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A cyberphysical-based approach to leader-following consensus for distributed multi-agent systems①

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Abstract

This paper considers a leader-following consensus problem for first-order linear multi-agent systems with constant input and communication time-delays. Based on the idea of cyber physical systems, the use of control states of neighboring agents, a new distributed control protocol is presented. Furthermore, in terms of linear matrix inequalities, sufficient conditions of leader-following consensus for multi-agent systems with constant input, communication and input time-delays are presented respectively. Numerical simulations on multi-agent systems are presented to demonstrate the efficiency of the proposed criteria.

Key words: leader-follower, cyber-physical system, multi-agent systems

0 Introduction

Recent years have witnessed increasing interest in multi-agent network cooperative systems partly due to its broad applications in unmanned aerial vehicles, mobile robots, automated highway systems, formation control, etc.[1-3]. Early work with multi-agent systems is presented in Refs[4,5]. A fundamental approach to achieve cooperative control is consensus. Consensus means the agreement of a group of agents on their common states via information interaction[3]. Leaderless and leader-following consensus problem for agents with first-order, second-order and high-order dynamics was studied recently by authors[6-19], to name just a few. A theoretical framework for consensus problems of multi-agent network cooperative systems was presented in Refs[1,2]. The first-order consensus algorithm with time-delay was studied in Refs[7-10]. And also the consensus problem for second-order dynamics systems was discussed in Refs[11-17]. The effect of limited and unreliable information exchange on agents' consensus behavior was analyzed in Refs[20-24].

The leader-follower approach is an important strategy for coordinating a team of agents. To let the whole network converge to a specific trajectory that a 'leader' can be added, for example the biological systems[25] and vehicle formation[12,26-29], and so on. The different type of leader for multi-agent systems was studied in Ref. [30]. Ni and Cheng[19] used the technique including algebraic graph theory, Riccati inequality and Lyapunov theory, leader-following consensus under fixed and switching topologies. The leader-following consensus under an undirected switching graph topology was studied in Ref. [31], which was inspired by the swarming behavior of silk worm moths. The control lability of a leader-following dynamic network with switching topologies was studied in Ref. [32], where the leader is a particular agent acting as an external input to steer the other agents, and it was also considered in Ref. [33] from a graph-theoretic perspective. For a leader with a nonzero and bounded control input, a distributed adaptive dynamic consensus protocol was proposed in Ref. [34]. Then the authors of Ref. [35] designed an adaptive nonlinear protocol by using the relative state information only for leader-following consensus under directed communication topology.

Note that almost all the protocols for the consensus problem rely on the relative states of neighboring agents, which however might not be sufficient in cyber physical system. Cyber physical system is an integration of computation and physical processes. Embedded computers and networks monitor and control the physical processes, usually with feedback loops where physical processes affect computations and vice versa[36,37].

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Multi-agent network cooperative system is an example of cyber physical system; therefore, it is a new approach to design the distributed coordination strategies for a network of agents in a cyber-physical environment.

In this paper, leader-following consensus problems are studied for continuous-time multi-agent systems in directed networks from a cyber-physical system perspective. Unlike most of papers, it is assumed that the information is not only the agent's state which is exchanged between an agent and all of its neighbors on the network, the information also includes the agent's control signal which would be easy in a cyber-physical system.

The remainder of this paper is organized as follows. In Section 1, some preliminaries on graph theory are provided and the formation of the leader-following consensus problem is given in a cyber-physical perspective. In Section 2, some results on the leader-following consensus problem with new consensus protocol are established. In Section 3, some numerical examples are simulated to verify the theoretical analysis. Finally, some concluding remarks are given in Section 4.

Throughout this paper, it is denoted by $\boldsymbol{I}_n$ the $n \times n$ identity matrix, $\boldsymbol{A}^{\mathrm{T}}$ means the transpose of matrix $\boldsymbol{A}$. $\boldsymbol{Q} < 0$ means that the matrix $\boldsymbol{Q}$ is negative-definite.

1 Preliminaries

1.1 Graph theory

Using the graph theory, the network topology can be modeled in a multi-agent systems consisting of n agents. A directed graph of order N is denoted by $G = (V, E, \boldsymbol{A})$, where the set of nodes $V = \{v_i : i \in N\}$ with $N = \{1, 2, \cdots, n\}$ and $n \geqslant 2$, the set of edges $E \subseteq V \times V$, $\boldsymbol{A} = [a_{ij}]$ is a weighted adjacency matrix. A directed edges of G is denoted by $e_{ij} = (v_i, v_j)$. The adjacency elements associated with the edges of the graph are not zeros, i. e. , $e_{ij} \in E$ if and only if $a_{ji} > 0$. Moreover, it is assumed $a_{ii} = 0$ for all $i \in N$. The set of neighbors node v_i is denoted by $N_i = \{v_j \in V; (v_j, v_i) \in E\}$. The Laplacian matrix $\boldsymbol{L}(G) = [l_{ij}]$ of digraph G is defined by $l_{ij} = -\sum_{k=1}^{n} a_{ik}$ for $i = j$ and $l_{ij} = a_{ij}$ for $i \neq j$. If (v_i, v_j) is an edge of G, v_i is called the parent of v_j and v_j is called the child of v_i. A directed tree is a directed graph, where every node, except one special node without any parent, which is called the root, has exactly one parent, and the root can be connected to any other nodes through paths. A spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph.

1.2 Multi-agent systems and consensus protocol

Suppose each node of digraph G is a dynamic agent with single-integrator kinematics as

$$\dot{x}_i(t) = u_i(t) \quad i = 0, 1, \cdots, n \tag{1}$$

where $x_i(t)$ and $u_i(t)$ are the state and the control input of the ith agent, respectively. The agent indexed by 0 with zero control input is called the leader, and the rest agents indexed by $i = 1, 2, \cdots, n$ are referred as the followers.

Definition 1 (Leader-following consensus problem). The distributed control law $u_i = f_i(z_i)$, $i = 1, \cdots, n$, where z_i denotes the local relative state information between ith agent and its neighboring agents. The typical control law can be described as

$$u_i(t) = \sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i) \tag{2}$$

where, $b_i > 0$ if the follower agent i has access to the leader's state x_0 and $b_i = 0$ otherwise. Such that for any initial condition $x_i(0)$, the following holds $\lim_{t \to \infty} e(t) = 0$, where $e = [e_1, \cdots, e_n]^{\mathrm{T}}$ and $e_i = x_i - x_0$.

Time-delay is frequently encountered in engineering systems involving multi-agent systems. It is well-known that introducing a delay generally leads to the reduction of performance or to instability. And the effect of time-delay in the consensus problem was studied in Refs[7-10,14,16-18]. Early work generally referred to consensus problems with two kinds of time-delays[38,39]. Communication time-delay may frequently occur due to agents moving, asymmetric interactions, communication congestion, or finite transmission speed. And the input time-delay intrinsically and inevitably exists in real-world dynamical systems, such as the inertia of the agent dynamics system and the actuator's time-delay. There is also time-delay caused by the sensor in the data sampling and data processing (Fig. 1).

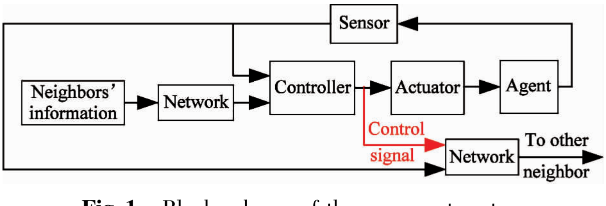

Fig. 1 Block scheme of the one-agent system

For typical protocol Eq. (2), suppose each agent has constant input time-delay but without communication and sensing time-delay. Then a typical control law is as

$$\hat{u}_i(t) = \sum_{j=1}^{n} a_{ij}(x_j(t-\tau_i) - x_i(t-\tau_i)) + b_i(x_0(t-\tau_i) - x_i(t-\tau_i)) \quad (3)$$

where τ_i denotes the ith agent's input time-delay which include the agent's actuator time-delay and system dynamics inertia.

Similarly, typical protocol Eq. (2) is considered with constant input time-delay and communication time-delay. Then a typical control law is

$$\tilde{u}_i(t) = \sum_{j=1}^{n} a_{ij}(x_j(t-\tau_i-\tau_{ij}^c) - x_i(t-\tau_i)) + b_i(x_0(t-\tau_i-\tau_{i0}^c) - x_i(t-\tau_i)) \quad (4)$$

where τ_{ij}^c denotes the communication delay from agent j to agent i, τ_{i0}^c represents the communication delay from the leader to agent i.

1.3 A new consensus protocol

For a network of agents in a cyber-physical environment, the agents' control signals can be used as feedforward signals for their neighbors (see Fig. 1). Two scenarios are considered in this paper.

(1) Fixed topology and zero communication time-delay, each agent with constant input time-delay: a new consensus protocol is used:

$$u_i(t) = \hat{u}_i(t) + k\sum_{j\in N_i} u_j(t) \quad (5)$$

where N_i denotes the set of neighbors of agent i, $\hat{u}_i(t)$ is defined in Eq. (3), k is the weighting factor of the feedforward control signal in cyber physical system.

(2) Fixed topology and constant communication time-delay, each agent with constant input time-delay: a new consensus protocol is used:

$$u_i(t) = \tilde{u}_i(t) + k\sum_{j\in N_i} u_j(t-\tau_{ij}^c) \quad (6)$$

where $\tilde{u}_i(t)$ is defined in Eq. (4).

Remark 1 For the leader-following consensus protocol Eqs(4) and (5) with less redundancy, digraph G is used to describe the interconnection topology of a multi-agent system which cannot consist of a loop, and means that there is no directed pathfrom v_i to v_i.

Assumption 1 All agents with the same input time-delay τ and the same communication delay τ^c, and $T = \tau + \tau^c$.

Defining error variable as $e_i(t) = x_i(t) - x_0(t)$, $e(t) = [e_1(t) \quad \cdots \quad e_n(t)]^{\mathrm{T}}$ and Eq. (7) is got along with Eqs(5) and (6):

$$\dot{e}(t) = \boldsymbol{L}e(t-\tau) - \boldsymbol{B}e(t-\tau) + k\boldsymbol{A}\dot{e}(t) \quad (7)$$

$$\dot{e}(t) = \boldsymbol{A}e(t-T) - \boldsymbol{D}e(t-\tau) - \boldsymbol{B}e(t-\tau) + k\boldsymbol{A}\dot{e}(t-\tau^c) \quad (8)$$

where, the leader adjacency matrix $\boldsymbol{B}$ is an $n\times n$ diagonal matrix whose ith diagonal element is b_i and is utilized to represent the connections between the followers and the leader. The degree matrix of G is $\boldsymbol{D} = diag\{d_1 \quad \cdots \quad d_n\} \in \mathbb{R}^{n\times n}$ in which $d_i = \sum_{j\in N_i} a_{ij}$ for $i = 1,\cdots,n$. The initial condition of the state $e(t)$ is supplemented as $e(\theta) = e(0)$, $\theta \in [-T \quad 0]$.

2 Main results

In this section, a stability criterion for system Eqs(7) and (8) will be derived. Before presenting the main results, the following lemmas are introduced.

Lemma 1 For any real differentiable vector function $x(t) \in \mathbb{R}^n$ and any $n\times n$ constant matrix $\boldsymbol{W} = \boldsymbol{W}^{\mathrm{T}} > 0$, the following inequality is got:

$$(x(t) - x(t-\tau))^{\mathrm{T}}\boldsymbol{W}(x(t) - x(t-\tau)) \leqslant \tau\int_{t-\tau}^{t} \dot{x}(s)^{\mathrm{T}}\boldsymbol{W}\dot{x}(s)\mathrm{d}s \quad (9)$$

for $t \geqslant 0$ and $\tau \geqslant 0$.

At first, the convergence analysis of the consensus problem in directed networks with fixed topology and constant input time-delay Eq. (7) is provided.

Theorem 1 For network Eq. (7), the following leader problem can be solved if there exist some positive matrices $\boldsymbol{P}$, $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$, such that the following LMI is satisfied:

$$\boldsymbol{M} = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} < 0 \quad (10)$$

where,

$M_{11} = \boldsymbol{Q} - \frac{1}{\tau}\boldsymbol{R}$, $M_{12} = \boldsymbol{P}(I - k\boldsymbol{A})^{-1}(\boldsymbol{L} - \boldsymbol{B}) + \frac{1}{\tau}\boldsymbol{R}$, $M_{22} = -\boldsymbol{Q} + \tau[(I - k\boldsymbol{A})^{-1}(\boldsymbol{L} - \boldsymbol{B})]^{\mathrm{T}}\boldsymbol{R}(\boldsymbol{I} - k\boldsymbol{A})^{-1}(\boldsymbol{L} - \boldsymbol{B}) - \frac{1}{\tau}\boldsymbol{R}$.

In Eq. (10), $*$ stands for the symmetric part of the matrix.

Proof Lyapunov-Krasovskii function for Eq. (7) is defined as follows

$$V(t) = e^T(t)\boldsymbol{P}e(t) + \int_{t-\tau}^{t} e^T(s)\boldsymbol{Q}e(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s\mathrm{d}\theta \quad (11)$$

where $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{R} \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices. The time derivative of $V(t)$ is

$$\dot{V}(t) = \dot{e}^T(t)\boldsymbol{P}e(t) + e^T(t)\boldsymbol{P}\dot{e}(t) + e^T(t)\boldsymbol{Q}e(t) - e^T(t-\tau)\boldsymbol{Q}e(t-\tau) + \tau\dot{e}^T(t)\boldsymbol{R}\dot{e}(t) - \int_{t-\tau}^{t} \dot{e}^T(s)\boldsymbol{R}\dot{e}(s)\mathrm{d}s \quad (12)$$

From Eq. (7) and Lemma 1, the following can be obtained:

$$
\begin{aligned}
\dot{V}(t) \leqslant\ & e^T(t)\boldsymbol{Q}e(t) \\
& + e^T(t-\tau)[(\boldsymbol{I}-k\boldsymbol{A})^{-1}(\boldsymbol{L}-\boldsymbol{B})]^T\boldsymbol{P}e(t) \\
& + e^T(t)\boldsymbol{P}(\boldsymbol{I}-k\boldsymbol{A})^{-1}(\boldsymbol{L}-\boldsymbol{B})e(t-\tau) \\
& - e^T(t-\tau)\boldsymbol{Q}e(t-\tau) \\
& + \tau e^T(t-\tau)[(\boldsymbol{I}-k\boldsymbol{A})^{-1}(\boldsymbol{L}-\boldsymbol{B})]^T \\
& \boldsymbol{R}(\boldsymbol{I}-k\boldsymbol{A})^{-1}(\boldsymbol{L}-\boldsymbol{B})e(t-\tau) \\
& \frac{1}{\tau}(e(t)-e(t-\tau))^T\boldsymbol{R}(e(t)-e(t-\tau)) \\
& := \boldsymbol{\gamma}^T(t)\boldsymbol{M}\boldsymbol{\gamma}(t)
\end{aligned} \tag{13}
$$

where $\boldsymbol{\gamma}(t) = [e^T(t), e^T(t-\tau)]^T$. Then Eq. (10) is obtained immediately, which completes the proof.

Theorem 2 For network system Eq. (8), the leader following problem can be solved if operator D is stable and there exist $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$, $\boldsymbol{R}_1 = \boldsymbol{R}_1^{\mathrm{T}} > 0$, $\boldsymbol{R}_2 = \boldsymbol{R}_2^{\mathrm{T}} > 0$, $\boldsymbol{Z}_1 = \boldsymbol{Z}_1^T > 0$, $\boldsymbol{Z}_2 = \boldsymbol{Z}_2^{\mathrm{T}} > 0$, $\boldsymbol{X} = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ * & X_{22} & X_{23} & X_{24} \\ * & * & X_{33} & X_{34} \\ * & * & * & X_{44} \end{bmatrix} \geqslant 0$, $\boldsymbol{Y} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ * & Y_{22} & Y_{23} & Y_{24} \\ * & * & Y_{33} & Y_{34} \\ * & * & * & Y_{44} \end{bmatrix} \geqslant 0$, and any matrices $\boldsymbol{N}_i$ and $M_i (i = 1, \cdots, 4)$ with appropriate dimensions such that the following LMIs hold:

$$
\boldsymbol{\Phi} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ * & * & \Phi_{33} & \Phi_{34} \\ * & * & * & \Phi_{44} \end{bmatrix} < 0 \tag{14}
$$

$$
\boldsymbol{\Psi} = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & N_1 \\ * & X_{22} & X_{23} & X_{24} & N_2 \\ * & * & X_{33} & X_{34} & N_3 \\ * & * & * & X_{44} & N_4 \\ * & * & * & * & Z_1 \end{bmatrix} \geqslant 0 \tag{15}
$$

$$
\boldsymbol{\Pi} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & M_1 \\ * & Y_{22} & Y_{23} & Y_{24} & M_2 \\ * & * & Y_{33} & Y_{34} & M_3 \\ * & * & * & Y_{44} & M_4 \\ * & * & * & * & Z_2 \end{bmatrix} \geqslant 0 \tag{16}
$$

where,

$$
\begin{aligned}
\Phi_{11} &= R_1 + R_2 + 2N_1 + 2M_1, \\
\Phi_{12} &= -\boldsymbol{P}(\boldsymbol{D}+\boldsymbol{B}) - N_1 + N_2^T - M_1 + M_2^T, \\
\Phi_{13} &= \boldsymbol{PA} + N_3^T + M_3^T \\
\Phi_{14} &= k\boldsymbol{PA} + N_4^T + M_4^T, \\
\Phi_{22} &= -R_1 + (\boldsymbol{D}+\boldsymbol{B})^T(\boldsymbol{Q}+\tau Z_1 + TZ_2)(\boldsymbol{D}+\boldsymbol{B}) \\
&\quad - 2N_2 - 2M_2 \\
\Phi_{23} &= -(\boldsymbol{D}+\boldsymbol{B})^T(\boldsymbol{Q}+\tau Z_1 + TZ_2)\boldsymbol{A} - N_3^T - M_3^T, \\
\Phi_{24} &= -k(\boldsymbol{D}+\boldsymbol{B})^T(\boldsymbol{Q}+\tau Z_1 + TZ_2)\boldsymbol{A} - N_4^T - M_4^T \\
\Phi_{33} &= -R_2 + \boldsymbol{A}^T(\boldsymbol{Q}+\tau Z_1 + TZ_2)\boldsymbol{A}, \\
\Phi_{34} &= k\boldsymbol{A}^T(\boldsymbol{Q}+\tau Z_1 + TZ_2)\boldsymbol{A}, \\
\Phi_{44} &= -\boldsymbol{Q} + k^2\boldsymbol{A}^T(\boldsymbol{Q}+\tau Z_1 + TZ_2)\boldsymbol{A}
\end{aligned}
$$

Proof Choose a Lyapunov functional candidate to be

$$
\begin{aligned}
V(t) =\ & e^T(t)Pe(t) + \int_{t-\tau^c}^{t} \dot{e}^T(s)Q\dot{e}(s)\,\mathrm{d}s \\
& + \int_{t-\tau}^{t} e^T(s)R_1 e(s)\,\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Z_1\dot{e}(s)\,\mathrm{d}s\mathrm{d}\theta \\
& + \int_{t-T}^{t} e^T(s)R_2 e(s)\,\mathrm{d}s + \int_{-T}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Z_2\dot{e}(s)\,\mathrm{d}s\mathrm{d}\theta
\end{aligned} \tag{17}
$$

From the Newton-Leibniz formula, the following equations are true for any matrices N_i and $M_i (i = 1, \cdots, 4)$

$$
2\begin{bmatrix} e^T(t)N_1 + e^T(t-\tau)N_2 \\ + e^T(t-T)N_3 \\ + \dot{e}^T(t-\tau^c)N_4 \end{bmatrix} \times \begin{bmatrix} e(t) - e(t-\tau) \\ - \int_{t-\tau}^{t} \dot{e}(s)\,\mathrm{d}s \end{bmatrix} = 0 \tag{18}
$$

$$
2\begin{bmatrix} e^T(t)M_1 + e^T(t-\tau)M_2 \\ + e^T(t-T)M_3 \\ + \dot{e}^T(t-\tau^c)M_4 \end{bmatrix} \times \begin{bmatrix} e(t) - e(t-T) \\ - \int_{t-T}^{t} \dot{e}(s)\,\mathrm{d}s \end{bmatrix} = 0 \tag{19}
$$

Calculating the derivative of $V(t)$ along the solution of Eq. (8) yields:

$$
\begin{aligned}
\dot{V}(t) =\ & 2e^T(t)P(Ae(t-T) - (D+B)e(t-\tau) \\
& + kA\dot{e}(t-\tau^c)) + \dot{e}^T(t)Q\dot{e}(t) \\
& - \dot{e}^T(t-\tau^c)Q\dot{e}(t-\tau^c) + e^T(t)R_1e(t) \\
& - e^T(t-\tau)R_1e(t-\tau) + \tau\dot{e}^T(t)Z_1\dot{e}(t) \\
& - \int_{t-\tau}^{t}\dot{e}^T(s)Z_1\dot{e}(s)\,\mathrm{d}s + e^T(t)R_2e(t) \\
& - e^T(t-T)R_2e(t-T) + T\dot{e}^T(t)Z_2\dot{e}(t) \\
& - \int_{t-T}^{t}\dot{e}^T(s)Z_2\dot{e}(s)\,\mathrm{d}s \\
& + 2\begin{bmatrix} e^T(t)N_1 + e^T(t-\tau)N_2 \\ + e^T(t-T)N_3 + \dot{e}^T(t-\tau^c)N_4 \end{bmatrix} \\
& \times \begin{bmatrix} e(t) - e(t-\tau) \\ - \int_{t-\tau}^{t}\dot{e}(s)\,\mathrm{d}s \end{bmatrix} \\
& + 2\begin{bmatrix} e^T(t)M_1 + e^T(t-\tau)M_2 \\ + e^T(t-T)M_3 + \dot{e}^T(t-\tau^c)M_4 \end{bmatrix} \\
& \times \begin{bmatrix} e(t) - e(t-T) \\ - \int_{t-T}^{t}\dot{e}(s)\,\mathrm{d}s \end{bmatrix} \\
& := \zeta^T(t)\Phi\zeta(t) - \int_{t-\tau}^{t}\eta^T(t,s)\Psi\eta(t,s)\,\mathrm{d}s \\
& \quad - \int_{t-T}^{t}\eta^T(t,s)\Pi\eta(t,s)\,\mathrm{d}s
\end{aligned} \tag{20}
$$

where $\zeta(t) = [e^T(t), e^T(t-\tau), e^T(t-T), \dot{e}^T(t-\tau^c)]^T$, $\eta(t,s) = [e^T(t), e^T(t-\tau), e^T(t-T), \dot{e}^T(t$

$-\tau^{c})$, $\dot{e}(s)]^{\mathrm{T}}$ and Φ, Ψ, Π is defined in Eqs(14-16).

Remark 2 First-order multi-agent systems with consensus protocol Eqs(5) and (6) can reach consensus if and only if the digraph has a directed spanning tree where all nodes in the graph are reachable from a root node by following the edge arrows. Thus, it is not difficult to get $\det(\boldsymbol{L}-\boldsymbol{B})\neq 0$ and $\det(\boldsymbol{D}+\boldsymbol{B})\neq 0$.

Remark 3 The consensus protocol in Eq. (6), where $T=\tau+\tau^{c}$, then the term $\int_{-\tau^{c}}^{0}\int_{t+\theta}^{t}\dot{e}^{T}(s)Z\dot{e}(s)\mathrm{d}s\mathrm{d}\theta$ is omitted so that there is no term in Eqs(14 – 16) directly related with τ^{c}. But the time-dependent stabilities in Eqs(14 – 16) are constrained by τ^{c} and τ.

3 Simulation

In this section, numerical example is presented to illustrate the effectiveness of the oretical results. Consider group of eight agents whose dynamics can be expressed by Eq. (1), and the communication topology to be a complicated one, as shown in Fig. 2, and the communication weight $a_{ji}=1$ if $e_{ij}\in E$, $a_{ji}=0$ otherwise. The initial states are $x_i(0)=i$ $(i=0,\cdots,7)$, where $x_0(0)=0$ is the leader's initial state.

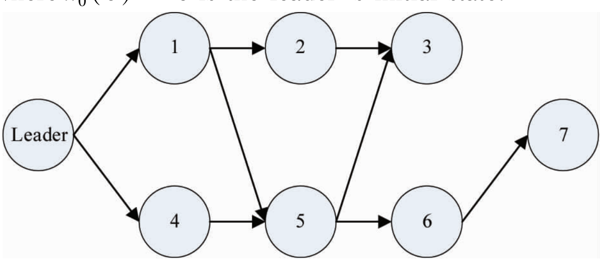

Fig. 2 The communication topology among the multi-agent

Example 1 A constant input time-delay $\tau_i=0.4\mathrm{s}$ $(i=1,\cdots 7)$ is taken and $k=0.7$ is set for consensus protocol Eq. (5). It is not difficult to find that LMI in Eq. (10) can be solved by using the MATLAB LMI tool. The state trajectories of the multi-agent systems under consensus protocol Eq. (5) are given in Fig. 4. In such a case, $k=0$ is set that consensus protocol Eq. (5) degenerates to typical protocol Eq. (4). It can be seen that the multi-agent systems achieve leader-following consensus. Comparing Fig. 3 with Fig. 4, the new consensus protocol shows faster convergence speed.

Example 2 Constant input time-delay $\tau_i=0.4$ s and communication time-delay $\tau_i^{c}=0.1$ s are considered for each following agent and $k=0.7$ is chosen for protocol Eq. (6). It is also not difficult to find that LMIs Eqs(14 – 16) can be solved by using the Matlab LMI tool. Similarly, $k=0$ is chosen that protocol Eq. (6) degenerates to typical protocol Eq. (4).

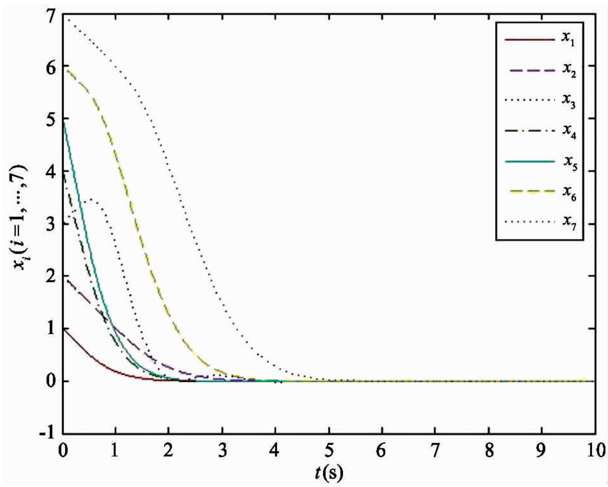

Fig. 3 State trajectories for consensus protocol Eq. (3)

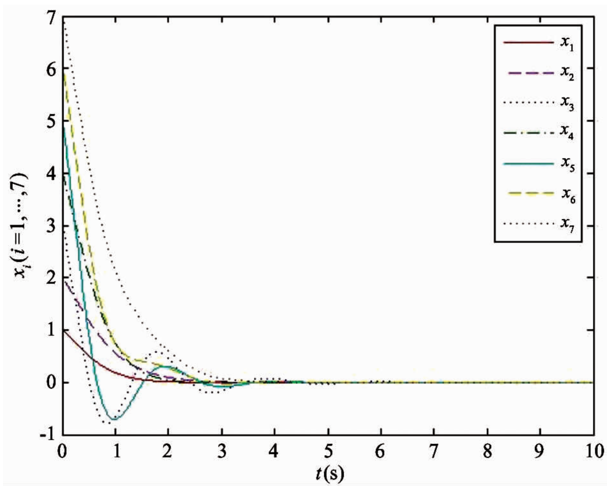

Fig. 4 State trajectories for consensus protocol Eq. (5)

It is clearly shown in Fig. 5 and Fig. 6 that the trajectories of the multi-agent converge to zero (consensus on the leader's state). Faster convergence speed for new protocol also can be demonstrated by simulation results shown in Fig. 5 and Fig. 6.

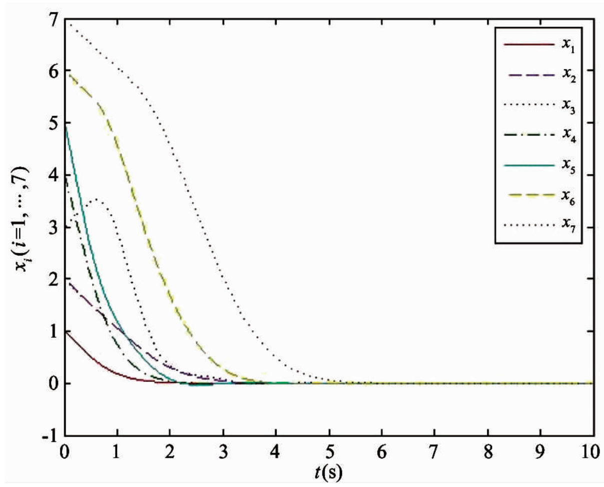

Fig. 5 State trajectories for consensus protocol Eq. (4)

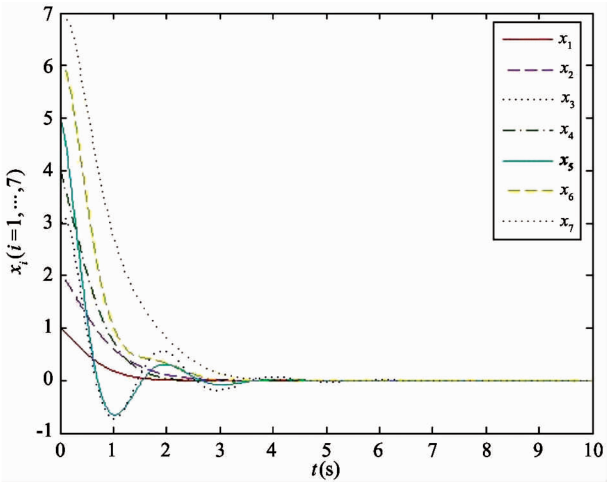

Fig. 6 State trajectories for consensus protocol Eq. (6)

Remark 4 From the simulation results, one can see that new protocol has faster convergence speed than the typical protocol. In fact, the agent with inertia and lag (input time-delay) shows that the agent's states change lag behind the control input. So by the use of neighbors' control signal, the agents could adjust their control signal before changing their neighbors' states.

4 Conclusions

The consensus problem of first-order multi-agent systems with fixed communication topology is discussed in this paper. A new consensus protocol is proposed for solving such a problem in a cyber-physical system. Then the leader-following consensus problem for the proposed consensus protocol is investigated based on graph theory, linear matrix in equality technique and Lyapunov approach. Two sufficient conditions are established in terms of a set of linear matrix in equalities for multi-agent systems with input time-delay and communication and input time-delay. Finally, numerical examples are given to illustrate the effectiveness of the obtained results. Future research will focus on extending the current work to the case where each agent with general dynamic and multi-agent systems has switching topologies and time-varying delays.

References

[1] Olfati-Saber R, Murray R M. Consensus problems in networks of agents with switching topology and time-delays [J]. *IEEE Transactions on Automatic Control*, 2004, 49(9):1520-1533

[2] Olfati-Saber R, Fax J A, Murray R M. Consensus and Cooperation in Networked Multi-Agent Systems[J]. *Proceedings of the IEEE*, 2007, 95(1):215-233

[3] Cao Y, Yu W, Ren W, et al. An overview of recent progress in the study of distributed multi-agent coordination [J]. *IEEE Transactions on Industrial Informatics*, 2013, 9(1):427-438

[4] Reynolds, Craig W. Flocks, herds and schools: a distributed behavioral model [J]. *ACM Siggraph Computer Graphics*, 1987, 21(4):25-34

[5] Vicsek T, Czirók A, Ben-Jacob E, et al. Novel type of phase transition in a system of self-driven particles[J]. *Physical Review Letters*, 2006, 75(6):1226-1226

[6] Olfati-Saber R. Flocking for multi-agent dynamic systems: algorithms and theory[J]. *IEEE Transactions on Automatic Control*, 2006, 51(3):401-420

[7] Wang L. State consensus for multi-agent systems with switching topologies and time-varying delays[J]. *International Journal of Control*, 2006, 79(10):1277-1284

[8] Tian Y P, Yang H Y. Stability of the Internet Congestion Control with Diverse Delays[M]. Pergamon Press, Inc. 2004

[9] Tian Y P, Liu C L. Consensus of multi-agent systems with diverse input and communication delays[J]. *IEEE Transactions on Automatic Control*, 2008, 53(9):2122-2128

[10] Bliman P A, Ferrari-Trecate G. Average consensus problems in networks of agents with delayed communications [J]. *Automatica*, 2008,44(4):1985-1995

[11] Lin P, Jia Y. Average consensus in networks of multi-agents with both switching topology and coupling time-delay[J]. *Physica A Statistical Mechanics & Its Applications*, 2008, 387(1):303-313

[12] Ren W. Consensus strategies for cooperative control of vehicle formations[J]. *IET Control Theory & Applications*, 2007, 1(2):505-512

[13] Xie G, Wang L. Consensus control for a class of networks of dynamic agents [J]. *International Journal of Robust and Nonlinear Control*, 2007,17(3):941-959

[14] Lin P, Jia Y M. Technical communique: consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies [J]. *Automatica*, 2009, 45(9):2154-2158

[15] Song Q, Cao J, Yu W. Second-order leader-following consensus of nonlinear multi-agent systems via pinning control[J]. *Systems & Control Letters*, 2010, 59(9): 553-562

[16] Yu W, Chen G, Cao M. Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems[J]. *Automatica*, 2010, 46(6):1089-1095

[17] Zhu W, Cheng D Z. Leader-following consensus of second-order agents with multiple time-varying delays[J]. *Automatica*, 2010, 46(12):1994-1999

[18] Zhou B, Lin Z. Consensus of high-order multi-agent systems with large input and communication delays[J]. *Automatica*, 2014,50(2):452-464

[19] Ni W, Cheng D. Leader-following consensus of multi-agent systems under fixed and switching topologies[J]. *Systems & Control Letters*, 2010, 59(3):209-217

[20] Hu J P, Feng G. Distributed tracking control of leader - follower multi-agent systems under noisy measurement [J]. *Automatica*, 2010, 46(8):1382-1387

[21] Zhang H, Zhou J. Distributed impulsive consensus for second-order multi-agent systems with input delays[J]. *IET Control Theory & Applications*, 2013, 7(16):1978-1983

[22] Sun F L, Guan Z H, Zhan X S, et al. Consensus of second-order and high-order discrete-time multi-agent systems with random networks[J]. *Nonlinear Analysis Real World Applications*, 2012, 13(5):1979-1990

[23] Mo L, Niu Y, Pan T. Consensus of heterogeneous multi-agent systems with switching jointly-connected interconnection[J]. *Physica A Statistical Mechanics & Its Applications*, 2015, 427:132-140

[24] Hu J, Hong Y. Leader-following coordination of multi-agent systems with coupling time delays[J]. *Physica A Statistical Mechanics & Its Applications*, 2007, 374(2):853-863

[25] Couzin I D, Krause J, Franks N R, et al. Effective leadership and decision-making in animal groups on the move[J]. *Nature*, 2005, 433(7025):513-516

[26] Fax J A, Murray R M. Information flow and cooperative control of vehicle formations[J]. *IEEE Transactions on Automatic Control*, 2004, 49(1):115-120

[27] Bando M, Hasebe K, Nakayama A, et al. Dynamical model of traffic congestion and numerical simulation[J]. *Physical Review E Statistical Physics Plasmas Fluids & Related Interdisciplinary Topics*, 1995, 51(2):1035

[28] Jiang R, Wu Q, Zhu Z. Full velocity difference model for a car-following theory[J]. *Phys Rev E Stat Nonlin Soft Matter Phys*, 2001, 64(1 Pt 2):017101

[29] Sun D H, Liu H, Zhang G, et al. The new car following model considering vehicle dynamics influence and numerical simulation[J]. *International Journal of Modern Physics C*, 2015, 26(07):1309

[30] Wei W, Slotine J J E. A theoretical study of different leader roles in networks[J]. *IEEE Transactions on Automatic Control*, 2006, 51(7):1156-1161

[31] Notarstefano G, Egerstedt M, Haque M. Containment in leader-follower networks with switching communication topologies[J]. *Automatica*, 2011, 47(5):1035-1040

[32] Liu B, Chu T, Wang L, et al. Controllability of a leader-follower dynamic network with switching topology[J]. *IEEE Transactions on Automatic Control*, 2008, 53(4):1009-1013

[33] Rahman A, Ji M, Mesbahi M, et al. Conteroliability of multi-agnet systems from a graph-theoretic perspective[J]. *Siam Journal on Control and Optimization*, 2009, 48(4):162-186

[34] Li Z, Ren W, Liu X, et al. Distributed consensus of linear multi-agent systems with adaptive dynamic protocols[J]. *Automatica*, 2013, 49(7):1986-1995

[35] Wang C, Ji H. Leader-following consensus of multi-agent systems under directed communication topology via distributed adaptive nonlinear protocol[J]. *Systems & Control Letters*, 2014, 70(7):23-29

[36] Derler P, Lee E A, Vincentelli A S. Modeling cyber-physical systems[J]. *Proceedings of the IEEE*, 2012, 100(1):13-28

[37] Kim K D, Kumar P R. Cyber-physical systems: a perspective at the Centennial[J]. *Proceedings of the IEEE*, 2012, 100(10):1287-1308

[38] Xiao F, Wang L. Consensus protocols for discrete-time multi-agent systems with time-varying delays[J]. *Automatica*, 2008, 44(10):2577-2582

[39] Meng Z, Ren W, Cao Y, et al. Leaderless and leader-following consensus with communication and input delays under a directed network topology[J]. *IEEE Transactions on Systems Man & Cybernetics Part B Cybernetics A Publication of the IEEE Systems Man & Cybernetics Society*, 2011, 41(1):75-88

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