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An application of insurance mechanism in risk management for energy internet[①]

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Abstract

As a completely new residential distribution infrastructure, energy internet facilitates transactions of equipment, energy and services. However, there is security risk under all the facilities. This paper proposes an electricity pricing model based on insurance from the perspective of maximizing the benefits of Energy Internet service providers by using the principal-agent theory. The consumer prepays the provider insurance premiums and signs a contract. The provider sets electricity price according to the premiums and therefore provides differentiated electric services for the consumer. Loss suffered by the consumer due to the power failure is compensated by the provider according to the contract. The equivalent model is presented and a necessary condition of the optimal strategy is obtained on the basis of Pontryagin's maximum principle. At last, a numerical example is presented, which illustrates the effectiveness of the proposed model.

Key words: energy internet, insurance, risk management, optimal control, fuzzy theory

0 Introduction

Energy internet is based on power grids which utilize renewable energy technologies, smart-grid technologies and Internet technologies. It is a kind of energy interconnected and shared network, which fuses gas networks, hydrogen power networks and electrified transportation networks[1]. As a completely new residential distribution infrastructure, energy internet facilitates transactions of equipment, energy and services. The centralized computer mainframes give way to a distributed computing infrastructure which allows users to access via a worldwide Internet. However, there is security risk under all the facilities[2]. Energy internet's large unified power system will bring robustness problems. If a node or link faults, there may be a domino effect turning the whole network into paralysis[3]. Uncertainty of renewable energy generation and extreme weather such as rain and snow will cause enormous economic losses. The risk of energy internet is often accompanied by a series of risks.

On the other hand, user-centric is the key to the success of energy internet in business. This paradigm shift can only be achieved by encouraging and facilitating participation of individual residential customers with a future power distribution infrastructure[4]. Designing incentive mechanisms to encourage consumers to participate energy internet actively is needed[5].

Insurance companies have rich experience of serving energy enterprises. Insurance mechanism in risk management has made more and more extensive application of a number of research results[6]. Fumagalli, et al.[7,8] proposed an insurance scheme for reliability which provides economically efficient investment incentives and alleviates consumers' reliability risk. Xie and Wei[9] analyzed four possible application modes of reliability insurance and calculation models of insurance premium under each application mode put forward respectively. Wang, et al.[10] proposed different reliability levels of the insurance mechanism which allows power companies to buy electricity from wind power plants at different prices. Karimi, et al.[11] examined the use of financial tools to manage dispatchable distributed generation economic risks and proposed a comprehensive framework considering various economic

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risks to distributed generation owners. Haring, et al.[12] used insurance theory to evaluate the advantages of demand side elasticity and differentiation in power quality. Nitanan and Audomvongseree[13,14] proposed electricity insurance pricing for bilateral contract market and the pricing scheme is based on the fair game principle. However, among all the literature, they characterized loss of consumers as random variables. Since the consumer's loss is often fuzziness by uncertain factors and cannot be exactly estimated by the insurance company, it is rational to indicate the loss by a fuzzy variable.

In this paper, an electricity price model is proposed based on insurance from the perspective of maximizing the benefits of the Energy Internet service provider by using the principal-agent theory. The consumer prepays the provider insurance premiums and signs a contract; the provider sets electricity price according to the premiums and therefore provides differentiated electric services for the consumer; loss suffered from the consumer due to the power failure is compensated by the provider according to the contract. This method can achieve effective management of energy resources and optimize resource allocation.

Deductible insurance is used to cover consumers' loss, taking into account the adverse selection and moral hazard caused by asymmetric information[15-18] in the insurance field. Because energy internet is more open and interconnected than the smart grid[19], which leads to more uncertain factors threatening the safety of users and further leads to more losses, therefore fuzzy variables are used to represent subjective estimates of consumers' average loss and use the fuzzy theory to solve the[20] optimization problem with subjective uncertainty.

The following primary research contributions is made in this study.

- Insurance mechanism is introduced into the problem of the pricing of Energy Internet services and an electricity price model based on insurance from the perspective of maximizing the benefits of Energy Internet service provider by using the principal-agent theory.
- Presenting the equivalent model is necessary condition of the optimal strategy which is on the basis of Pontryagin's maximum principle.
- A numerical example, which illustrates the effectiveness of the proposed model.

The rest of this paper is organized as follows: Section 1 establishes an electricity pricing model based on insurance mechanism. Section 2 presents the equivalent model. Section 3 obtains the optimal solution for the equivalent model. Section 4 gives an numerical example to illustrate the effectiveness of the proposed model. finally, Section 5 summarizes main results of the paper.

1 An electricity pricing model based on insurance mechanism

In this section, an optimal electricity pricing problem is considered, in which there are two participants, the provider and the consumer, among them, the provider is the principal who provides electricity service with insurance to the consumer and the consumer is the provider's agent[21]. Thus, there exists a principal-agent relationship: the relationship between the provider and the consumer. The consumer needs to buy insurance from the provider and sign a contract. The provider makes a pricing of the electricity service according to the premium and provides differentiated electricity services to the consumer. Because the consumer should pay electric fees and premiums to the provider, the provider needs to design an electricity pricing mechanism and an insurance product pricing mechanism.

The consumer gets access to the information of his own loss, which is not available to the provider, before buying the electricity service. Due to lacking relevant data of the loss, the provider can only estimate the loss based on expertise. This subjective assessment is denoted by a fuzzy variable ξ. Loss ξ has a continuous membership function with support $[0, L]$, where L denotes the consumer's maximum loss. The provider needs to design an insurance contract under adverse selection because of lacking relevant data of the consumer's loss. Moreover, in order to stimulate the consumer to reveal the factual information of his own loss, the insurance contract should contain a deductible. Let $x \in [0, L]$ denote a certain amount of the consumer's loss. The deductible and the premium are denoted by $D(x)$ and $\pi(x)$ respectively. For the purpose of avoiding moral hazard, it is assumed that $\frac{\mathrm{d}D(x)}{\mathrm{d}x} \geqslant 0$. In other words, the deductible increases with the increase of the consumer's loss. This kind of insurance contract will penalize the consumer for a large loss.

The insurance payout is dependent on the deductible and the consumer's loss. The relationship between them may be not linear. Let $I(D(x), x)$ denote the payout. The electricity pricing is dependent on the premium and denoted by $P(\pi(x))$. The consumer's benefit from power utilization is denoted by $q(x)$.

Thus, given that the consumer's initial wealth is

$w_0(w_0 \geqslant L)$, the consumer's welfare is

$$W(\pi(x), D(x), P(\pi(x)), q(x), x) = w_0 - \pi(x) - P(\pi(x)) - x + I(D(x), x) + q(x) \quad \forall x \in [0, L] \quad (1)$$

Truthful announcement of loss implies that the incentive compatible constraint of the consumer must be satisfied. $W(P(\pi(y)), \pi(y), D(y), q(x), x)$ are used to indicate that the consumer's loss is x but he says it's y. The incentive compatible constraint of the consumer is that if for each $x \in [0, L]$, the consumer with loss x will choose $(\pi(x), D(x), P(\pi(x)))$ to maximize his own welfare, i. e.,

$$W(\pi(x), D(x), P(\pi(x)), q(x), x) \geqslant W(\pi(y), D(y), P(\pi(y)), q(x), x) \quad \forall x, y \in [0, L] \quad (2)$$

In addition, the contract must be individually rational so that the consumer wants to participate in the trade. u_0 is used to indicate the consumer's welfare without purchase. Then the individually rational constraint is

$$W(\pi(x), D(x), P(\pi(x)), q(x), x) \geqslant u_0 \quad \forall x \in [0, L] \quad (3)$$

$V(\pi(\xi), D(\xi)) = \pi(\xi) - I(D(\xi), \xi)$ are used to indicate the provider's benefit obtained from the insurance contract, where c is the social average cost. The provider's welfare is

$$B(V(\pi(x), D(x)), P(\pi(x))) = P(\pi(x)) - c + V(\pi(x), D(x)) \quad \forall x \in [0, L] \quad (4)$$

In the optimal electricity pricing problem, the provider aims to maximize the expected social welfare by designing an electricity price for the consumer under the incentive compatibility constraint and participation constraint. As a consequence, the electricity pricing model can be formulated as follows:

$$\begin{cases} \max\limits_{(\pi(\cdot), D(\cdot), P(\pi(\cdot)))} E[B(V(\pi(\xi), D(\xi)), P(\pi(\xi)))] \\ \text{subject to:} \\ W(\pi(x), D(x), P(\pi(x)), q(x), x) \geqslant \\ W(\pi(y), D(y), P(\pi(y)), q(x), x), \\ W(\pi(x), D(x), P(\pi(x)), q(x), x) \geqslant u_0 \end{cases} \quad \forall x, y \in [0, L] \quad (5)$$

2 Model analyzing

In this section, an equivalent model of Eq. (5) is considered.

Proposition 1 If $\frac{\partial^2 I(D, x)}{\partial D \partial x} \geqslant 0$, the incentive compatibility constraint (2) can be written as

$$\begin{cases} \frac{dP(\pi(x))}{d\pi}\frac{d\pi(x)}{dx} + \frac{d\pi(x)}{dx} = \frac{\partial I(D(x), x)}{\partial D}\frac{dD(x)}{dx} \\ \frac{dD(x)}{dx} \geqslant 0 \end{cases} \quad \forall x \in [0, L] \quad (6)$$

Proof By Eq. (1), the incentive compatibility constraint (2) can be written as

$$w_0 - P(\pi(x)) - \pi(x) - x + I(D(x), x) + p(x) \geqslant w_0 - x - P(\pi(y)) - \pi(y) + I(D(y), x) + p(x) \quad \forall x, y \in [0, L] \quad (7)$$

Let $J(x, y) = w_0 + I(D(y), x) - P(\pi(y)) - \pi(y) - x + p(x)$, for any given x, Inequality (7) can be written as

$$J(x, x) \geqslant J(x, y) \quad \forall x, y \in [0, L] \quad (8)$$

It means that $J(x, y)$ obtains its maximal value at $y = x$. Thus, $J(x, y)$ satisfies the first-order condition $\left.\frac{\partial J(x, y)}{\partial y}\right|_{y=x} = 0$ and the second-order condition $\left.\frac{\partial^2 J(x, y)}{\partial y^2}\right|_{y=x} < 0$. It follows from the first-order condition that

$$\frac{dP(\pi(x))}{d\pi} \cdot \frac{d\pi(x)}{dx} + \frac{d\pi(x)}{dx} = \frac{\partial I(D(x), x)}{\partial D} \cdot \frac{dD(x)}{dx} \quad (9)$$

and

$$\frac{\partial^2 I(D(x), x)}{\partial D^2} \cdot \left(\frac{dD(x)}{dx}\right)^2 + \frac{\partial I(D(x), x)}{\partial D} \cdot \frac{d^2 D(x)}{dx^2} < \frac{d^2 P(\pi(x))}{d\pi^2} \cdot \left(\frac{d\pi(x)}{dx}\right)^2 + \frac{dP(\pi(x))}{d\pi} \cdot \frac{d^2\pi(x)}{dx^2} + \frac{d^2\pi(x)}{dx^2} \quad (10)$$

By differentiating Eq. (9) with respect to x:

$$\frac{d^2 P(\pi(x))}{d\pi^2} \cdot \left(\frac{d\pi(x)}{dx}\right)^2 + \frac{dP(\pi(x))}{d\pi} \cdot \frac{d^2\pi(x)}{dx^2} + \frac{d^2\pi(x)}{dx^2} = \frac{\partial^2 I(D(x), x)}{\partial D^2} \cdot \left(\frac{dD(x)}{dx}\right)^2 + \frac{\partial^2 I(D(x), x)}{\partial D \partial x} \cdot \frac{dD(x)}{dx} + \frac{\partial I(D(x), x)}{\partial D} \cdot \frac{d^2 D(x)}{dx^2} \quad (11)$$

Applying Eq. (10) to Eq. (11) yields:

$$\frac{\partial^2 I(D(x), x)}{\partial D \partial x} \cdot \frac{dD(x)}{dx} > 0$$

Thus:

$$\begin{cases} \frac{dD(x)}{dx} \geqslant 0, \text{ if } \frac{\partial^2 I(D(x), x)}{\partial D \partial x} \geqslant 0 \\ \frac{dD(x)}{dx} \leqslant 0, \text{ if } \frac{\partial^2 I(D(x), x)}{\partial D \partial x} < 0 \end{cases}$$

Note that $\frac{dD(x)}{dx} \geqslant 0$, thus Eq. (2)⇒Eq. (6).

On the other hand, by $\frac{dD(x)}{dx} \geqslant 0$ and

$\frac{\partial^2 I(D(x), x)}{\partial D \partial x} \geqslant 0$, when $y > x$:

$$P(\pi(x)) - P(\pi(y)) + \pi(x) - \pi(y)$$
$$= -\int_x^y \frac{\partial I(D(s), s)}{\partial D} \cdot \frac{dD(s)}{ds} ds$$
$$\leqslant -\int_x^y \frac{\partial I(D(s), x)}{\partial D} \cdot \frac{dD(s)}{ds} ds$$
$$= I(D(x), x) - I(D(y), x)$$

and when $y < x$:

$$P(\pi(x)) - P(\pi(y)) + \pi(x) - \pi(y)$$
$$= -\int_x^y \frac{\partial I(D(s), s)}{\partial D} \cdot \frac{dD(s)}{ds} ds$$
$$\leqslant -\int_x^y \frac{\partial I(D(s), x)}{\partial D} \cdot \frac{dD(s)}{ds} ds$$
$$= I(D(x), x) - I(D(y), x)$$

Therefore, when $\frac{\partial^2 I(D, x)}{\partial D \partial x} \geqslant 0$, Eq. (6) is satisfied and incentive constraint (2) is satisfied.

Therefore the proof of Proposition 1 is completed.

Proposition 2 Participation constraint (3) can be written as

$$\pi(L) = w_0 - P(\pi(L)) - L + I(D(L), L) + q(L) - u_0 \quad (12)$$

Proof By Eq. (1), hence

$$\frac{dW(\pi(x), D(x), P(\pi(x)), q(x), x)}{dx} = -1 + \frac{dq(x)}{dx} - \frac{d\pi(x)}{dx} + \frac{\partial I(D(x), x)}{\partial D} \cdot \frac{dD(x)}{dx} - \frac{dP(\pi(x))}{d\pi} \cdot \frac{d\pi(x)}{dx}$$

and applying Eq. (9) yields:

$$\frac{dW(\pi(x), D(x), P(\pi(x)), q(x), x)}{dx} = \frac{dp(x)}{dx} - 1$$

Assuming that $\frac{dp(x)}{dx} < 1$, thus:

$$\frac{dW(\pi(x), D(x), P(\pi(x)), q(x), x)}{dx} < 0$$

Which means that $W(\pi(x), D(x), P(\pi(x)), q(x), x)$ is decreasing with respect to x. Thus, participation constraint (3) is equivalent to:

$$W(\pi(L), D(L), P(\pi(L)), q(L), L) = w_0 - \pi(L) - L - P(\pi(L)) + I(D(L), L) + q(L) \leqslant u_0$$

It is easy to testify that the optimal insurance contract satisfies

$$w_0 - \pi(L) - L - P(\pi(L)) + I(D(L), L) + q(L) = u_0$$

Otherwise, let:

$$\begin{cases} \pi_1(L) = w_0 - P(\pi(L)) - L + q(L) + I(D(L), L) - u_0 \\ \frac{d\pi_1(x)}{dx} = \frac{d\pi(x)}{dx}, \ \forall x \in [0, L] \end{cases}$$

New insurance contract $(\pi_1(\cdot), D(\cdot))$ can be established, where $\pi_1(x) \geqslant \pi(x)$. The new insurance contract satisfies Eq. (9). Consequently, the new insurance contract is also feasible for model Eq. (5). $V(\pi, D)$ is strictly increasing with respect to π, hence:

$$V(\pi_1(x), D(x)) \geqslant V(\pi(x), D(x))$$

It means that the provider will choose $(\pi_1(\cdot), D(\cdot))$ rather than $(\pi(\cdot), D(\cdot))$.

Thus, an optimal insurance contract should satisfy Eq. (12).

Therefore, the proof of proposition 2 is completed.

Proposition 3 The objective function of the model Eq. (5) can be written as

$$\max_{D(\cdot)} \int_0^L \left\{ [w_0 - u_0 - L + q(L) - c]\varphi(x) + \Phi(x) \frac{\partial I(D(x), x)}{\partial x} \right\} dx \quad (13)$$

$\Phi(x)$ and $\varphi(x)$ are credibility density function and density function of ξ respectively.

Proof Integrating Eq. (9) and then applying Eq. (12) yields

$$P(\pi(x)) = \int_x^L \frac{\partial I(D(s), s)}{\partial s} ds + I(D(x), x) - \pi(x) + w_0 - u_0 - L + q(L).$$

Because the provider's welfare is

$$B(V(\pi(x), D(x)), P(\pi(x))) = P(\pi(x)) - c + V(\pi(x), D(x))$$

the following can be obtained:

$$B(V(\pi(x), D(x)), P(\pi(x))) = w_0 - u_0 - L + q(L) - c + \int_x^L \frac{\partial I(D(s), s)}{\partial s} ds$$

Moreover,

$$\frac{dB(V(\pi(x), D(x)), P(\pi(x)))}{dx} = -\frac{\partial I(D(x), x)}{\partial x} \leqslant 0$$

which means that $B(V(\pi(x), D(x)), P(\pi(x)))$ is decreasing with respect to x. By Refs[22,23],

$$E[B(V(\pi(\xi), D(\xi)), P(\pi(\xi)))] = \int_0^L \left[w_0 - u_0 - L + q(L) - c + \int_x^L \frac{\partial I(D(s), s)}{\partial s} ds \right] \varphi(x) dx$$

is got.

Thus, the provider's welfare is

$$E[B(V(\pi(x), D(x)), P(\pi(x)))] = \int_0^L \left\{ [w_0 - u_0 - L + q(L) - c]\varphi(x) + \Phi(x) \frac{\partial I(D(x), x)}{\partial x} \right\} dx$$

Note that the provider's welfare is irrelevant to $\pi(\cdot)$ and $P(\pi(\cdot))$, thus, decision vector $(\pi(\cdot),$

$D(\cdot)$, $P(\pi(\cdot)))$ is simplified as $D(\cdot)$.

Therefore, the proof of proposition 3 is completed.

Theorem 1 For any given function $P(\pi(\cdot))$, Eq. (5) is equivalent to

$$\begin{cases} \max\limits_{D(\cdot)} \int_0^L \left\{ [w_0 - u_0 - L + q(L) - c]\varphi(x) \right. \\ \qquad \left. + \Phi(x) \dfrac{\partial I(D(x), x)}{\partial x} \right\} dx \\ \text{subject to:} \\ \dfrac{dD(x)}{dx} \geqslant 0, \ \forall x \in [0, L] \end{cases} \tag{14}$$

Proof It is easy to verify the theorem according to Propositions 1-3.

3 Optimal solution for the equivalent model

In this section, the optimal solution of the equivalent model is obtained. Rewrite this problem as a control problem as follows.

Let

$$N(D(L), L) = \pi(L) - w_0 + P(\pi(L)) + L - q(L) - I(D(L), L) + u_0$$

and

$$u(x) = \frac{dD(x)}{dx}.$$

Furthermore, let $u(\cdot)$ be the control variable, $c(\cdot)$ be the state variable and $N(D(L), L) = 0$ be the constraint.

Thus, Eq. (14) can be written as:

$$\begin{cases} \max\limits_{u(\cdot)} \int_0^L \left\{ [w_0 - u_0 - L + q(L) - c]\varphi(x) \right. \\ \qquad \left. + \Phi(x) \dfrac{\partial I(D(x), x)}{\partial x} \right\} dx \\ \text{subject to:} \\ \dfrac{dD(x)}{dx} = u(x), \\ u(x) \geqslant 0, \\ N(D(L), L) = 0 \end{cases} \quad \forall x \in [0, L] \tag{15}$$

Hamiltonian

$$H(D(x), u(x), \lambda(x), x) = \lambda(x)u(x) + [w_0 - u_0 - L + q(L) - c]\varphi(x) + \Phi(x) \cdot \frac{\partial I(D(x), x)}{\partial x} \tag{16}$$

where $\lambda(\cdot)$ is an adjoint state.

According to Pontryagin's maximum principle[24,25], if optimal solution of Eq. (15) exists, then there exists adjoint state $\lambda(\cdot)$ such that the following conditions hold:

1) u^* maximizes the Hamiltonian Eq. (16), i.e.,

$$H(D^*(x), u^*, \lambda(x), x) = \max_{u \geqslant 0} \left(\lambda u + [w_0 - u_0 - L + q(L) - c]\varphi(x) + \Phi(x) \frac{\partial I(D(x), x)}{\partial x} \right) \tag{17}$$

2) $(u^*(\cdot), D^*(\cdot), \lambda(\cdot))$ satisfies

$$\begin{cases} \dfrac{d\lambda(x)}{dx} = -\Phi(x) \dfrac{\partial I^2(D(x), x)}{\partial D \partial x} \\ \dfrac{dD(x)}{dx} = u^*(x) \end{cases} \tag{18}$$

3) The boundary conditions are

$$\begin{cases} N(D(L), L) = 0 \\ \lambda(L) = \dfrac{\partial I(D(L), L)}{\partial D} \\ \cdot \dfrac{\dfrac{\partial I(D(L), L)}{\partial L} + [w_0 - u_0 - L - c + q(L)]\varphi(L)}{1 - \dfrac{\partial I(D(L), L)}{\partial L} - \dfrac{dD(L)}{dL} \cdot \dfrac{\partial I(D(L), L)}{\partial D}} \end{cases} \tag{19}$$

If $(u^*(\cdot), D^*(\cdot))$ is obtained by Eqs (17), (18) and (19), then the optimal premium and the optimal electricity price are

$$P^*(\pi^*(x)) + \pi^*(x) = w_0 - u_0 - L + q(L) - c + \int_x^L \frac{\partial I(D^*(s), s)}{\partial s} ds + I(D^*(x), x)$$

4 Numerical example

In this section, a numerical example is presented to obtain the optimal solutions for the electricity pricing model by analytical methods and testify the effectiveness.

It is considered the consumer's loss as fuzzy variable ξ with support $[0, L]$, where L denotes the maximum of the consumer's loss. Without loss of generality, triangular fuzzy variable $\xi = (0, a, L)$ is used to denote the provider's subjective assessment of the consumer's loss, with the credibility density function

$$\phi(x) = \begin{cases} \dfrac{1}{2a}, & 0 \leqslant x < a \\ \dfrac{1}{2(L-a)}, & a \leqslant x \leqslant L \\ 0, & \text{otherwise} \end{cases}$$

and the distribution function

$$\Phi(x) = \begin{cases} \dfrac{x}{2a}, & 0 \leqslant x < a \\ \dfrac{x + L - 2a}{2(L-a)}, & a \leqslant x \leqslant L \\ 0, & \text{otherwise} \end{cases}$$

Thus, the provider's optimal electricity pricing problem can be formulated as follows:

$$\begin{cases} \max\limits_{D(\cdot)} \int_0^L \{ \Phi(x) D(x) - (u_0 + c) \varphi(x) \} \mathrm{d}x \\ \text{subject to:} \\ \dfrac{\mathrm{d}D(x)}{\mathrm{d}x} \geqslant 0, \\ N(D(L), L) = 0 \end{cases} \quad (20)$$

where

$$N(D(L), L) = \pi(L) + P(\pi(L)) + u_0 - I(D(L), L)$$

and

$$P(\pi(x)) + \pi(x) = \int_x^L \frac{\partial I(D(s), s)}{\partial s} \mathrm{d}s + I(D(x), x) - u_0 - c$$

Eq. (20) can be formulated as follows:

$$\begin{cases} \max\limits_{u(\cdot)} \int_0^L \{ \Phi(x) D(x) - (u_0 + c) \phi(x) \} \mathrm{d}x \\ \text{subject to:} \\ \dfrac{\mathrm{d}D(x)}{\mathrm{d}x} = u(x), \\ u(x) \geqslant 0, \\ N(D(L), L) = 0. \end{cases}$$

Hamiltonian

$$H(D(x), u(x), (x), x) = D(x)\Phi(x) - (u_0 + c)\phi(x) + \lambda(x) u(x)$$

If the payout function is $I(D(x), x) = x - D(x)$, according to Eqs (17), (18) and (19), $D^*(x) = D(0)$ is obtained

and

$$P^*(\pi^*(x)) + \pi^*(x) = w_0 - u_0 - c - D(0)$$

That means that if the deductible is larger, the premium and the electricity price is lower; if the deductible are lower, the premium and the electricity price are higher. The optimal contract stimulates the consumer to reveal factual information.

5 Conclusions

Energy Internet has become a focus of concern for international academia and industry since Huang of North Carolina State University introduced the concept of it. In this paper, an electricity pricing model is proposed based on the insurance from a perspective on maximizing the benefits of Energy Internet service provider for the purpose of managing risk of Energy Internet and designing incentive mechanisms to encourage consumers to participate Energy Internet actively. The equivalent model is presented and a necessary condition of the optimal strategy is obtained on the basis of Pontryagin's maximum principle. At last, a numerical example is presented which illustrates the effectiveness of the proposed model. The results demonstrate that the insurance contract stimulates consumers to reveal information actually and the optimal electricity price encourages consumers to participate Energy Internet actively.

References

[1] Tian S M, Luan W P, Zhang D X, et al. Technical forms and key technologies on energy internet[J]. *Zhongguo Dianji Gongcheng Xuebao/proceedings of the Chinese Society of Electrical Engineering*, 2015, 35(14):3482-3494

[2] Su W C, Huang A Q, Michigan-Dearborn U O. The energy internet and electricity market in the United States[J]. *Chinese Science Bulletin*, 2016, 61(11):1210-1221

[3] Huang A Q, Crow M L, Heydt G T, et al. The future renewable electric energy delivery and management (FREEDM) system: the energy internet[J]. *Proceedings of the IEEE*, 2010, 99(1):133-148

[4] Sun H B, Guo Q L, Pan Z G. Energy internet: concept, architecture and frontier outlook[J]. *Automation of Electric Power Systems*, 2015, 39(19):1-8

[5] Wang J, Meng K, Cao J, et al. Information technology for energy internet: a survey[J]. *Journal of Computer Research and Development*, 2015, 52(5): 1109-1126

[6] Dou C, Yue D, Han Q L, et al. A Multi-Agent system based Event-triggered hybrid control scheme for energy internet[J]. *IEEE Access*, 2017, 5: 3263-3272

[7] Fumagalli E, Black J W, Vogelsang I, et al. Quality of service provision in electric power distribution systems through reliability insurance[J]. *IEEE Transactions on Power Systems*, 2004, 19(3): 1286-1293

[8] Allcott H. Rethinking real-time electricity pricing[J]. *Resource & Energy Economics*, 2011, 33(4):820-842

[9] Xie C S, Wei X M. Research on application mode of power supply reliability insurance to electricity market in China[J]. *Power System Technology*, 2005, 29(18): 67-73

[10] Xu W, Jiang C, Yuan R, et al. Application of insurance mechanism in new energy power generation[J]. *Dianli Xitong Zidonghua/automation of Electric Power Systems*, 2012, 36(14):41-45

[11] Karimi S A, Rajabi-Ghahnavieh A, Azad H. Financial tools to manage dispatchable distributed generation economic risks[C]. In: Proceedings of the 2015 IEEE International Conference on Smart Energy Grid Engineering (SEGE), Oshawa, Canada, 2015. 1-9

[12] Haring T, Lygeros J, Andersson G. Welfare assessment of decentralized pricing of operating reserves[C]. In: Proceedings of the 2013 IEEE Grenoble Conference, Grenoble, France, 2013. 1-6

[13] Nitanan A, Audomvongseree K. Electricity insurance pricing for the bilateral contract market[C]. In: Proceedings of the 8th Electrical Engineering/ Electronics, Computer, Telecommunications and Information Technology (ECTI) Association of Thailand - Conference 2011, Khon Kaen, Thailand, 2011. 812-815

[14] Yang P, Tang G, Nehorai A. A game-theoretic approach for optimal time-of-use electricity pricing[J]. *IEEE Transactions on Power Systems*, 2013, 28(2):884-892

[15] Healy P M, Palepu K G. Information asymmetry, corporate disclosure, and the capital markets: a review of the empirical disclosure literature[J]. *Journal of accounting and economics*, 2001, 31(1): 405-440
[16] Song Q F, Wang J Y, Shi K. A new fuzzy deductible contract model[J]. *Xitong Gongcheng Lilun Yu Shijian/system Engineering Theory & Practice*, 2014, 34(10): 2645-2649
[17] Arrow K J. Optimal insurance and generalized deductibles[J]. *Scandinavian Actuarial Journal*, 1974, 1974(1): 1-42
[18] Dimitriyadis I, Öney Ü N. Deductibles in health insurance[J]. *Journal of computational and applied mathematics*, 2009, 233(1): 51-60
[19] Sun H, Guo Q, Pan S. Energy internet: concept, architecture and frontier outlook[J]. *Automat Elect Power Systems*, 2015, 39(19): 1-8
[20] Zimmermann H J. An application-oriented view of modeling uncertainty[J]. *European Journal of operational research*, 2000, 122(2): 190-198
[21] Zhang W Y. Games Theory and Information Economics [M]. Shanghai: Truth & Wisdom Press, 2012. 47-66
[22] Xue F, Tang W, Zhao R. The expected value of a function of a fuzzy variable with a continuous membership function[J]. *Computers & Mathematics with Applications*, 2008, 55(6): 1215-1224
[23] Lan Y, Zhao R, Tang W. A bilevel fuzzy principal-agent model for optimal nonlinear taxation problems[J]. *Fuzzy Optimization and Decision Making*, 2011, 10(3): 211-232
[24] Pucci P, Serrin J. The strong maximum principle revisited[J]. *Journal of Differential Equations*, 2004, 196(1): 1-66
[25] Arutyunov A V, Karamzin D Y, Pereira F. Pontryagin's maximum principle for optimal impulsive control problems [J]. *Doklady Mathematics*, 2010, 81(3):418-421

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