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The rough representation and measurement of quotient structure in algebraic quotient space model[①]

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Abstract

Granular computing is a very hot research field in recent years. In our previous work an algebraic quotient space model was proposed, where the quotient structure could not be deduced if the granulation was based on an equivalence relation. In this paper, definitions were given and formulas of the lower quotient congruence and upper quotient congruence were calculated to roughly represent the quotient structure. Then the accuracy and roughness were defined to measure the quotient structure in quantification. Finally, a numerical example was given to demonstrate that the rough representation and measuring methods are efficient and applicable. The work has greatly enriched the algebraic quotient space model and granular computing theory.

Key words: granular computing, algebraic quotient space model, quotient structure, upper (lower) congruence relation

0 Introduction

Granular computing, which was first introduced in 1997 by Lin[1,2], is an emerging computing paradigm of information processing, and is now viewed as a superset of models including rough set, topology quotient space[3], fuzzy set[4], word theory, etc. Granular computing has been widely applied in image processing, data mining, complex problem solving, pattern recognition, intelligent control, artificial neural network, knowledge acquisition, and so on[5-8].

Being a widely applied structure in data coding, formal language and electronic circuit design, algebra is used broadly to describe the granule structure[9-10]. Based on the topology quotient space model (U,F,T) proposed in Ref. [11], and supposing the granule structure T as an algebraic operator $\circ$, in Refs [12,13] of our previous work, algebraic quotient space model $(U,F,\circ)$ was proposed.

In the rough set theory, it is granulated by equivalence relation R, and lower approximation $\underline{RX}$ and upper approximation $\overline{RX}$ are defined to approximately represent rough set $X \subseteq U$. But in algebraic quotient space model in Refs [12,13], only if granulation rule R is a congruence relation, one can get the quotient universe $[U]$, the quotient attribute $[F]$ and the quotient structure $[\circ]$ on granularity $(U,F,\circ)$, i. e., in the algebraic quotient space model, it is granulated by a congruence relation. But in the majority of granular computing models, taking the rough set and topological quotient space for example, the granulation rule is an equivalence relation.

In the algebraic quotient space theory, given equivalence relation R on granularity $(U,F,\circ)$, one can only get the quotient universe $[U]$ and quotient attribute $[F]$ according to Refs [3,11], but can not get the quotient structure $[\circ]$ by the conclusion of Refs [12,13]. Thus, is there a method of roughly representing the quotient structure? If yes, how to measure the new method?

In this paper, an equivalence relation R is given as a granulation rule in the algebraic quotient space model. Inspired by the lower approximation and upper approximation to approximately represent a rough set, a rough representation method of quotient structure is shown in algebraic quotient space model. Rough set theory and algebraic quotient space model in Refs [12,13] are simply introduced in Section 1. in Section 2, the lower quotient structure and upper quotient structure are defined to roughly represent the quotient structure. Section 3 gives the measurement of the

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above rough representation method. Section 4 presents a numeral example which shows that the rough representation and measuring methods are feasible and applicable.

1 Research basis

In this section, simple introductions are given to the rough set theory and the algebraic quotient space model in our previous work in Refs[12,13].

The rough set theory, first proposed by Polish scientist Pawlak[14,15] in 1982, is an effective mathematical tool for the characterization of incomplete and uncertain problems. The rough set is defined as follows.

Definition 1[14] Given equivalence relation R on approximate space (U,F) and subset $X \subseteq U$, the lower approximation $\underline{RX}$ and upper approximation $\overline{RX}$ are defined as

$$\underline{RX} = \cup \{Y \mid (\forall Y \in U/R) \wedge (Y \subseteq X)\} \quad (1)$$

$$\overline{RX} = \cup \{Y \mid (\forall Y \in U/R) \wedge (Y \cap X \neq \varphi)\} \quad (2)$$

Then if $\underline{RX} = \overline{RX}$, X is called an exact set, otherwise, X is called a rough set.

In Definition 1, $[U] = U/R$ is the quotient set of universe U by R, namely the quotient universe. If X equals to the intersection of a subset of U/R, i. e., target knowledge X can be exactly represented by U/R, then X is called an exact set. Otherwise, X is called a rough set, and by defining the lower approximation $\underline{RX}$ and upper approximation $\overline{RX}$, interval $(\underline{RX}, \overline{RX})$ is used to approximately represent target knowledge X.

The algebraic quotient space model $(U,F,\circ)$ in Refs [12,13] was proposed as follows.

Definition 2[12,13] Given congruence relation R on granularity $(U,F,\circ)$, where U is the universe, $F:U \rightarrow Y$ is the attribute function, and $\circ$ is the granule structure on U. It defines: the quotient universe $[U]$ as $p:U \rightarrow U/R$, the quotient attribute $[F]$ as $[F]:[U] \rightarrow 2^Y$, where

$$\forall x \in [U], [F](x) = F(p^{-1}(x)) = \{F(y) \mid y \in p^{-1}(x)\} \quad (3)$$

the quotient structure $[\circ]$ as

$$\forall x,y \in U, p(x \circ y) = p(x)[\circ]p(y) \quad (4)$$

then $([U],[F],[\circ])$ is defined as an algebraic quotient space of $(U,F,\circ)$.

In Definition 2, it gives the mapping functions of the quotient universe $[U]$, the quotient attribute $[F]$ and the quotient structure $[\circ]$. The quotient universe is a natural mapping, and the quotient attribute provides a general solution, because $[F]:[U] \rightarrow 2^Y$ must satisfy some optimization principle specified as a certain value such as a statistic number, the average, the maximum, the sum, the intersection or the union, etc.[13]. The quotient structure is a homomorphic mapping between the original structure and quotient structure.

In Definition 2, the sufficient condition of having the algebraic quotient space $([U],[F],[\circ])$ is that R is a congruence relation, which is proved in theorem in Refs [12,13]. From the view point of the algebraic theory, in order to keep the original structure $\circ$ and quotient structure $[\circ]$ being homomorphic, the granulation rule must be a congruence relation, i. e., only if congruence relation R is given on granularity $(U,F,\circ)$, one can get the quotient space $[\circ]$.

2 Rough representation of quotient structure

Given equivalence relation R on granularity $(U,F,\circ)$, by Definition 2 the quotient structure $[\circ]$ does not exist, and the key of getting an approximate quotient structure is to find an approximate congruence relation. Hence, the properties of congruence relation and equivalence relation are first discussed.

On universe U, by Refs [3,11] all the equivalence relations form a complete semi-order lattice, and by Refs [12,13] all the congruence relations form a complete semi-order lattice. Meanwhile, it is known that the congruence relation is a special case of the equivalence relation, i. e., a congruence relation must be an equivalence relation, but an equivalence relation may not be a congruence relation.

When an equivalence relation R is given on granularity $(U,F,\circ)$, it is the key to get the approximate congruence relation and approximate quotient structure. Inspired by the lower approximation and upper approximation to approximately represent a rough set, there are two kinds of approximate methods: (1) Trying to find a coarsest congruence relation $\underline{R}$ among all the congruence relations which are finer than R. (2) Trying to find a finest congruence relation $\overline{R}$ among all the congruence relations which are coarser than R. If there exists $\underline{R}$ and $\overline{R}$, then based on Definition 1 the quotient structure $\underline{[\circ]}$ and $\overline{[\circ]}$ can be deduced by $\underline{R}$ and $\overline{R}$. Therefore, $\underline{[\circ]}$ and $\overline{[\circ]}$ can be used to roughly represent the quotient structure.

On granularity $(U,F,\circ)$, let $\mathfrak{R}$ be all the equivalence relations, Ω be all the congruence relations, and $R \in \mathfrak{R}$. Then, under the containment order of equivalence relation, $(\mathfrak{R}, \subseteq)$ is a complete semi-order lattice, and $(\Omega, \subseteq)$ is a complete semi-order lattice.

The lattice Hasse graphic is shown in Fig. 1, where the hollow point is an equivalence relation, and the solid point is a congruence relation.

In the following, the $\underline{R}$, $\overline{R}$, $\underline{[\circ]}$, $\overline{[\circ]}$ is first defined, then the theorem are given to prove that they must exist and are unique.

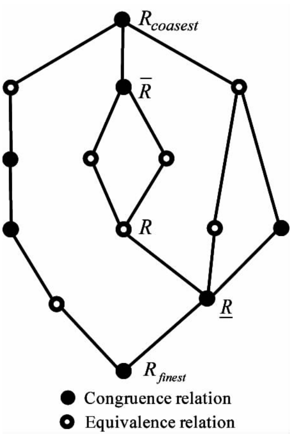

Fig. 1 The lattice Hasse graph of R and Ω

Definition 3 Let R be an equivalence relation on granularity $(U, F, \circ)$. If there exists a congruence relation $\underline{R} \subseteq R(\overline{R} \supseteq R)$, and for any congruence relation $R' \subseteq R(R' \supseteq R)$, there exists $\underline{R} \supseteq R'(\overline{R} \subseteq R')$, then $\underline{R}(\overline{R})$ is defined as the lower(upper) congruence relation of R, and by Definition 2 the quotient structure $\underline{[\circ]}(\overline{[\circ]})$ can be deduced, then $\underline{[\circ]}(\overline{[\circ]})$ is defined as the lower(upper) quotient structure of $\circ$.

Theorem 1 On granularity $(U, F, \circ)$, let R be an equivalence relation, and Ω be all the congruence relations. Then there must exist and uniquely exists the lower congruence relation $\underline{R}$ and the upper congruence relation $\overline{R}$, where

$$\underline{R} = \cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha \tag{5}$$

$$\overline{R} = t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta) \tag{6}$$

Proof: (1) It proves $\underline{R} = \cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$.

Firstly, $\forall (x,y) \in \cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$, so $\forall R_0 \in \{R_\alpha \mid R_\alpha \in \Omega, R_\alpha \subseteq R\}_\alpha$, $(x,y) \in R_0$, thus, $\forall z \in U$, $(x \circ z, y \circ z), (z \circ x, z \circ y) \in R_0 \in \cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$, therefore, $\cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$ is a congruence relation.

Secondly, it is proved that $\underline{R}$ is unique, i. e., $\cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$ is the greatest lower bound of $\{R_\alpha \mid R_\alpha \in \Omega, R_\alpha \subseteq R\}_\alpha$. $\forall R_0 \in \{R_\alpha \mid R_\alpha \in \Omega, R_\alpha \subseteq R\}_\alpha$, $\cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha \subseteq R_0$, so $\cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$ is one lower bound of $\{R_\alpha \mid R_\alpha \in \Omega, R_\alpha \subseteq R\}_\alpha$. Let R' be any lower bound of $\{R_\alpha \mid R_\alpha \in \Omega, R_\alpha \subseteq R\}_\alpha$, then $\forall R_0 \in \{R_\alpha \mid R_\alpha \in \Omega, R_\alpha \subseteq R\}_\alpha$, $R' \subseteq R_0$, thus $R' \subseteq \cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$, therefore, $\cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$ is the greatest lower bound of $\{R_\alpha \mid R_\alpha \in \Omega, R_\alpha \subseteq R\}_\alpha$.

Based on the above and Definition 3, $\underline{R} = \cap_{R_\alpha \in \Omega, R_\alpha \subseteq R} R_\alpha$.

(2) It is proved $\overline{R} = t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$.

Firstly, it is proved that $t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$ is a congruence relation. For $\forall (x,y) \in t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$, it has two cases: On the one hand, $(x,y) \in \cup_{R \subseteq R_\beta \in \Omega} R_\beta$. Then $\exists R_0 \in \{R_\beta \mid R \subseteq R_\beta \in \Omega\}$, where $(x,y) \in R_0$, so $\forall z \in U, (x \circ z, y \circ z), (z \circ x, z \circ y) \in R_0 \subseteq \cup_{R \subseteq R_\beta \in \Omega} R_\beta$. On the other hand, $(x,y) \notin \cup_{R \subseteq R_\beta \in \Omega} R_\beta$, then, $\exists (x = p_1), p_2, \cdots, (p_m = y)$, where $(p_i, p_{i+1}) \in R_0 \in \{R_\beta \mid R \subseteq R_\beta \in \Omega\}$, so $\forall z \in U$, $(p_i \circ z, p_{i+1} \circ z), (z \circ p_i, z \circ p_{i+1}) \in R_0 \subseteq t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$. Therefore, $t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$ is a congruence relation.

Secondly, it is proved $\overline{R}$ is unique, i. e., $t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$ is the least upper bound of $\{R_\beta \mid R \subseteq R_\beta \in \Omega\}$. $\forall R_0 \in \{R_\beta \mid R \subseteq R_\beta \in \Omega\}$, $R_0 \subseteq \cup_{R \subseteq R_\beta \in \Omega} R_\beta \subseteq t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$, so $t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$ is one upper bound of $\{R_\beta \mid R \subseteq R_\beta \in \Omega\}$. On the other hand, let R' be any upper bound of $t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$, thus $\forall R_0 \in \{R_\beta \mid R \subseteq R_\beta \in \Omega\}$, $R_0 \subseteq R'$, so $\cup_{R \subseteq R_\beta \in \Omega} R_\beta \subseteq R'$. R' is transitive, so $t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta) \subseteq t(R') = R'$. So $t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$ is the least upper bound of $\{R_\beta \mid R \subseteq R_\beta \in \Omega\}$.

Based on the above and Definition 3, $\overline{R} = t(\cup_{R \subseteq R_\beta \in \Omega} R_\beta)$.

In Theorem 1 it is proved that there exist the lower congruence relation $\underline{R}$ and the upper congruence relation $\overline{R}$, and by Definition 3 it is clear that they exist uniquely. In fact, the lower congruence relation $\underline{R}$ is the coarsest congruence relation among all the congruence relations which are finer than R, and the upper congruence relation $\overline{R}$ is the finest congruence relation among all the congruence relations which are coarser than R.

Having the lower congruence relation $\underline{R}$ and upper congruence relation $\overline{R}$ and by Definition 2 and Definition 3, the lower quotient structure $\underline{[\circ]}$ and upper quotient structure $\overline{[\circ]}$ can be deduced uniquely.

Thus, the lower quotient structure or upper quotient structure can be used to roughly represent the quotient structure which are not actually exist.

3 Rough measurement of quotient structure

In the front section, just the rough representing method of quotient structure is discussed, i. e., the concepts of lower congruence relation and upper congruence relation are defined and their calculating formulas are given. In the following, the rough measuring method of quotient structure is discussed in detail by defining the accuracy and roughness of quotient structure.

The lower quotient structure $\underline{[\circ]}$ is an algebraic operator on the lower quotient universe $\underline{[U]}$, and the upper quotient structure $\overline{[\circ]}$ is an algebraic operator on the upper quotient universe $\overline{[U]}$. It is not easy to directly measure the quality superiority of $\underline{[\circ]}$, $\overline{[\circ]}$, which can be discussed by $\underline{R}, \overline{R}, \underline{[U]}, \overline{[U]}$ indirectly.

In order to describe the quotient structure in quantification more exactly, accuracy $\alpha_R[\circ]$ and roughness $\rho_R[\circ]$ are defined as follows from the reverse side.

Definition 4 Let R be an equivalence relation on granularity $(U, F, \circ)$, $\underline{R}$ be the lower congruence relation and $\overline{R}$ be the upper congruence relation R of R. Then accuracy $\alpha_R[\circ]$ and roughness $\rho_R[\circ]$ of the quotient structure is defined as

$$\alpha_R[\circ] = \frac{|U/\overline{R}|}{|U/\underline{R}|} \tag{7}$$

$$\rho_R[\circ] = 1 - \alpha_R[\circ] \tag{8}$$

Clearly, accuracy $0 < \alpha_R[\circ] \leqslant 1$, and the larger the accuracy $\alpha_R[\circ]$ is, the smaller the roughness $\rho_R[\circ]$ is.

At the best, $\underline{R} = \overline{R}$, i. e., R is a congruence relation, then the accuracy is $\alpha_R[\circ] = 1$, and the roughness is $\rho_R[\circ] = 0$.

At the worst, the lower quotient structure is on the finest granularity, i. e., $|U/\underline{R}| = |U|$, and the upper quotient structure is on the coarsest granularity, i. e., $|U/\overline{R}| = |\{U\}| = 1$. Then the accuracy is $\alpha_R[\circ] = \frac{1}{|U|}$, and the roughness is $\rho_R[\circ] = 1 - \frac{1}{|U|}$.

4 Numerical example

In this section, a numerical example is demonstrated which shows that the rough representation and measuring methods above are feasible and applicable.

Supposing R is an equivalence relation on granularity $(U, F, \circ)$, where $X = \{0,1,2,3,4,5,6,7\}$, $X/R = \{\{0,2\},\{1,5\},\{3,7\},\{4,6\}\}$ and the algebraic operation $\circ$ is shown in Table 1.

Seeing from Table 1, the binary algebraic operation $x \circ y$ means $(x \times y)$ mod8. It can be proved that R is only an equivalence relation but not a congruence relation, then by Definition 2 there does not exist the quotient structure.

Table 1 Algebraic operation $\circ$ on U

$\circ$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

By Theorem 1 the lower quotient congruence relation $\underline{R}$ can be got, and the lower quotient universe $\underline{[U]} = U/\underline{R} = \{\{0\},\{1,5\},\{2\},\{3,7\},\{4\},\{6\}\}$ are gained. By Eq. (4) of Definition 2 the lower quotient operation $\underline{[\circ]}$ is got shown in Table 2. If the isomorphic mapping is given as $\{0\} = 0$, $\{1,5\} = 1$, $\{2\} = 2$, $\{3,7\} = 3$, $\{4\} = 4$, $\{6\} = 5$, then the lower quotient operation $\underline{[\circ]}$ is as shown in Table 3.

Table 2 Algebraic operation $\underline{[\circ]}$ on $\underline{[U]}$

$\underline{[\circ]}$	$\{0\}$	$\{1,5\}$	$\{2\}$	$\{3,7\}$	$\{4\}$	$\{6\}$
$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
$\{1,5\}$	$\{0\}$	$\{1,5\}$	$\{2\}$	$\{3,7\}$	$\{4\}$	$\{6\}$
$\{2\}$	$\{0\}$	$\{2\}$	$\{4\}$	$\{6\}$	$\{0\}$	$\{4\}$
$\{3,7\}$	$\{0\}$	$\{3,7\}$	$\{6\}$	$\{1,5\}$	$\{4\}$	$\{2\}$
$\{4\}$	$\{0\}$	$\{4\}$	$\{0\}$	$\{4\}$	$\{0\}$	$\{0\}$
$\{6\}$	$\{0\}$	$\{6\}$	$\{4\}$	$\{2\}$	$\{0\}$	$\{2\}$

Table 3 On $\underline{[U]}$ after isomorphic mapping

$\underline{[\circ]}$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	5	0	4
3	0	3	5	1	4	2
4	0	4	0	4	0	0
5	0	5	4	2	0	2

By Theorem 1 the upper quotient congruence relation $\overline{X}$ is got, and the upper quotient universe $\overline{[U]} = U/\overline{X} = \{\{0,2,4,6\},\{1,5\},\{3,7\}\}$ is obtained. By Eq. (4) of Definition 2 the upper quotient operation $\overline{[\circ]}$ is got shown in Table 4. If the isomorphic mapping is given as $\{0,2,4,6\} = 0$, $\{1,5\} = 1$, $\{3,7\} = 2$, then the upper quotient operation $\overline{[\circ]}$ is as shown in Table 5, clearly it is a binary operation $(x \times y)$ mod 3.

Table 4 Algebraic operation $\overline{[\circ]}$ on $\overline{[U]}$

$\overline{[\circ]}$	$\{0,2,4,6\}$	$\{1,5\}$	$\{3,7\}$
$\{0,2,4,6\}$	$\{0,4\}$	$\{0,2,4,6\}$	$\{0,6,4,2\}$
$\{1,5\}$	$\{0,2,4,6\}$	$\{1,5\}$	$\{3,7\}$
$\{3,7\}$	$\{0,6,4,2\}$	$\{3,7\}$	$\{1,5\}$

Table 5 $\overline{[\circ]}$ on $\overline{[U]}$ after isomorphic mapping

$\overline{[\circ]}$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

According to Eq. (7) of Definition 4, the accuracy $\alpha_R[\circ]$ of quotient structure is

$$\alpha_R[\circ] = \frac{|U/\overline{R}|}{|U/\underline{R}|} = \frac{|\{\{0,2,4,6\},\{1,5\},\{3,7\}\}|}{|\{\{0\},\{1,5\},\{2\},\{3,7\},\{4\},\{6\}\}|} = \frac{3}{6},$$

so, $\alpha_R[\circ] = 50\%$, and the roughness $\rho_R[\circ]$ of quotient structure is

$$\rho_R[\circ] = 1 - \alpha_R[\circ] = 1 - 50\% = 50\%.$$

5 Conclusion

In Refs [12,13] of the previous work, it is granulated by a congruence relation in the algebraic quotient space model. In this paper, granulated by an equivalence relation, one can only get the quotient universe and quotient attribute but can't get the quotient structure according to Refs [3, 11] in the algebraic quotient space model.

Inspired by the definitions of lower approximation and upper approximation in the rough set theory, this paper gives the definitions and calculating formulas of the lower quotient congruence relation and upper quotient congruence relation, based on which one can easily get the lower quotient structure and upper quotient structure, which can be used to roughly represent the quotient structure.

Then the accuracy and roughness are defined to measure the quotient structure in quantification. Finally, a numerical example is given to demonstrate that the rough representation and measuring methods are efficient and applicable.

This work enriches the algebraic quotient space model and granular computing models greatly.

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