k} + \boldsymbol{v}_{k}^{n}$ is subject to the zero-mean observation noise with covariance $2\boldsymbol{\sigma}_{\boldsymbol{v}_k}^{2}$. In order to represent all observation information in observation bootstrapping strategy, let $\boldsymbol{\Theta}_{k}^{n}$ denote the observation set consisting of the real observation and all bootstrapping observations.

$$\boldsymbol{\Theta}_{k}^{n} \triangleq \{z_{k}, \tilde{z}_{k}^{1}, \cdots, \tilde{z}_{k}^{n}, \cdots, \tilde{z}_{k}^{N}\} \tag{16}$$

In order to unify the element expression in $\boldsymbol{\Theta}_{k}^{n}$, let $\tilde{z}_{k}^{0} = z_{k}$. So $\boldsymbol{\Theta}_{k}^{n}$ is rewritten as

$$\boldsymbol{\Theta}_{k}^{n} \triangleq \{\tilde{z}_{k}^{0}, \tilde{z}_{k}^{1}, \cdots, \tilde{z}_{k}^{n}, \cdots, \tilde{z}_{k}^{N}\} \tag{17}$$

According to the sampling mechanism of bootstrapping observation, the variance of $\tilde{z}_{k}^{n}$ is twice as large as the variance of z_{k}. Thus, instead of Eq. (11), observation error covariance matrix $\tilde{\boldsymbol{P}}_{zz}^{n}$ is calculated as

$$\tilde{\boldsymbol{P}}_{zz}^{n} = \begin{cases} \sum_{l=0}^{2L} \omega^{l} (\boldsymbol{\zeta}_{k|k-1}^{l} - \hat{z}_{k|k-1})(\boldsymbol{\zeta}_{k|k-1}^{l} - \hat{z}_{k|k-1})^{\mathrm{T}} + \boldsymbol{\sigma}_{\boldsymbol{v}_k}^{2} & n = 0 \\ \sum_{l=0}^{2L} \omega^{l} (\boldsymbol{\zeta}_{k|k-1}^{l} - \hat{z}_{k|k-1})(\boldsymbol{\zeta}_{k|k-1}^{l} - \hat{z}_{k|k-1})^{\mathrm{T}} + (2\boldsymbol{\sigma}_{\boldsymbol{v}_k})^{2} & n = 1,2,\cdots,N \end{cases} \tag{18}$$

The design idea of CDKF-OBS is to replace z_{k} used in CDKF with $\boldsymbol{\Theta}_{k}^{n}$. And then combining with the realization step of standard CDKF, gain matrix $\boldsymbol{K}_{k}^{n}$, state estimation $\hat{\boldsymbol{x}}_{k|k}^{n}$ and estimation error covariance matrix $\boldsymbol{P}_{k|k}^{n}$ are calculated on the basis of $\boldsymbol{\Theta}_{k}^{n}$, respectively.

$$\hat{\boldsymbol{x}}_{k|k}^{n} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_{k}^{n} (\boldsymbol{\Theta}_{k}^{n} - \hat{z}_{k|k-1}) \tag{19}$$

$$\boldsymbol{P}_{k|k}^{n} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_{k}^{n} \boldsymbol{P}_{zz} (\boldsymbol{K}_{k}^{n})^{\mathrm{T}} \tag{20}$$

$$\boldsymbol{K}_{k}^{n} = \boldsymbol{P}_{xz} (\boldsymbol{P}_{zz}^{n})^{-1} \tag{21}$$

Finally, using the way of weight distribution given in Eq. (22), $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$ are solved by

$$\boldsymbol{\varpi}_{k}^{n} = [\sum_{n=0}^{N} (\boldsymbol{P}_{k|k}^{n})^{-1}]^{-1} (\boldsymbol{P}_{k|k}^{n})^{-1} \tag{22}$$

$$\hat{\boldsymbol{x}}_{k|k} = \sum_{n=0}^{N} \boldsymbol{\varpi}_{k}^{n} \hat{\boldsymbol{x}}_{k|k}^{n} \tag{23}$$

$$\boldsymbol{P}_{k|k} = [\sum_{n=0}^{N} (\boldsymbol{P}_{k|k}^{n})^{-1}]^{-1} \tag{24}$$

$\boldsymbol{\varpi}_{k}^{n}$ denotes the weight coefficient in fusion process of $\hat{\boldsymbol{x}}_{k|k}^{n}$.

2 Maneuvering target tracking algorithm based on central difference Kalman filter in observation bootstrapping strategy

2.1 Interacting multiple model

The key of IMM is that multiple models working in parallel are respectively used to match different modes of maneuvering target. Among models, they are transferred according to probability matrix. Based on cutting and merging the hypothesis of each model, the estimation of multiple parallel filters is synthesized. IMM overcomes the influence of error caused by the mismatch between motion state and model when single model is used to describe the estimated system. Considering the following multi-model system with model switching characteristics

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \gamma_k, \boldsymbol{u}_{k-1}) \tag{25}$$

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k, \boldsymbol{v}_k) \tag{26}$$

$$\gamma_k \sim p(\gamma_k \mid \gamma_{k-1}) \tag{27}$$

where $\boldsymbol{u}_{k-1}$ and $\boldsymbol{v}_k$ denote the process noise and the observation noise, which meet zero-mean Gaussian distribution with variance $\boldsymbol{\sigma}^2_{\boldsymbol{u}_k}$ and $\boldsymbol{\sigma}^2_{\boldsymbol{v}_k}$, respectively. γ_k denotes the target motion model at time k, and it is subject to first-order Markov chain with the characteristic of discrete time and finite state. $\mu_0^i = P_\gamma\{\gamma_0 = i\}$ denotes the initial probability distribution. $\pi_{ij} = P_\gamma\{\gamma_{k-1} = j \mid \gamma_k = i\}$ and $\boldsymbol{\Pi} = [\boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \cdots, \boldsymbol{\pi}_J]^{\mathrm{T}}$ denote the state transition probability of priori model and the transition probability matrix, respectively, and $\boldsymbol{\pi}_i = [\pi_{i1}, \pi_{i2}, \cdots, \pi_{ij}]$, $\sum_{j=1}^{d} \pi_{ij} = 1$, $i, j, d \in J$. J denotes the number of elements in model set. IMM adopts the recursive form. The recursive process at each time mainly includes four steps: input interaction, model filtering, model probability updating and output interaction. In the input interaction stage, the prediction probability of each model $\bar{\mu}^i_{k-1}$ and the model mixed probability $\mu^{ij}_{k-1|k-1}$ are firstly calculated, and on this basis the state mixed estimation $\bar{\boldsymbol{x}}^i_{k-1|k-1}$ and the mixed estimation error covariance matrix $\bar{\boldsymbol{P}}^i_{k-1|k-1}$ can be solved. In the model filtering stage, by selecting suitable model filters, the state estimation $\hat{\boldsymbol{x}}^i_{k|k}$ and the estimation error covariance matrix $\boldsymbol{P}^i_{k|k}$ are solved in parallel filtering mechanism for each model. In the model probability updating stage, it needs to calculate the model likelihood l^i_k and the model updating probability μ^i_k, respectively. In the output interaction stage, combining with μ^i_k, $\hat{\boldsymbol{x}}^i_{k|k}$ and $\boldsymbol{P}^i_{k|k}$ obtained above three stages, $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$ can be obtained by the following expressions.

$$\hat{\boldsymbol{x}}_{k|k} = \sum_{i=1}^{J} \hat{\boldsymbol{x}}^i_{k|k} \mu^i_k, \ i = 1, 2, \cdots, J \tag{28}$$

$$\boldsymbol{P}_{k|k} = \sum_{i=1}^{J} [\boldsymbol{P}^i_{k|k} + (\hat{\boldsymbol{x}}_{k|k} - \hat{\boldsymbol{x}}^i_{k|k})(\hat{\boldsymbol{x}}_{k|k} - \hat{\boldsymbol{x}}^i_{k|k})^{\mathrm{T}}] \mu^i_k \tag{29}$$

2.2 Interacting multiple model based on cubature Kalman filter with observation iterated update

Considering that CDKF-OBS has high estimation precision, CDKF-OBS is selected as the model filter in IMM. The objective is to improve the overall performance of IMM by promoting the state estimation of each pattern. On the basis of that, the IMM algorithm based on CDKF in observation bootstrapping strategy (IMMCDKF- OBS) is proposed. In order to facilitate understanding the concrete implementation of IMMCDKF-OBS, the form of pseudo code is given.

Initialization: $\hat{\boldsymbol{x}}^i_{0|0} = \boldsymbol{x}_0, \boldsymbol{P}^i_{0|0} = \boldsymbol{P}_0, \boldsymbol{\pi}_{ij} = \boldsymbol{\pi}_0, \mu^i_0 = \mu_0$

1) Input interaction

$\bar{\mu}^i_{k-1}$, $\mu^{ij}_{k-1|k-1}$, $\bar{\boldsymbol{x}}^i_{k-1|k-1}$ and $\bar{\boldsymbol{P}}^i_{k-1|k-1}$ are calculated by

$$\bar{\mu}^i_{k-1} = \sum_{i=1}^{J} \pi_{ij} \mu^j_{k-1}$$

$$\mu^{ij}_{k-1|k-1} = \pi_{ij} \mu^j_{k-1} / \bar{\mu}^i_{k-1}$$

$$\bar{\boldsymbol{x}}^j_{k-1|k-1} = \sum_{i=1}^{J} \hat{\boldsymbol{x}}^i_{k-1|k-1} \mu^{ij}_{k-1|k-1}$$

$$\bar{\boldsymbol{P}}^j_{k-1|k-1} = \sum_{i=1}^{J} [\boldsymbol{P}^i_{k-1|k-1} + (\hat{\boldsymbol{x}}^i_{k-1|k-1} - \bar{\boldsymbol{x}}^j_{k-1|k-1}) (\hat{\boldsymbol{x}}^i_{k-1|k-1} - \bar{\boldsymbol{x}}^j_{k-1|k-1})^{\mathrm{T}}] \mu^{ij}_{k-1|k-1}$$

μ^j_{k-1} denotes the model probability of model j at time $k-1$, and π_{ij} denotes the transition probability from model i to model j.

2) Model filtering

Taking $\bar{\boldsymbol{x}}^i_{k-1|k-1}$ and $\bar{\boldsymbol{P}}^i_{k-1|k-1}$ as $\hat{\boldsymbol{x}}_{k-1|k-1}$ and $\boldsymbol{P}_{k-1|k-1}$ in Eq. (1), calculate $\hat{\boldsymbol{x}}_{k|k-1}$ and $\boldsymbol{P}_{k|k-1}$ can be calculated in accordance with Eq. (2) to Eq. (7). Combining with the bootstrapping observation, $\hat{\boldsymbol{x}}^{n,i}_{k|k}$ and $\boldsymbol{P}^{n,i}_{k|k}$ on the basis of $\boldsymbol{\Theta}^n_k$, model i can be solved by Eq. (8) to Eq. (10) and Eq. (18) to Eq. (21). Then, according to Eq. (22) to Eq. (24), $\hat{\boldsymbol{x}}^i_{k|k}$ and $\boldsymbol{P}^i_{k|k}$ can be obtained for each model.

3) Model probability updating

l^i_k is firstly calculated by following equations.

$$l^{n,i}_k = |(2\pi)\boldsymbol{P}^{n,i}_{zz}|^{-\frac{1}{2}} \exp\{-[(\boldsymbol{\Theta}^{n,i}_k - \hat{\boldsymbol{z}}^i_{k|k-1})^{\mathrm{T}} (\boldsymbol{P}^{n,i}_{zz})^{-1} (\boldsymbol{\Theta}^{n,i}_k - \hat{\boldsymbol{z}}^i_{k|k-1})]/2\}$$

$$l^i_k = \sum_{n=1}^{N} {}^{n,i}_{k} l^{n,i}_k$$

And then model probability μ^i_k after updating is expressed as

$$\mu^i_k = \bar{\mu}^i_{k-1} l^i_k / \sum_{j=1}^{J} (\bar{\mu}^j_{k-1} l^j_k)$$

4) Output interaction

Combining with Eq. (28) and Eq. (29), $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$ can be calculated.

5) Let $k = k + 1$, return to step 1).

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3 Simulation result and analysis

To verify the feasibility and availability of the proposed algorithm, the simulation scenario is set as the maneuvering target tracking by using the observations of two-coordinate radar. The sampling interval τ is 1s and the sampling steps are 35. The number of Monte Carlo simulation is 100. The experiment platform adopts PC, Pentium4 (CPU), 3.26GHz dominant frequency, 2G memory, Windows 7, and the programming language is Matlab2012b. The mode of target motion in radar scanning area is as follows. The estimated target move in uniform circular mode in the first 10 sampling periods, and its turning angular velocity is +0.3rad/s. In the sampling periods from 10 to 25 and from 11 to 35, its turning angular velocities are −0.15rad/s and +0.3rad/s, respectively, where "+" and "−" denote that the estimated target move the direction of anticlockwise and clockwise, respectively. Combining with the dynamic characteristics of maneuvering target motion and the physical characteristic of radar sensors, the system state equation and the observation equation of estimated target are as follows.

$$\boldsymbol{x}_k = \begin{cases} \boldsymbol{F}_1\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}\boldsymbol{u}_{1,k-1} & 1 \leqslant k \leqslant 10 \\ \boldsymbol{F}_2\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}\boldsymbol{u}_{2,k-1} & 11 \leqslant k \leqslant 25 \\ \boldsymbol{F}_1\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}\boldsymbol{u}_{1,k-1} & 26 \leqslant k \leqslant 35 \end{cases}$$

$$\boldsymbol{z}_k = \begin{bmatrix} \boldsymbol{\gamma}_k \\ \boldsymbol{\theta}_k \end{bmatrix} = \begin{bmatrix} \sqrt{\boldsymbol{x}_k^2 + \boldsymbol{y}_k^2} \\ \arctan \dfrac{\boldsymbol{y}_k}{\boldsymbol{x}_k} \end{bmatrix} + \begin{bmatrix} \boldsymbol{v}_{\gamma_k} \\ \boldsymbol{v}_{\theta_k} \end{bmatrix}$$

where $\boldsymbol{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^{\mathrm{T}}$ is the system state vector, $[x_k, y_k]^{\mathrm{T}}$ and $[\dot{x}_k, \dot{y}_k]^{\mathrm{T}}$ are position component and velocity component of $\boldsymbol{x}_k$, respectively. $\boldsymbol{F}_m = \begin{bmatrix} 1 & \sin(w_m\tau)/w_m & 0 & (1-\cos(w_m\tau))/w_m \\ 0 & \cos(w_m\tau) & 0 & -\sin(w_m\tau) \\ 0 & (1-\cos(w_m\tau))/w_m & 1 & \sin(w_m\tau)/w_m \\ 0 & \sin(w_m\tau) & 0 & \cos(w_m\tau) \end{bmatrix}$ is the system state transition matrix, and $m = 1,2$. $w_1 = +0.3$rad/s and $w_2 = -0.15$rad/s denote the turning angular velocity in $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, respectively. The transition probability matrix $\boldsymbol{\Pi}$ of models is $\begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$. The system noise $\boldsymbol{u}_{1,k}$ and $\boldsymbol{u}_{2,k}$ are subject to zero-mean Gaussian distribution with the variances $\boldsymbol{\sigma}_{\boldsymbol{u}_{1,k}}^2$ and $\boldsymbol{\sigma}_{\boldsymbol{u}_{2,k}}^2$. $\boldsymbol{\sigma}_{\boldsymbol{u}_{1,k}}^2$ and $\boldsymbol{\sigma}_{\boldsymbol{u}_{2,k}}^2$ are $0.2\boldsymbol{I}$ and $0.4\boldsymbol{I}$, respectively, and $\boldsymbol{I}$ denotes an unit matrix with suitable dimension. The observation noise $\boldsymbol{\sigma}_{v_k}^2$ is subject to zero-mean Gaussian distribution with the standard deviation $\begin{bmatrix} 0.1\text{km} & 0 \\ 0 & 0.1^\circ \end{bmatrix}$. $\boldsymbol{\Gamma} = \begin{bmatrix} 0 & 0 & \tau/2 & \tau \\ \tau/2 & \tau & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ is the system noise matrix. Number N of bootstrapping observations is 2. The initial state is $\boldsymbol{x}_0 = [8 \quad 0.4 \quad 6 \quad 0.2]^{\mathrm{T}}$. In this case, five algorithms including IMMUKF, IMMEKF, IMMCKF, IMMCDKF and IMMCDKF-OBS are compared in the simulations, among them, the top four types of algorithms use UKF, EKF, CKF and CDKF as the model filter in IMM.

Results from Fig. 1 to Fig. 5 show the model matching probability of five algorithms. In total, it is easy to see the model matching probability of IMMCDKF and IMMCDKF-OBS are superior to IMMUKF, IMMEKF and IMMCKF, furthermore, IMMCDKF-OBS is better than IMMCDKF. The reason is that the pros and cons of model filter selection directly effect the reliability of model identification in IMM. Because of introducing observation bootstrapping strategy in IMMCDKF-OBS, the performance of CDKF-OBS is superior to CDKF. When the feature is introduced into IMM, it reflects the improvement of real-time, precision and stability of models identification. Fig. 6 and Fig. 7 show the RMSE comparison of five algorithms. It is clear that the RMSE of IMMCDKF-OBS is less than other four algorithms, that is, the precision of IMMCDKF-OBS is the highest. From the figures one can also know that RMSE of IMMCDKF-OBS always keeps at low level and relatively stabilized. Table 1 quantitatively gives the mean of RMSE and the average time over 100 independent runs. It can be clearly found that the data of mean of RMSE describing algorithm filtering precision verifies the above analyzed results. The time cost is used to assess the computational complexity of these algorithms. The above results are conducive to reasonable selection of filters in practical engineering applications. It can be seen that the run time of IMMEKF is minimum, but its precision is also the lowest. The run time of IMMCDKF-OBS is slightly increased relative to IMMCDKF. However, its precision is certainly

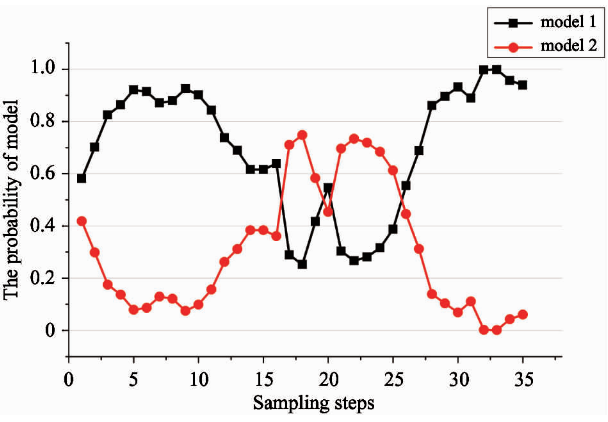

Fig. 1 IMMUKF

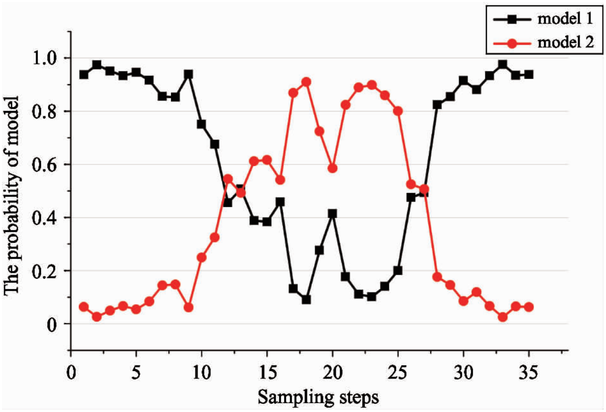

Fig. 2 IMMEKF

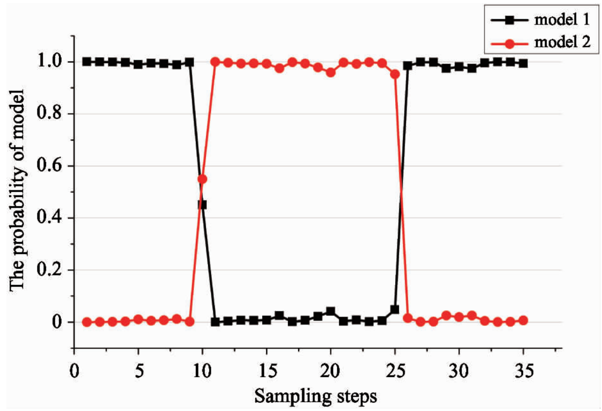

Fig. 5 IMMCDKF-OBS

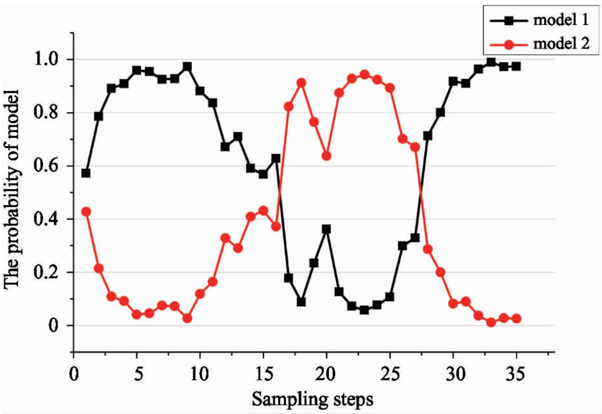

Fig. 3 IMMCKF

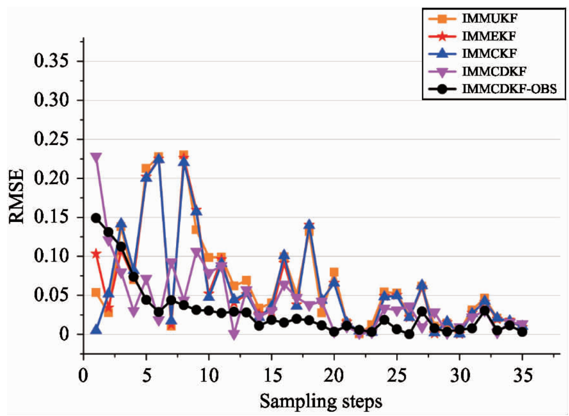

Fig. 6 Horizontal direction

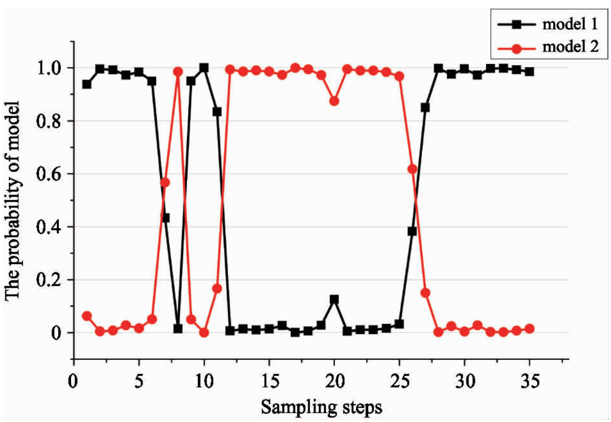

Fig. 4 IMMCDKF

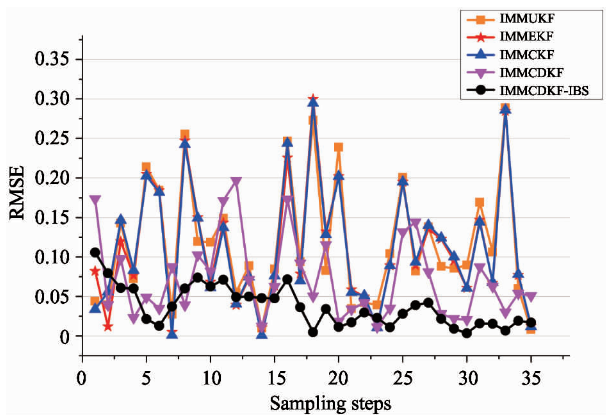

Fig. 7 Vertical direction

Table1 The comparison for the mean of RMSE and the average time over 100 independent runs

Algorithm	Horizontal direction (km)	Vertical direction (km)	Time cost (s)
IMMUKF	0.0648	0.1165	0.0096
IMMEKF	0.0615	0.1118	0.0025
IMMCKF	0.0605	0.1123	0.0138
IMMCDKF	0.0433	0.0721	0.0096
IMMCDKF-OBS	0.0289	0.0379	0.0352

superior to the other algorithms. According to the above results shown in this paper, the five types of maneuvering target tracking algorithms provide guidance significance in the practical engineering application.

The significance of above results gives the reasonable selection direction of model filter in the maneuvering target tracking system.

4 Conclusions

The method of interactive multiple model solves the model matching problem by sacrificing filtering precision, a maneuvering target tracking algorithm based on CDKF in observation bootstrapping strategy is proposed. In process of IMMCDKF-OBS, through the dynamic connection among observation bootstrapping strategy, central difference Kalman filter and interacting multiple model, the valid identification and estimation of the mode and state for maneuvering target tracking are realized. Compared with the existing processing method, the advantages of the new algorithm are as follows: Firstly, based on the method of interacting multiple model, the problem of state estimation in multi-model system is solved in the process of IMMCDKF-OBS. Secondly, observation bootstrapping strategy is used to simulate observation information of multiple sensors and the information will be extracted and utilized by weight fusion strategy. New algorithm improves filtering precision on condition that hardware cost of the system is of no growth.

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Hu Zhentao, born in 1979. He received his Ph. D degrees in Control Science and Engineering from Northwestern Polytechnical University in 2010. He also received his B. S. and M. S. degrees from Henan University in 2003 and 2006 respectively. Now, he is an assistant professor of college of computer and information engineering, Henan University. His research interests include complex system modeling and estimation, target tracking and particle filter, etc.