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Closed-form interference alignment with heterogeneous degrees of freedom①

Yang Xiumei (杨秀梅)②* **, Wang Jiang* **, Wang Rui**, Zhou Ting* **
(* Key Laboratory of Wireless Sensor Network & Communication, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, Shanghai 200050, P. R. China)
(** Shanghai Research Center for Wireless Communications, Shanghai 201210, P. R. China)

Abstract

Interference alignment (IA) has been recognized as a promising technique for obtaining the optimal degrees of freedom (DOF) in interference networks. A closed-form interference alignment design is proposed for three-cell uplink transmissions with heterogeneous DOFs. By exploiting the heterogeneity of DOFs from different transmission links, full or partial interference alignment precoders are calculated at each base station (BS). Aided by information exchange among BSs, the precoders can be finally obtained for all transmission links. Comparing to existing IA methods, the proposed scheme has a closed-form expression. Furthermore, there is no need to go through the iterative adaptation or jointly calculate the precoder and the equalizer. Simulation results show that the proposed design is able to achieve the optimal DOF performance with the advantage of perfect alignment capabilities.

Key words: interference alignment (IA), heterogeneous DOFs, precoder design

0 Introduction

Interference alignment (IA) has been developed as a promising technique for interference management in wireless networks. By aligning interference signals from different links into a subspace with reduced dimensions, the dimensions of the subspace for the desired signals can be enlarged so as to further improve network capacity[1]. Since IA is able to achieve new breakthroughs on the optimal DOF, a great deal of research activities have been inspired to investigate various aspects of IA from theoretical analysis to practical design[2,3].

Among various design approaches, heterogeneity of network topologies has been utilized to obtain precoders for the wireless transmissions. In Ref. [4], a hierarchical interference alignment scheme was proposed for heterogeneous networks by exploiting different numbers of transmit antennas of picocells and macrocells. In Ref. [5], characteristics of the partial connectivity for downlink multiple-input multiple output (MIMO) heterogeneous networks were investigated. The work in Ref. [6] used the heterogeneous path loss and spatial correlations to design the interference alignment precoders in MIMO interference networks. To further reduce the signaling overhead, work in Ref. [7] proposed a cognitive radio (CR) based interference alignment scheme by treating the macrocell and small cells as the primary system and secondary systems, respectively. Besides the precoder design in interference networks, recently, research work focusing on network performance analysis with co-channel interference modeling and spatial spectrum analysis was presented under random cellular networks[8,9], which provide new insights for the analysis of interference networks.

Differing from the aforementioned work, the heterogeneity of DOFs among different transmission links is exploited to design the IA precoders. The channel propagation for different links can own much disparity, resulting in different ranks of channel matrices[10]. Even for a full rank channel, the transmitter might only support a smaller number of independent data streams in order to effectively avoid crosslink interference. It also differs in transmission rates when further considering the traffic dynamics of interfering users, where each transmission link will support a different number of independent data streams in practical scenarios[11]. From the perspective of interference network capacity, the number of independent data streams or multiplexing

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② To whom correspondence should be addressed. E-mail: xiumei. yang@ wico. sh

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gain can be characterized by the degrees of freedom[1]. Therefore, the interference system also inherits heterogeneity property from the perspective of the DOF per link. This work, analyzes subspace relationships induced by the DOF's heterogeneity to present a closed-form precoder design method for two different DOF cases, which fully utilizes the DOF's heterogeneity either to reduce signaling overhead or achieve a higher sum rate. Unlike it often does with joint calculations between transmitter and receiver[4] or iterations[12,13], proposed precoders are obtained only by requiring calculations at each BS and limited information exchange among BSs.

The remainder of this paper is organized as follows. Section 1 introduces the system model for three-cell uplink transmission. Section 2 analyzes the relationship of the heterogeneous DOFs and highlights the closed-form IA precoder expressions. Section 3 provides the simulation results and analysis. And the final section concludes this work.

1 System model

A system model of the three-cell uplink MIMO system with heterogeneous DOFs is used in this work and it is assumed for the i^{th} transmission link that the i^{th} user (UE) is equipped with $N_t^{(i)}$ transmit antennas, the i^{th} BS has $N_r^{(i)}$ receive antennas, and the supported DOF is d_i. The received signal at the i^{th} receiver is expressed as

$$\boldsymbol{y}_i = \boldsymbol{H}_{ii}\boldsymbol{W}_i\boldsymbol{s}_i + \sum_{j=1,\, j\neq i}^{3} \boldsymbol{H}_{ij}\boldsymbol{W}_j\boldsymbol{s}_j + \boldsymbol{n}_i \tag{1}$$

where $\boldsymbol{H}_{ij} \in \mathbb{C}^{N_r^{(i)} \times N_t^{(j)}}$ denotes the channel matrix from the j^{th} UE to the i^{th} BS, whose entry is independent and identically distributed according to complex Gaussian distribution of $CN(0,1)$. For the i^{th} transmission link, $\boldsymbol{W}_i \in \mathbb{C}^{N_t^{(i)} \times d_i}$ is the normalized precoding matrix and $\boldsymbol{s}_i \in \mathbb{C}^{d_i \times 1}$ is the transmitted signal vector with independent elements. And $\boldsymbol{n} \in \mathbb{C}^{N_r^{(i)} \times 1}$ represents the additive white Gaussian noise vector observed at each receiver, with elements following $CN(0,\sigma^2)$. The achievable sum rate of the system can be expressed as

$$R(\rho) = \sum_{i=1}^{3} \sum_{k=1}^{d_i} \log_2(1 + \gamma_{ik}) \tag{2}$$

where ρ is the average signal-to-noise ratio (SNR) per link with $\rho = 1/\sigma^2$ under the above normalized parameter configurations. γ_{ik} is the post signal-to-noise-plus-interference ratio (SINR) of the k^{th} data stream at the i^{th} receiver. Therefore, the total DOF of the system, also known as the capacity per-log or the multiplexing gain[1], is

$$d \triangleq \lim_{\rho\to\infty} \frac{R(\rho)}{\log_2(\rho)} = \sum_{i=1}^{3} d_i \tag{3}$$

Since the focus is to exploit the heterogeneity of DOFs instead of the number of antennas, for simplicity, the number of transmit and receive antennas for each link are set to be the same, i. e., $N_t^{(i)} = N_r^{(i)} = N$. However, the number of the DOF supported by each transmission link will be different.

2 IA Precoder design

2.1 Heterogeneous DOFs

For heterogeneous DOFs among different transmission links, without loss of generality, it is assumed

$$d_3 \leqslant d_2 \leqslant d_1 < N \tag{4}$$

Thereafter the maximal number of dimensions available for interfering signals of three links are $N - d_1$, $N - d_2$ and $N - d_3$, respectively. When interference alignment is introduced into the above three-link system, it is claimed that the DOF per link should further satisfy constraint in the following proposition. The proof is straightforward and provided simply in the following.

Proposition 1 The DOFs of d_1 and d_2 in the three-link interference network, under the assumptions of $d_3 \leqslant d_2 \leqslant d_1 < N$, should further satisfy the following constraint if interference alignment is adopted,

$$d_1 + d_2 \leqslant N \tag{5}$$

Proof With interference alignment, the number of dimensions occupied by the aligned interference signals should be no larger than the number of dimensions for the null pace of the desired signal at each receiver[1,14]. For instance, at receiver #1, there is $\max\{d_2, d_3\} \leqslant (N - d_1)$. Therefore, the DOF set in a three-link transmission system should satisfy

$$\begin{cases} \max\{d_2, d_3\} \leqslant N - d_1 \\ \max\{d_1, d_3\} \leqslant N - d_2 \\ \max\{d_1, d_2\} \leqslant N - d_3 \end{cases} \tag{6}$$

Combining with Eq. (4), the following is

$$\begin{cases} d_2 \leqslant N - d_1 \\ d_1 \leqslant N - d_2 \\ d_1 \leqslant N - d_3 \end{cases} \tag{7}$$

Considering $d_3 \leqslant d_2$, the relationship of $d_1 + d_2 \leqslant N$ becomes a more strict constraint compared to $d_1 + d_3 \leqslant N$.

Motivated by Proposition 1, the achievable sum DOFs of the above interference network can be obtained. In Table 1, several parameter sets of the achievable DOFs under different antenna configurations are illustrated when equal mark being guaranteed in Eq. (5). The DOF sets are further divided into two different cases, with Case *B*. 1 satisfying $d_2 + d_3 \leqslant d_1$ and Case *B*. 2 satisfying $d_2 + d_3 > d_1$. Such two cases will result in different closed-form design procedures as well as the requirements for the channel state information (CSI).

Table 1 DOF parameters vs. antenna parameters

N	DOF set (d_1, d_2, d_3)	
	Case *B*. 1	Case *B*. 2
3	(2,1,1)	
4	(3,1,1)	(2,2,1),(2,2,2)
5	(4,1,1),(3,2,1)	(3,2,2)
6	(5,1,1),(4,2,2),(4,2,1)	(3,3,1),(3,3,2),(3,3,3)
7	(6,1,1),(5,2,1),(5,2,2),(4,3,1)	(4,3,2),(4,3,3)

2.2 Proposed design

In this subsection, a closed-form design of the IA precoders for the mentioned two different DOF cases is presented.

Case *B*. 1 $d_2 + d_3 \leqslant d_1$

In the DOF region of $\{d_1, d_2 d_3 : d_2 + d_3 \leqslant d_1\}$, partial or full precoders for all three transmission links can be firstly calculated based on the estimated CSI at each own receiver.

Stage 1 Firstly, the full interference signals from UE #3 and the partial interference signals from UE #2 with dimensions of d_3 are aligned at the receiver of link #1. That is,

$$Span(\boldsymbol{H}_{13}\boldsymbol{W}_3) = Span(\boldsymbol{H}_{12}\boldsymbol{W}_2^{d_3}) \tag{8}$$

where the superscript d_3 refers to arbitrary d_3 columns of matrix $\boldsymbol{W}_2$. Either $\boldsymbol{W}_3$ or $\boldsymbol{W}_2^{d_3}$ can be designed flexibly and the other one is obtained by taking $\boldsymbol{W}_3$ for instance,

$$\boldsymbol{W}_3 = \boldsymbol{H}_{13}^{-1}\boldsymbol{H}_{12}\boldsymbol{W}_2^{d_3} \tag{9}$$

here, $Span(\cdot)$ is the spanning set of the columns of a matrix. And $(\cdot)^{-1}$ denotes the inversion of a square matrix. From Eq. (8) to Eq. (9), the full precoder for UE #3 and partial precoder for UE #2 are obtained at BS #1. Then both precoders are respectively signaled to BS #2 and BS #3 in some manners, e. g., through signaling broadcasting or via dedicated channels.

Stage 2 Then partial dimensions of precoder $\boldsymbol{W}_1$ are calculated at both BS #2 and BS #3, respectively. By aligning $\boldsymbol{W}_1^{d_3}$ and $\boldsymbol{W}_3$ into the interfering subspace at BS #2,

$$Span(\boldsymbol{H}_{21}\boldsymbol{W}_1^{d_3}) = Span(\boldsymbol{H}_{23}\boldsymbol{W}_3) \tag{10}$$

$\boldsymbol{W}_1^{d_3}$ can be calculated by solving Eq. (10),

$$\boldsymbol{W}_1^{d_3} = \boldsymbol{H}_{21}^{-1}\boldsymbol{H}_{23}\boldsymbol{W}_3 \tag{11}$$

The interference links at BS #2 will just occupy d_1 dimensions of the matrix space so that the desired signal can be easily identified with the zero-forcing (ZF) detection since d_2 satisfies $d_2 \leqslant N - d_1$ according to Eq. (5). The rest part of $\boldsymbol{W}_2$ with $(d_2 - d_3)$ dimensions, which is not calculated can be flexibly designed, e. g., for better diversity gain.

Similarly, another part of $\boldsymbol{W}_1$ can be calculated in parallel at BS #3 following the same operations as those at BS #2,

$$\boldsymbol{W}_1^{d_2} = \boldsymbol{H}_{31}^{-1}\boldsymbol{H}_{32}\boldsymbol{W}_2 \tag{12}$$

After that, BS #2 and BS #3 signal the partial precoders $\boldsymbol{W}_1^{d_2}$ and $\boldsymbol{W}_1^{d_3}$ to BS #1, respectively.

Stage 3 Finally, BS #1 combines the received partial precoders and finally obtains full precoder $\boldsymbol{W}_1$. The remaining vectors with $(d_1 - d_2 - d_3)$ dimensions of $\boldsymbol{W}_1$ can be generated flexibly, similar to $\boldsymbol{W}_2$.

Remark 1 All three precoders can be obtained in closed forms after one stage of calculations at BS #1 and the parallel stage of calculations both at BS #2 and BS #3. Therefore the DOF's heterogeneity contributes to simplify the derivation of the IA precoders.

Remark 2 Only local CSI is needed to perform the proposed algorithm at each receiver. The local CSI consists of the channel state information from both the desired link and the interfering links arrived at the same receiver. The CSI from interfering links at the same receiver can be achieved by channel estimation under cooperative transmission systems. Without the need for global and full CSI sharing among all transmission links, the local CSI feature provides one of the most important advantages for the proposed design.

Case *B*. 2 $d_2 + d_3 > d_1$

When the heterogeneous DOFs fall in the region of $\{d_1, d_2 d_3 : d_2 + d_3 > d_1\}$, however, the design is a little complicated. This is because the flexibility remaining for $\boldsymbol{W}_1$ is reduced, and it is not able to be achieved by directly combining the partial vectors from the other two receivers as in Case B. 1. In such case, all three precoders should satisfy according to the dimension relationships of the subspaces in Eq. (4) and Eq. (5),

$$\begin{cases} Span(\boldsymbol{H}_{13}\boldsymbol{W}_3) \subset Span(\boldsymbol{H}_{12}\boldsymbol{W}_2) \\ Span(\boldsymbol{H}_{23}\boldsymbol{W}_3) \subset Span(\boldsymbol{H}_{21}\boldsymbol{W}_1) \\ Span(\boldsymbol{H}_{32}\boldsymbol{W}_2) \subset Span(\boldsymbol{H}_{31}\boldsymbol{W}_1) \end{cases} \tag{13}$$

According to (13), the following is got:

$$\begin{cases} \boldsymbol{W}_3 \subset Span(\boldsymbol{H}_{13}^{-1}\boldsymbol{H}_{12}\boldsymbol{W}_2) \\ Span(\boldsymbol{H}_{21}^{-1}\boldsymbol{H}_{23}\boldsymbol{W}_3) \subset Span(\boldsymbol{W}_1) \\ Span(\boldsymbol{H}_{31}^{-1}\boldsymbol{H}_{32}\boldsymbol{W}_2) \subset Span(\boldsymbol{W}_1) \end{cases} \tag{14}$$

and it further obtains

$$W_2 = H_{32}^{-1} H_{31} H_{21}^{-1} H_{23} H_{13}^{-1} H_{12} W_2 \quad (15)$$

It can be solved by the generalized eigen-problem as

$$W_2 = eig(H_{32}^{-1} H_{31} H_{21}^{-1} H_{23} H_{13}^{-1} H_{12}) \quad (16)$$

where $eig(\cdot)$ means the eigenvectors of a matrix, with the number of eigenvectors being d_2 for solutions. Furthermore, W_1 can be obtained as

$$W_1 = [H_{31}^{-1} H_{32} W_2 \quad v] \quad (17)$$

where $v \in \mathbb{C}^{N\times(d_1-d_2)}$ can be flexibly determined, like that in Case *B*. 1. As for W_3, it can be easily constructed from $H_{13}^{-1} H_{12} W_2$, e. g. , a submatrix with d_3 columns.

Remark 3 CSI sharing is necessary for the precoder calculations in Case *B*. 2. And higher sum rate performance can be expected than that in Case *B*. 1 due to the much stricter CSI requirement. The cooperative transmission improves the network capacity, which is also verified through the capacity analysis in Ref. [8].

Remark 4 In principle, either uplink or downlink transmissions can use the proposed designs. According to current standard specifications, however, it seems more suitable for uplink considering the requirement for information sharing among receivers. The information exchange between BSs has already been supported through X2 interface while it is somewhat challenging for UEs to exchange control information directly[15]. That is also the reason why the uplink transmission is selected as the system model in the above descriptions.

3 Performance results

The performance of the ergodic sum rate is presented under a three-cell uplink model via Monte Carlo simulations. Three aspects of performance verifications are considered for 1) Case B. 1; 2) Case B. 2 and 3) comparison with a typical time division multiple access (TDMA) MIMO transmissions. The sum DOF as defined by Eq. (3), which is the slope of the performance curve versus $\log_2(SNR)$, is also illustrated. To fully evaluate the performance of the proposed designs, the bit error rate (BER) performance is further provided for analysis.

In two IA cases, precoders are designed according to the proposed schemes while ZF detectors are used at receivers. Note that the precoding matrix is applied with normalization. In TDMA benchmarks, the antenna configurations are the same as those in IA schemes. However, the value of the DOF per link equals to the number of antennas and each link occupies one time slot for transmission while the other two links keep silent.

The sum rate performance for Case *B*. 1 is shown in Fig. 1. For illustration, the number of antennas is configured as $N = 5,6,7$. It can be seen that the sum rate as well as the sum DOF (d_{sum}) is increased with the increase of the number of antennas. And the interference signals are finely aligned and thus removed from the receiver. Without the need of sharing CSI among BSs, the signaling overhead is reduced in this case.

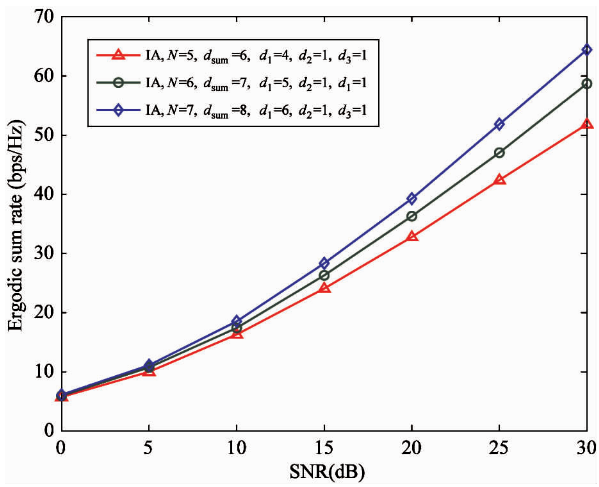

Fig. 1 Ergodic sum rate performance with the number of antennas $N \in \{5,6,7\}$ and the DOF set falling in the region of $\{d_1, d_2, d_3 : d_2 + d_3 \leqslant d_1\}$

In Fig. 2, performance of the sum rate under Case *B*. 2 is provided. Under the same antenna configurations, it can be observed that the sum DOF is a little higher than that in Fig. 1 due to the benefit of fully

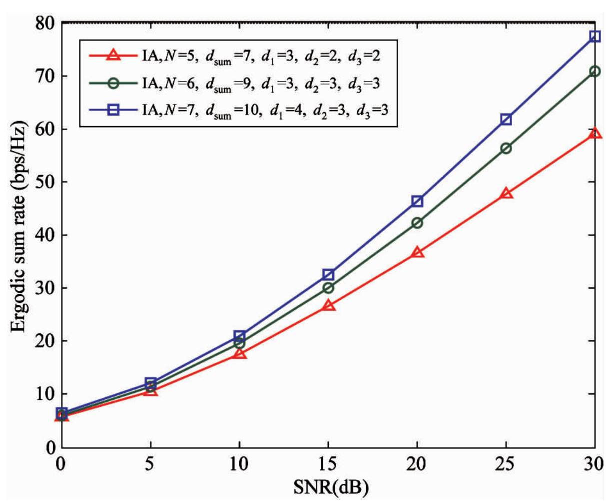

Fig. 2 Ergodic sum rate performance with the number of antennas $N \in \{5,6,7\}$ and the DOF set falling in the region of $\{d_1, d_2, d_3 : d_2 + d_3 > d_1\}$

shared CSI. For example, the sum DOF is 7 while it is 6 for the system in Case *B*. 1 with $N = 5$. With the increase of the number of antennas, the advantage from the perspective of the sum DOF becomes more obvious.

The proposed closed-form IA design also has superior sum DOF performance than TDMA schemes, as shown by Fig. 3. In this simulation, two MIMO schemes as TDMA benchmarks are performed. One is the spatial multiplexing TDMA scheme without precoding, while the other one adopts the eigen beam-forming based on singular value decomposition (SVD). Simulation results show that the proposed design is able to achieve higher sum DOF than both conventional MIMO schemes. It also achieves better sum rate performance compared to the TDMA scheme without precoding. However, in the low SNR region, the proposed design is inferior to the SVD scheme. This is due to the perfect match filtering for the precoding and receiving in the SVD scheme while the proposed scheme uses the precoder and the ZF detector, still remaining further optimization opportunities for better diversity gain.

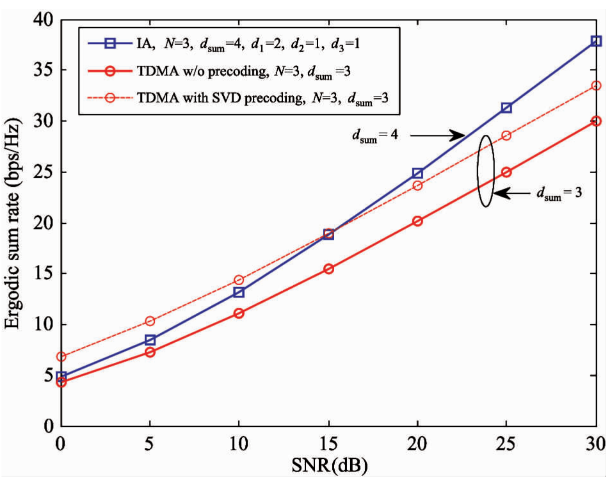

Fig. 3 Ergodic sum rate performance comparison between the proposed IA design and the TDMA benchmarks with and without precoding ($N = 3$)

To fully evaluate the performance of the IA schemes, the BER performance in Fig. 4 is further presented. In this simulation, the number of antennas is set at the transmitter and receiver as $N = 3, 5, 7$ and the corresponding DOFs are $d_{sum} = 4, 6, 8$ respectively. Binary phase shift keying (BPSK) is adopted as the modulation scheme and ZF detectors are used for interference cancellation at the receivers. From the simulation results, it shows that the interference signals from the co-channel links can be perfectly nulled and the BER performance is improved as SNR increases. Furthermore, for different antenna configurations, the BER performance slightly decreases as the number of antennas increases. However, the sum DOF is higher for cases with more antennas which is consistent with the results from aforementioned simulation results.

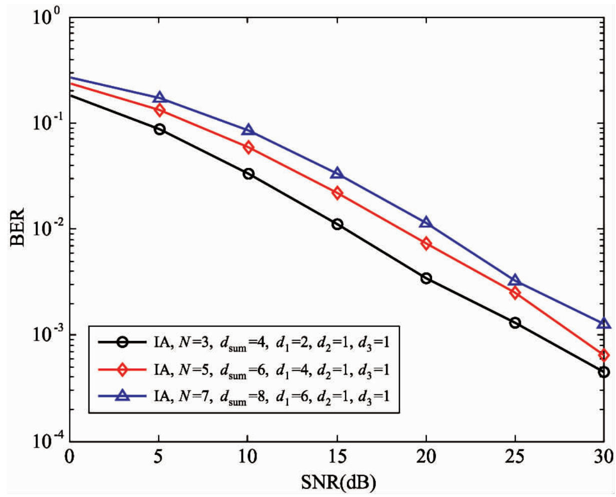

Fig. 4 Bit error rate performance of the proposed IA design

4 Conclusion

This work investigates the IA schemes with heterogeneous DOFs for a three-cell uplink transmission system. After analyzing the relationship of the subspace dimensions among the desired and interfering links, the closed-form IA design procedures are described in detail for two different DOF cases, respectively. The features of two specific design methods are further discussed. Based on the feature analysis, the proposed designs have the advantages that either local CSI requirements can reduce the signaling burden or the higher sum DOF contributes to the improvement of the sum rate performance. In essence, the proposed designs can be used in both uplink and downlink transmissions, and in this work the uplink transmission is taken as the implementation example. Although here the three-cell interference model is adopted which is more consistent to the current cellular topology, its extension to a general case with an arbitrary number of links will be more meaningful in theory but will be rather more complicated.

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Yang Xiumei, born in 1979. She received the Bachelor's and Master's degrees in electrical engineering from Shandong University in 2001 and 2004, respectively, and the Ph. D. degree in communications and information systems from the Shanghai Institute of Microsystem and Information Technology (SIMIT), Chinese Academy of Sciences (CAS) in 2011. Her research focuses on interference management, cooperative communications, and heterogeneous wireless networks.