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A new diagnosis strategy under the PMC model and applications①

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Abstract

A new diagnosis method, called Double-Syndrome diagnostic, is proposed, which can identify faulty nodes by comparing 2 different syndromes. For the same system, the average number of faulty nodes identified correctly by the Double-Syndrome diagnostic is much greater than the t-diagnosability and the (t_1/t_1) -diagnosability of the system. Furthermore, in order to identify the remaining faulty nodes in the system, two strategies of fault diagnostic are proposed, one is called (k, t) -fault diagnosable strategy, another is called $(k, t/t)$ - fault diagnosable strategy. Besides, the conditional (k, t) -diagnosable ($(k, t/t)$ -diagnosable) system is introduced. Furthermore, the conditional diagnosabilities are proved for some regular (k, t) -diagnosable and $(k, t/t)$ -diagnosable networks such as n-dimensional hypercube network and n-dimensional star network. And then, for a system, its (k,t) -conditional diagnosability and its $(k, t/t)$ -conditional diagnosability are identical, and in the worst case, they are equal to their traditional conditional diagnosability.

Key words: Double-Syndrome diagnostic, (k, t) -diagnosable, $(k, t/t)$ -diagnosable, hypercube, 2D(3D) mesh, permutation star graph

0 Introduction

With the rapid development of multiprocessors, multiprocessor computer systems contain hundreds and thousands of processors now[1]. It is inevitable that some processors in such a system may fail. To ensure reliability, the system should have the ability to identify the faulty processors which are then isolated from the system or replaced by additional fault-free ones[2]. In order to maintain the reliability of the system, automatic diagnosis procedures were proposed by Preparata et al.[3] and Somani et al.[4], which is known as system-level diagnosis. Preparata et al.[3] proposed the first system-level diagnosis model, namely the PMC model, which can be represented by a digraph $G = (V,E)$ and the edge (i, j) means node i tests node j. A test result $\omega(i, j)$ is associated with each (i, j) and $\omega(i, j) = 1(0)$ if i evaluates j to be faulty (fault-free). A complete set of test results associated with the edges of the system is called a syndrome[5-7]. For a syndrome σ, let $\omega(\sigma:i, j) = \omega(i, j)$ where $\omega(i, j) \in \sigma$. Under the PMC model, there are 2 fundamentally different strategies to system-level diagnosis: t-diagnosis[3] and t/t -diagnosis[3-9]. A system is t -diagnosable if and only if all the nodes can be identified by the system correctly in the presence of most t faulty nodes[10]. And a system is t/t -diagnosable if and only if all the faulty nodes can be isolated by it to within a set of size at most t in the presence of at most t faulty nodes[11,12]. However, the diagnosability (t-diagnosable and t/t -diagnosable) of a system given by $G = (V, E)$ is nearly depending on the degree of the graph G, which results in that the improvement of the diagnosability of one system by using traditional method becomes increasingly difficult[11-16]. Therefore, this provides a strong motivation to discover a new diagnosis method, for which more faulty nodes can be identified correctly. Next section will present a new diagnosis method, called Double-Syndrome diagnostic, under the PMC model, for which more faulty nodes can be identified correctly. Section 2 proposes a new system called (k, t) -diagnosable and the characterization and some properties of such systems are also presented. Section 3 proposes a new system called $(k, t/t)$-diagnosable system and the characterization and some properties of such a system are also presented. Section 4 uses properties of these 2 systems and Double-Syndrome diagnostic to further increase the number

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of faulty nodes which can be identified correctly. In Section 5, a further study is proposed on above 2 diagnosable systems under the conditional diagnosis and figure out some special conditional diagnosability of above 2 diagnosable systems. In the last section, a conclusion is drawn.

1 Double-Syndrome diagnostic

Under PMC model, for a system given by $G = (V, E)$, let $\Gamma u = \{v \mid (u,v) \in E, u, v \in V\}$ and $\Gamma u^{-1} = \{v \mid (v,u) \in E, u, v \in V\}$. Similarly, for any subset $X \subset V$, $\Gamma X = \cup_{u \in X} \Gamma u - X$ and $\Gamma X^{-1} = \cup_{u \in X} \Gamma u^{-1} - X$. Without loss of generality, let $\Gamma u = \{v_0, v_1, v_2, v_3, \cdots, v_m\}$ and for a syndrome σ_i, let $\omega(u, \sigma_i) = (\omega(\sigma_i : u, v_0), \omega(\sigma_i : u, v_1), \omega(\sigma_i : u, v_2), \cdots, \omega(\sigma_i : u, v_m))$.

Lemma 1 For a system given by $G = (V, E)$ and 2 different syndromes σ_1 and σ_2, $u \in V$, if $\omega(u, \sigma_1)$, $\omega(u, \sigma_2)$, then u is a faulty node.

Proof Suppose that, to the contrary, u is fault-free. Since $\omega(u, \sigma_1)$, $\omega(u, \sigma_1)$, there exists some $\omega(\sigma_1 : u, v_k)$ such that $\omega(\sigma_1 : u, v_k)$, $\omega(\sigma_2 : u, v_k)$. Without loss of generality, let $\omega(\sigma_1 : u, v_0) = 1$ and $\omega(\sigma_2 : u, v_0) = 0$. $\omega(\sigma_1 : u, v_0) = 1$ implies v_0 is faulty. On the other hand, $\omega(\sigma_2 : u, v_0) = 0$ implies v_0 is fault-free, a contradiction complete the proof.

Now, Double-Syndrome diagnostic (Algorithm 1) is introduced as follows.

Algorithm 1 Double-Syndrome diagnostic

Require:

A system given by $G = (V, E)$ with n nodes and 2 different syndromes σ_1 and σ_2.

Ensure:

A set of faulty nodes.

1) For each node $v_i \in V (0 \leqslant i \leqslant n-1)$, if $\omega(v_i, \sigma_1) = \omega(v_i, \sigma_2)$, continue the Double-Syndrome diagnostic, otherwise, mark v_i with fault and continue the Double-Syndrome diagnostic.

2) Output the nodes marked with fault.

Under the PMC model, the test result of one faulty node testing the other nodes is unreliable[15,16]. In other words, the value of $\omega(u, v)$ is stochastic where u is a faulty node. For convenience, the possibility of test result 1 (or 0) of each faulty node testing other nodes is equivalent and let $P(u, v:1) = \alpha (P(u, v:0) = 1 - \alpha)$ be the possibility of test result 1 (0) of one faulty node u testing another node v (v can be faulty or fault-free).

Definition 1 Let A be a event and $P(A)$ be the possibility of the event A happened.

Property 1 For a system given by $G = (V, E)$, suppose that $u \in V$ is a faulty node with $\mid \Gamma u \mid = m$. For any 2 stochastic syndromes σ_1 and σ_2, let $P(u)$ be the possibility that u is not marked with fault by Double-Syndrome diagnostic. Then $P(u) = P(\omega(u, \sigma_1) = \omega(u, \sigma_2)) = \alpha^l \times (1 - \alpha)^k$ with $l + k = m$. Without loss of generality, let $\alpha \geqslant 0.5$, then $P(u) \leqslant \alpha^m$.

Property 2 For a system given by $G = (V, E)$ with n nodes and t faulty nodes. Let $E(G)$ be the mean number of faulty nodes which can be identified by Double-Syndrome diagnostic. Let $F = \{v_i, 0 \leqslant i \leqslant t-1\}$ be the set of the faulty nodes in the system, then $E(G) = \sum_{i=0}^{t-1} [1 - P(v_i)]$.

Definition 2 A regular graph is a graph, in which each vertex has the same number of neighbors. Let $D(G)$ be the number of neighbors of each vertex in $G = (V, E)$.

Property 3 For a system given by $G = (V, E)$ with n nodes and t faulty nodes. If $G = (V, E)$ is a regular graph, then $E(G) = t(1 - p(v))$, where $v \in V$ is a faulty node.

The lower bounds of $E(G)$ under the n-dimensional hypercube with a different α are shown in Table 1. Here, t denotes the exact faulty number in the system.

Table 1 The changes of $E(G)$ of n-dimensional hypercube

$E(G) \geqslant$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	n
$\alpha = 0.5$	$0.98t$	$0.99t$	$0.996t$	$0.998t$	$(1 - 0.5^n)t$
$\alpha = 0.7$	$0.88t$	$0.918t$	$0.942t$	$0.96t$	$(1 - 0.7^n)t$
$\alpha = 0.9$	$0.47t$	$0.52t$	$0.57t$	$0.61t$	$(1 - 0.9^n)t$

For a system given by $G = (V, E)$, average $E(G)$ faulty nodes can be identified by Double-Syndrome diagnostic correctly. For given 2 syndromes σ_1 and σ_2, there may exist some faulty nodes which cannot be identified by Double-Syndrome diagnostic. In next section, another diagnosable method is proposed to deal with these unidentified faulty nodes.

2 Two-step (k, t)-diagnosable system

For a system given by $G = (V, E)$, under the assumption that k-faulty nodes have been identified, it is a very interesting problem to recognize the remaining faulty nodes as much as possible. It is worth noting that with the different distribution of the k-identified nodes, the number of remaining faulty nodes which can be identified may be different[17].

Definition 3 A system is one-step t-diagnosable if all faulty nodes can be recognized without replacement provided the number of faulty nodes does not exceed t[3].

Definition 4 Given a system by $G = (V, E)$ and a syndrome σ, a set $X \subseteq V$ is called an allowable fault set (AFS) of the system for syndrome σ if for any 2 nodes i, j such that $(i, j) \in E$, the following conditions hold: if $i, j \in V - X$ then $\omega(\sigma : i, j) = 0$, and if $i \in V - X$ and $j \in X$ then $\omega(\sigma : i, j) = 118$.

It is worth noting that given a system by $G = (V, E)$, a syndrome σ and a fault set F, then there must exist an allowable fault set F', such that $F \subseteq F'$. In other words, there must exist a subset $S \subset V$ such that $F \cup S$ is an allowable fault set for syndrome σ.

Definition 5 A system is two-step (k, t)-diagnosable if under the condition that k faulty nodes have been already recognized, the all remaining faulty nodes can be identified provided the number of faulty nodes in the system does not exceed $k + t$.

It is worth noting that according to Definition 3 and Definition 5, a one-step $(k + t)$-diagnosable system must be two-step (k, t)-diagnosable system, but the inverse is not true. Now an example is given which is two-step (k, t)-diagnosable but not one-step $(k + t)$-diagnosable.

Consider a system $G = (V, E)$ shown in Fig. 1, it is a two-step $(2, 1)$-diagnosable system. In fact, for any given syndrome σ produced by the system in the presence of the fault set F with $|F| \leqslant 3$, if $|F| \leqslant 2$, then the conclusion is true according to the definition. We shall show it is also true when $|F| = 3$. Now we only need to consider following 3 cases due to the symmetry of the system. Let $F_c \subset F$ be the possible identified faults set.

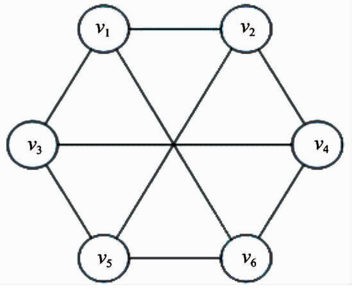

Fig. 1 An example of a two-step $(2, 1)$-diagnosable system

Case 1 $F_c = \{v_1, v_2\}$.

There is only one faulty node in $\{v_3, v_4, v_5, v_6\}$ and subgraph induced by $\{v_3, v_4, v_5, v_6\}$ is connected. For the given syndrome σ, there always exist 2 adjacent nodes $u, v \in \{v_3, v_4, v_5, v_6\}$ such that at least one of $\omega(\sigma : u, v)$ and $\omega(\sigma : u, v)$ is 1. Then the faulty node belongs to $\{u, v\}$ and $\{v_3, v_4, v_5, v_6\}$-$\{u, v\}$ are all fault-free. Therefore, the remaining faulty node can be identified by the test results of their neighbors testing them.

Case 2 $F_c = \{v_1, v_3\}$.

For any 2 adjacent nodes $u, v \in \{v_4, v_5, v_6\}$, if $\omega(\sigma : u, v) = 0$, then v_5 is the remaining faulty node. Otherwise, v_5 is fault-free and v_5 can be identified correctly. Furthermore, v_4, v_6 can also be identified correctly.

Case 3 $F_c = \{v_1, v_4\}$.

Note that the subgraph induced by $\{v_2, v_3, v_5, v_6\}$ is isomorphic to the subgraph induced by $\{v_3, v_4, v_5, v_6\}$. A similar argument of Case 1 can be used.

Above all, the system shown in Fig. 1 is a two-step $(2, 1)$-diagnosable system. However, it is not a one-step 3-diagnosable system due to the fact that $|V| = 6 < 2 \times 3 + 1$.

With the definition of the two-step (k, t)-diagnosable system, the characterization of this kind of system is presented.

Theorem 1 A system given by $G = (V, E)$ is two-step (k, t)-diagnosable if and only if for each subset $F_c \subset V$ with $|F_c| = k$ and any 2 distinct subsets $|S_1| \leqslant t$, $|S_2| \leqslant t$ with $|S_1| \leqslant t$, $|S_2| \leqslant t$, there exists an edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$.

Proof Necessity: suppose that a system is two-step (k, t)-diagnosable, there exist some $F_c \subset V$ with $|F_c| = k$ and some pair of subsets $S_1, S_2 \subset V - F_c$ with $S_1 \neq S_2$, $|S_1| \leqslant t$, $|S_2| \leqslant t$ such that there are no edges from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Consider a syndrome σ such that for each $(i, j) \in E$: if $i, j \in V - S_1 - S_2 - F_c$, then $\omega(\sigma : i, j) = 0$, if $i \in V - S_1 - S_2 - F_c$ and $j \in F_c \cup S_1 \cup S_2$, then $\omega(\sigma : i, j) = 1$, other possible test results can be arbitrary.

For such syndrome σ and the identified fault set F_c, both $F_c \cup S_1$ and $F_c \cup S_2$ are all allowable fault sets of cardinality at most $t + k$, which is a contradiction to the hypothesis.

Sufficiency: suppose that, to the contrary, the system is not two-step (k, t)-diagnosable, implying that there exists a syndrome σ by which a k-node fault set F_c can be identified, and that there exist 2 distinct subsets S_1, $S_2 \subset V - F_c$ of cardinality at most t such that $F_c \cup S_1$ and $F_c \cup S_2$ are allowable fault sets. Noting that there exists an edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Without loss of generality, let $i \in V - S_1 - S_2 - F_c$, $j \in (S_1 - S_2)$ with $(i, j) \in E$. If $\omega(\sigma : i, j) = 1$, then $F_c \cup S_2$ is not an allowable fault set. If $\omega(\sigma : i, j) = 0$, $F_c \cup S_1$ is not an allowable fault set. This is a contradiction.

Note that two-step (k, t)-diagnosable system can be considered to be a generalization of t-diagnosable system, since if $k = 0$, two-step (k, t)-diagnosable system corresponds directly to t-diagnosable system.

Corollary 1 If a system is two-step (k, t)-diagnosable, then the system is also two-step $(k, t-1)$-diagnosable.

Proof According to Theorem 1, the result is true.

Corollary 2 If a system given by $G = (V, E)$ is two-step (k, t)-diagnosable, then the system is also two-step $(k-1, t)$-diagnosable.

Proof Assume that, to the contrary, the system is not two-step $(k-1, t)$-diagnosable, thus, there exist a subset $F_c \subset V$ with $|F|_c = k-1$ and a pair of subsets $S_1, S_2 \subset V - F_c$ with $S_1 \neq S_2$, $|S_1| \leqslant t$, $|S_2| \leqslant t$, such that there are no edges from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Let $v \in V - S_1 - S_2 - F_c$ and $F'_c = F_c + \{v\}$. Since there are no edges from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$, there are also no edges from $V - S_1 - S_2 - F'_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$ which is a contradiction to the assumption that the system is two-step (k, t)-diagnosable.

Corollary 3 If a system given by $G = (V, E)$ is two-step (k, t)-diagnosable $(k > 2, t > 1)$, then the system is also two-step $(k-2, t+1)$-diagnosable.

Proof Assume that, to the contrary, the system is not two-step $(k-2, t+1)$-diagnosable, thus, there exist a subset $F_c \subset V$ with $|F_c| = k-2$ and a pair of subsets $S_1, S_2 \subset V - F_c$ with $S_1 \neq S_2$, $|S_1| \leqslant t+1$, $|S_2| \leqslant t+1$, such that there are no edges from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$. According to Lemma 1, the system is two-step $(k-2, t)$-diagnosable, implying that either $|S_1| \leqslant t+1$ or $|S_2| \leqslant t+1$. Consider the following cases.

Case 4 $|S_1| = t+1$ and $|S_2| = t+1$.

Let $v \in S_1 - S_2$, $u \in S_2 - S_1$ and $F'_c = F_c \cup \{v\} \cup \{u\}$, $S'_1 = S_1 - \{v\}$, $S'_2 = S_2 - \{u\}$. Note that $|F_c| = k$ and $S'_1 \neq S'_2$, $|S'_1| \leqslant t$, $|S'_2| \leqslant t$, and $V - S'_1 - S'_2 - F'_c = V - S_1 - S_2 - F_c$, $(S'_1 - S'_2) \cup (S'_2 - S') \subseteq (S_1 - S_2) \cup (S_2 - S_1)$. Therefore, there are also no edges from $V - S'_1 - S'_2 - F'_c$ to $(S'_1 - S'_2) \cup (S'_2 - S'_1)$ which is a contradiction to the assumption that the system is two-step (k, t)-diagnosable.

Case 5 $|S_1| \leqslant t$ and $|S_2| = t+1$.

If $S_1 \subseteq S_2$, then $|S_1 \cap S_2| \geqslant 1$. Since a two-step (k, t)-diagnosable system has at least $t+k$ nodes and $|F_c \cup S_1 \cup S_2| = t+k-1$, there always exits a node $v \in V - S_1 - S_2 - F_c$. Let $u \in S_1 \cap S_2$ and $F'_c = F_c \cup \{v\} \cup \{u\}$, $S'_1 = S_1 - \{u\}$, $S'_2 = S_2 - \{u\}$. Note that $|F_c| = k$ and $S_1 \neq S_2$, $|S'_1| \leqslant t$, $|S'_2| \leqslant t$ and $V - S'_1 - S'_2 - F'_c \subseteq V - S_1 - S_2 - F_c$, $(S'_1 - S'_2) \cup (S'_2 - S') = (S_1 - S_2) \cup (S_2 - S_1)$. Therefore, there also are no edges from $V - S'_1 - S'_2 - F'_c$ to $(S'_1 - S'_2) \cup (S'_2 - S'_1)$ which is a contradiction to the assumption that the system is two-step (k, t)-diagnosable. And a similar argument of Case 4 can be used when $S_1 \not\subset S_2$.

Case 6 $|S_1| = t+1$ and $|S_2| \leqslant t$.

A similar argument of Case 6 can be used.

3 Two-step $(k, t/t)$-diagnosable system

In the previous section, the generalization of t-diagnosable system is discussed, namely the two-step (k, t)-diagnosable system. Next we shall consider the generalization of t/t-diagnosable system, namely the two-step $(k, t/t)$-diagnosable.

Definition 6 A system S is t/t-diagnosable if given any syndrome and a positive integer t, the faulty nodes can be isolated within a set of at most t nodes provided the number of faulty nodes does not exceed t[19].

Definition 7 A system is two-step $(k, t/t)$-diagnosable if and only if given any syndrome and a pair positive integers t, k, under the condition that k faulty nodes have been already identified correctly, all the remaining faulty nodes can be isolated within a set of size at most t in the presence of at most $t+k$ faulty nodes in all. With the definition of the two-step $(k, t/t)$-diagnosable system, we shall present the characterization of this kind of system.

Theorem 2 For a system S given by $G = (V, E)$, the following 3 statements are equivalent.

1) S is two-step $(k, t/t)$-diagnosable.

2) For each subset $F_c \subset V$ with $|F_c| = k$ and for any 2 distinct subsets $S_1, S_2 \subset V - F_c$ with $|S_1| = |S_2| = t$, there exists an edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$.

3) For each subset $F_c \subset V$ with $|F_c| = k$ and for any 2 distinct subsets $S_1, S_2 \subset V - F_c$ with $S_1 \not\subset S_2$, $S_2 \not\subset S_1$, $|S_1| \leqslant t$, $|S_2| \leqslant t$, there exists an edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$.

Proof The proof is similar to the proof of Lemma 1 in Ref. [10].

According to Theorem 6, the following corollaries can be concluded.

Corollary 4 If a system given by $G = (V, E)$ is two-step $(k, t/t)$-diagnosable, then the system is also two-step $(k, t-1/t-1)$-diagnosable.

Corollary 5 If a system given by $G = (V, E)$ is two-step $(k, t/t)$-diagnosable, then the system is also two-step $(k-1, t/t)$-diagnosable.

Corollary 6 If a system given by $G = (V, E)$ is two-step $(k, t/t)$-diagnosable, then the system is also two-step $(k-2, t+1/t+1)$-diagnosable.

Next section will analysis the specific situation by using the theories of two-step (k, t)-diagnosable system and two-step $(k, t/t)$-diagnosable system.

4 Combine to the Double-Syndrome diagnostic

For a system given by $G = (V, E)$ and a fault set F_c identified by Double-Syndrome diagnostic with $|F_c| = k$, if there exists a node $v \in V$ such that $\Gamma v^{-1} \subseteq F_c$, then the node v cannot be judged as faulty or fault-free. Therefore, whether all the remaining faulty nodes can be identified or not depends on the distribution of F_c. Next it will be discussed that with the changes of distribution of F_c, how many remaining faulty nodes can be identified.

Definition 8 For a two-step (k, t)-diagnosable system S and a fault set F_c identified by Double-Syndrome diagnostic or other methods, the diagnosability of the two-step (k, t)-diagnosable based on F_c, denoted by $T(S, F_c)$, is the maximum number of nodes which is guaranteed to be identified as faulty correctly. To facilitate the discussion, the following theorem is equivalent to Theorem 1.

Theorem 3 For a system given by $G = (V, E)$ and a fault set $F_c \subseteq V$ with $|F_c| = k$, then all the remaining faulty nodes at most t can be identified if and only if for any 2 subsets $S_1, S_2 \subset V$ with $S_1 \neq S_2$, $|S_1| \leqslant t$, $|S_2| \leqslant t$ such that $\Gamma(S_1 - S_2)^{-1} \not\subseteq F_c \cup S_2$ or $\Gamma(S_2 - S_1)^{-1} \not\subseteq F_c \cup S_1$.

Corollary 7 For a system given by $G = (V, E)$ and a fault set F_c with $|F_c| = k$, if for each subset $U \subset V - F_c$ with $|U| \leqslant 2t$, there exists no such subset $X \subseteq U$ such that $\Gamma X^{-1} \subseteq F_c \cup (U - X)$, then all the remaining faulty nodes can be identified provided the number of remaining faulty does not exceed t.

Proof For any distinct subsets $S_1, S_2 \subseteq V$ with $|S_1| \leqslant t$, $|S_2| \leqslant t$, let $U = S_1 \cup S_2$ and $X = (S_2 - S_1) \cup (S_1 - S_2)$. if $\Gamma X^{-1} \not\subseteq (F_c \cup (U - X))$, then it is easily seen that the condition of the Theorem 1 is satisfied.

The following theorem is equivalent to Theorem 2.

Theorem 4 For a system given by $G = (V, E)$ and a fault set F_c with $|F_c| = k$, all remaining faulty nodes at most t can be isolated within a set of size at most t if and only if for any 2 subsets $S_1, S_2 \subset V - F_c$ with $S_1 \not\subseteq S_2$, $S_2 \not\subseteq S_1$, $|S_1| \leqslant t$, $|S_2| \leqslant t$, such that $\Gamma(S_1 - S_2)^{-1} \not\subseteq F_c \cup S_2$ or $\Gamma(S_2 - S_1)^{-1} \not\subseteq F_c \cup S_1$.

Corollary 8 For a system given by $G = (V, E)$ and a fault set F_c with $|F_c| = k$, if for each subset $S \subset V$ with $|S| \leqslant 2t$, there exists no such a subset $X \subseteq S$ such that $\Gamma X^{-1} \subseteq F_c \cup (S - X)$, then all remaining faulty nodes can be isolated within a set of size at most t provided the number of remaining faulty does not exceed t.

A similar proof of Corollary 7 can be used.

Observe above theorems and corollaries, when the neighbors of some nodes are all faulty, it cannot be correctly identified such a node. Therefore the following conclusion.

Theorem 5 For a system S given by $G = (V, E)$, if S is t'-diagnosable (but not $t'+1$-diagnosable) and also two-step (k, t)-diagnosable (but neither two-step $(k, t+1)$-diagnosable nor two-step $(k+1, t)$-diagnosable) $(t' \geqslant k)$), then its diagnosability satisfies the following inequality: $T(S, F_c) \geqslant t'$ where $F_c \subseteq V$ and $|F_c| = k$.

Proof According to Definition 4 and Definition 8, for any given fault set $F_c \subseteq V$ with $|F_c| = k$, $T(S, F_c) \geqslant t + k$. Next, it can be shown that $t + k \geqslant t'$. Otherwise, $t + k < t'$. The system is neither two-step $(k, t+1)$-diagnosable nor two-step $(k+1, t)$-diagnosable implies that for some fault set $F_c \subseteq V$ with $|F_c| = k$ there exists 2 distinct subsets $S_1, S_2 \subset V - F_c$ with $|S_1| \leqslant t+1$, $|S_2| \leqslant t+1$, there exists no edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Let $U_1 = F_c \cup S_1$, $U_2 = F_c \cup S_2$. Then $|U_1|$, $|U_2| \leqslant t + k + 1$. Note that there exists no edge from $V - U_1 - U_2 - F_c$ to $(U_1 - U_2) \cup (U_2 - U_1)$. Therefore, the system is not $(t+k+1)$-diagnosable, which implies $t + k + 1 > t'$, this is a contradiction.

Note that the number of fault-free neighbors of each node is related to the diagnosability of kinds of diagnosable systems[20,21]. Ref. [20] considered the situation that each node has at least one fault-free neighbor in the system and proposed the concept of conditional diagnosability. The next section will extend the concept of conditional diagnosability to two-step (k, t)- diagnosable ($(k, t/t)$-diagnosable) system and present the characterizations of conditional two-step (k, t)- diagnosable (((k, t/t)- diagnosable) systems.

5 Conditional two-step (k-t)-diagnosable and two-step (k, t/t)-diagnosable

The hypercube structure is a well-known network model for multi-processor systems. Fault-tolerant computing for n-dimensional hypercube has been of interest to many researchers[17-19,21]. In this subsection, the conditional t/t-diagnosability of n – dimensional hypercube is studied.

Definition 9 For a system given by $G = (V, E)$, a subset $S \subset V$ is called a conditional subset if there exists no such a node $v \in V$ such that $\Gamma v^{-1} \subseteq S$.

Lemma 2 A system given by $G = (V, E)$ is conditionally t -diagnosable if and only if for any 2 conditional subsets $S_1, S_2 \subseteq V$ with $S_1 \neq S_2$, $|S_1| \leqslant t$, $|S_2| \leqslant t$, there exists an edge from $V - S_1 - S_2$ to $(S_1 - S_2) \cup (S_2 - S_1)$[22].

Definition 10 A system is conditionally two-step (k, t)- diagnosable under the condition that k faulty nodes have been already recognized, and all the remaining faulty nodes can be identified provided the number of faulty nodes in the system does not exceed $k + t$ and each node of the system has at least one fault-free neighbor.

Theorem 6 A system given by $G = (V, E)$ is conditional two-step (k, t)- diagnosable if and only if for any conditional subset F_c with $|F_c| = k$ and any 2 conditional subsets $S_1, S_2 \subset V - F_c$ with $S_1 \neq S_2$, $|S_1| \leqslant t$, $|S_2| \leqslant t$, $F_c \cup S_1$ and $F_c \cup S_2$ are conditional subsets, there exists an edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$.

Proof Necessity: suppose that a system is conditional two-step (k, t)- diagnosable and for some conditional subset F_c with $|F_c| = k$ and 2 conditional subsets $S_1, S_2 \subset V - F_c$ with $S_1 \neq S_2$, $|S_1| \leqslant t$, $|S_2| \leqslant t$, $F_c \cup S_1$ and $F_c \cup S_2$ are conditional subsets, there exists no edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Consider a syndrome σ satisfying the following conditions for all nodes i, j such that $(i, j) \in E$. If $i, j \in V - S_1 - S_2 - F_c$, then $\omega(\sigma : i, j) = 0$. If $i \in V - S_1 - S_2 - F_c$ and $j \in S_1 \cup S_2$, then $\omega(\sigma : i, j) = 1$. If $i \in (S_1 - S_2) \cup (S_2 - S_1)$ and $j \in F_c \cup (S_1 \cap S_2)$, then $\omega(\sigma : i, j) = 1$. Other possible test results can be arbitrary.

According to Definition 3, that S_1 and S_2 are all allowable fault sets. Therefore, it cannot be identified that which one is the real fault set which is a contradiction to the hypothesis.

Sufficiency Suppose that, to the contrary, the system is not conditional two-step (k, t)- diagnosable. Thus, there exist a conditional subset $F_c \subseteq V$ with $|F_c| = k$ and 2 conditional subsets $S_1, S_2 \subset V$ with $S_1 \neq S_2$, $|S_1| \leqslant t$, $|S_2| \leqslant t$, and $F_c \cup S_1$ and $F_c \cup S_2$ are conditional subsets, such that $F_c \cup S_1$ and $F_c \cup S_2$ are all allowable fault sets. Noting that there exists an edge from $V - S_1 - S_2 - F_c$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Without loss of generality, let $i \in V - S_1 - S_2 - F_c$, $j \in (S_1 - S_2)$ with $(i, j) \in E$. For a syndrome σ, if $\omega(\sigma : i, j) = 1$, then S_2 is not an allowable fault set, otherwise, S_1 is not an allowable fault set which is a contradiction to the hypothesis. Similarly, $j \in (S_1 - S_2)$ also leads to a contradiction to the hypothesis.

Definition 11 For a conditionally two-step (k, t)- diagnosable system S and a fault set F_c identified by Double-Syndrome diagnostic or other methods, the diagnosability of the conditionally two-step (k, t)- diagnosable systems based on F_c, denoted by $T_c(S, F_c)$, is the maximum number of nodes that are guaranteed to be identified as faulty correctly.

Theorem 7 For a system S given by $G = (V, E)$, if S is conditionally t' -diagnosable (but not conditionally $t' + 1$ -diagnosable) and also conditionally two-step (k, t) -diagnosable (but neither conditionally two-step $(k, t + 1)$ -diagnosable nor conditionally two-step $(k + 1, t)$ -diagnosable) $(t' \geqslant k)$), then its diagnosability satisfies following inequality: $T_c(S, F_c) \geqslant t$ where $F_c \subseteq V$ and $|F_c| = k$.

Proof A similar argument of Theorem 5 can be used.

Lemma 3 Suppose that an undirected graph $G = (V, E)$ denotes a system and that each node in $G = (V, E)$ has at least one fault-free neighbor. For any set $S \subset V$ with $|S| \leqslant 3$, if $N(S)$ are all faulty nodes, then each node of S can be identified correctly.

Proof Let $|S| = m$ and $S = \{v_i : 1 \leqslant i \leqslant m\}$. Now discuss the following cases:

Case 7 $m = 2$.

It is obvious that if $v_1(v_2)$ is faulty, then $N(v_2)(N(v_1))$ is all faulty nodes which is a contradiction to the condition. Therefore, v_1, v_2 are all fault-free.

Case 8 $m = 3$ and S can form a cycle.

There is at most 1 faulty node in S. Otherwise, there are at least 2 faulty nodes in S, without loss of generality, assume that v_1, v_2 are faulty nodes, then $N(v_3)$ are all faulty nodes which contradict the assumption. It is easy to judge the state (faulty or fault-free) of each node by observing the syndrome.

Case 9 $m = 3$ and S cannot form a cycle.

Since each node has at least one fault-free neighbor and $N(S)$ are all faulty nodes, S is connected. The middle node v_2 of S is fault-free, otherwise, $N(v_1)$ and $N(v_3)$ are all faulty nodes, which is a contradiction to the assumption. Furthermore, the other 2 nodes can be identified correctly.

5.1 *n*-dimensional hypercube

Lemma 4 Let $G = (V, E)$ be the graph of a hypercube of n dimension and $X \subseteq V$ with $|X| = k$, $1 \leqslant k \leqslant n+1$, then $N(X) > kn - \frac{k(k+1)}{2} + 1$[22].

Lemma 5 n-dimensional $(n \geqslant 5)$ hypercube is conditional $[4(n-2)+1]$-diagnosable[20].

Next, the n- dimensional $(n \geqslant 5)$ hypercube given by $G = (V, E)$ is not conditional $[4(n-2)+1]$-diagnosable. Let $S = \{v_0, v_1, v_2, v_3\}$ where S can form a cycle and $S_1 = N(S) \cup \{v_0, v_1\}$, $S_2 = N(S) \cup \{v_2, v_3\}$. Note that $|N(S)| = 4(n-2)$ and $S_1 = S_2 = 4(n-2)+2$. For subsets S_1, S_2, there exists no such a node v that $N(v) \subseteq S_1$ or $N(v) \subseteq S_2$. And for subsets S_1, S_2, there exists no edge from $V - S_1 - S_2$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Therefore, the system is not conditional $[4(n-2)+2]$-diagnosable.

Note that if a system is t-diagnosable, then such system must be t/t-diagnosable[23]. Therefore, n-dimensional $(n \geqslant 5)$ hypercube given by $G = (V, E)$ is conditional $(4n-7)/(4n-7)$-diagnosable[24]. Furthermore, the n-dimensional $(n \geqslant 5)$ hypercube given by $G = (V, E)$ is not conditional $(4n-6)/(4n-6)$-diagnosable.

Theorem 8 n-dimensional $(n \geqslant 5)$ hypercube is not conditional $(4n-6)/(4n-6)$-diagnosable.

Proof Let $S = \{v_0, v_1, v_2, v_3\}$ where S can form a cycle and $S_1 = N(S) \cup \{v_0, v_1\}$, $S_2 = N(S) \cup \{v_2, v_3\}$. Note that $|N(S)| = 4(n-2)$ and $S_1 = S_2 = 4(n-2)+2$. Now consider following syndrome σ under the condition that all nodes of $N(S)$ are faulty.

1) The test results from S to $N(S)$ are 1.

2) $\omega(\sigma: v_0, v_1) = 0$, $\omega(\sigma: v_1, v_0) = 0$, $\omega(\sigma: v_2, v_3) = 0$, $\omega(\sigma: v_3, v_2) = 0$, $\omega(\sigma: v_1, v_2) = 1$, $\omega(\sigma: v_2, v_1) = 1$, $\omega(\sigma: v_0, v_3) = 1$, $\omega(\sigma: v_3, v_0) = 1$.

3) The other possible test results are arbitrary.

For above syndrome σ, the system cannot isolate all faulty nodes within a set of size at most $4n-6$. Therefore, the n-dimensional $(n \geqslant 5)$ hypercube is not conditional $(4n-6)/(4n-6)$-diagnosable.

5.2 Permutation star graph

Lemma 6 Let $G = (V, E)$ be the graph of a star graph of $n(n \geqslant 4)$ dimension and $X \subseteq V$ with $|X| = 8$ and X can form an 8-node ring, then $|N(X)| = 8n-24$.

Proof According to the symmetry of star graph, each 8-node ring in $n(n \geqslant 4)$ dimensional star graph is equivalent. Therefore, consider following case as Fig. 2, the 1234A represents n-bit position of v_1 and A is a $(n-4)$-bit position which consists of $5, 6, \cdots, n$. Let $add(v, i, j)$ be the address of node v from number i bit to number j bit. Note that in Fig. 2, $add(v_i, 2, 4) \neq add(v_j, 2, 4)$ where $i, j \in [1, 2 \cdots, 7]$ and $i \neq j$. Therefore, for each node of Fig. 2, it has $(n-4)$ private neighbors. Note that each node of n-dimensional star graph has $(n-1)$ adjacent nodes and for each node of an 8-node ring, for example, v_1 has 2 adjacent nodes in the ring and $n-4$ private neighbors outside the ring. Then the address of the last neighbor of v_1 is 2134A which shows that last neighbor of v_1 is also the private neighbor of v_1. Similarly, the last neighbor of node $v_2(v_3 \cdots v_8)$ is also their private neighbors. Thus, each node of an 8-node ring has $n-3$ private neighbors and let X be an 8-node ring, then $|N(X)| = 8n-24$.

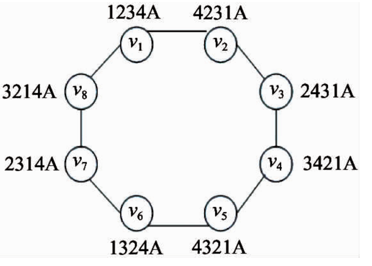

Fig. 2 An 8-node ring of n-dimensional star graph

Lemma 7 In an n-dimensional star graph, there are no odd cycles and there are even cycles with length l where $l \geqslant 6, l \leqslant n$[25].

Lemma 8 In an n-dimensional star graph, let u be a node and let u_1, $u_2 \cdots u_{n-1}$ be $n-1$ neighbors of it[22]. Then every pair u_i, u_j and node u form a loop with 3 other nodes which are unique.

Lemma 9 n-dimensional $(n \geqslant 5)$ star graph given by $G = (V, E)$ is conditional $[8(n-3)+3]$-diagnosable[26]. Secondly, the n-dimensional $(n \geqslant 5)$ star graph given by $G = (V, E)$ is not conditional $(8n-20)$-diagnosable.

Theorem 9 n-dimensional $(n \geqslant 5)$ star graph given by $G = (V, E)$ is not conditional $(8n-20)$-diagnosable.

Proof Let $S = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ where S can form a cycle in the clockwise and $S_1 = N(S) \cup \{v_1, v_2, v_5, v_6\}$, $S_2 = N(S) \cup \{v_3, v_4, v_7, v_8\}$. Note that according to Lemma 9, $|N(S)| = 8n-24$ and $|S_1| = |S_2| = 8n-20$. For subsets S_1, S_2, there exists no such a node v that $N(v) \subseteq S_1$ or $N(v) \subseteq S_1$. And for subsets S_1, S_2, there exists no edge from $V - S_1 - S_2$ to $(S_1 - S_2) \cup (S_2 - S_1)$. Therefore, the system is not conditional $(8n-20)$- diagnosable.

Theorem 10 n-dimensional $(n \geqslant 5)$ star graph given by $G = (V, E)$ is not conditional $(8n-20/8n-$

20)-diagnosable.

Proof Let $S = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ where S can form a cycle in the clockwise and $S_1 = N(S) \cup \{v_1, v_2, v_5, v_6\}$, $S_2 = N(S) \cup \{v_3, v_4, v_7, v_8\}$. Note that according to Lemma 9, $|N(S)| = 8n - 24$ and $|S_1| = |S_2| = 8n - 20$. Now consider following syndrome σ under the condition that all nodes of $N(S)$ are faulty.

1) The test results from S to $N(S)$ are 1.

2) $\omega(\sigma:v_1, v_2) = 0$, $\omega(\sigma:v_2, v_1) = 0$, $\omega(\sigma:v_3, v_4) = 0$, $\omega(\sigma:v_4, v_3) = 0$, $\omega(\sigma:v_5, v_6) = 0$, $\omega(\sigma:v_6, v_5) = 0$, $\omega(\sigma:v_7, v_8) = 0$, $\omega(\sigma:v_8, v_7) = 0$, $\omega(\sigma:v_2, v_3) = 1$, $\omega(\sigma:v_3, v_2) = 1$, $\omega(\sigma:v_4, v_5) = 1$, $\omega(\sigma:v_5, v_4) = 1$, $\omega(\sigma:v_6, v_7) = 1$, $\omega(\sigma:v_7, v_6) = 1$, $\omega(\sigma:v_1, v_8) = 1$, $\omega(\sigma:v_8, v_1) = 1$.

3) The other possible test results are arbitrary.

For above syndrome σ, the system cannot isolate all faulty nodes within a set of size at most $8n - 20$. Therefore, the n-dimensional ($n \geqslant 5$) star graph is not conditional $(8n - 20/8n - 20)$-diagnosable. The following is shown as Algorithm 2 and Algorithm 3.

Algorithm 2 Double-Syndrome conditional diagnosis: part 1

Algorithm: major neighbor:

Require:

A system by undirected graph $G = (V, E)$ with N nodes denoted by $\{u_1, u_2, \cdots, u_N\}$ and a subset $F_c \subseteq V$ with $|F_c| = k$ and $a_i = 0 (1 \leqslant i \leqslant n)$.

Ensure:

A set N_{F_c}.

1) For each node $v_i \in V (0 \leqslant i \leqslant k - 1)$, if $u_j \in N(v_i)$ $(1 \leqslant j \leqslant n)$, then $a_j = a_j + 1$.

2) Let $N_{F_c} = \{u_i \mid a_i \geqslant a_j$ where $1 \leqslant j \leqslant n\}$.

3) Output the set N_{F_c}.

Algorithm: depth-first search:

Require:

A system given by undirected graph $G = (V, E)$ with N nodes and a node $v \in V$. Let $S = \{v\}$.

Ensure:

Several node sets $M_i (1 \leqslant i \leqslant N)$.

1) $DFS(v)$:

For each $u \in N(v)$

If $w(u, v) = w(v, u) = 0$.

$S = S \cup \{u\}$ and $DFS(u)$.

2) Output the nodes set S.

Algorithm: test neighbor:

Require:

A system given by undirected graph $G = (V, E)$ with N nodes and a subset $X \subseteq V$ and three sets T, F_c and M. Let $S = \{v\}$.

Ensure:

The set T, F_c and M.

1) Test (X, T, F_c, M)

For each node of $u \in N(X)$

If the test result from u to $N(X)$ is 0, then $T = T \cup \{u\}$ and test $(\{u\}, T, F_c)$. Otherwise $F_c = F_c \cup \{u\}$ and $M = M - \{u\}$.

2) Output the sets T, F_c and M.

Algorithm: test component:

Require:

A system given by undirected graph $G = (V, E)$ with N nodes and three sets T, F_c and M.

Ensure:

The set T, F_c.

Test component (T, F_c, M):

1) For each node $w \in M$, if there exists a node $x \in M$ such that $N(x) \cap M = \{w\}$, then $T = T \cup \{w\}$ and Test $(\{w\}, T, F_c, M)$.

2) If there exist 3 nodes u, v, $w \in M$ such that the test results of them are all 0, then $T = T \cup \{u, v, w\}$ and Test $(\{u, v, w\}, T, F_c, M)$.

3) If there exist 2 pair adjacent nodes $\{u, v\}$, $\{w, x\} \in M$ such that the test results of u, v (and w, x) are all 0, then $T = T \cup \{u, v, w, x\}$ and test $(\{u, v, w, x\}, T, F_c, M)$.

4) Repeat step 1) to step 3), until $V = T \cup F_c$. Output the set T, F_c.

Algorithm 3 Double-Syndrome conditional diagnosis: part 2

Require:

A system given by undirected graph $G = (V, E)$ with N nodes and a fault node set $F_c \subseteq V$ with $|F_c| = k$ obtained from Double-Syndrome diagnostic or other methods. And a fault bound t (that the system is conditional t-diagnosable).

Ensure:

A faulty node set F_c and a fault-free nodes set T $(T \cup F_c = V)$.

1) N_{F_c} = major neighbor (G, F_c).

2) For each node $u_i \in V - \cup_{j=1}^{i} S_j - F_c$ ($u_i \in N_{F_c}$ is a priority).

Do $DFS(u_i)$.

$S_i = DFS(u_i)$

If $|S_j| \geq t - |F_c| + 1$, where $1 \leq j \leq i$.

$T = T \cup S_j$ and $F_c = F_c \cup N(T)$.

3) If $V = T \cup F_c$, then output the fault-free node set T and faulty nodes set F_c. Otherwise go to step 4).

4) Let $M = V - T - F_c$ and $M = \{C_i \mid C_i$ is a component of $M\}$.

Test component (T, F_c, M).

Output the fault-free nodes set T and the faulty nodes set F_c.

5.3 Algorithms for conditional two-step (k, t)-diagnosable systems

In the following, a diagnosis algorithm is proposed called Double-Syndrome conditional diagnosis (DSCD) which combines Double-Syndrome diagnostic and the theories of conditional two-step (k, t)-diagnosable system.

Consider step 4) in Algorithm 3, the neighbors of the nodes of set M are all faulty. Note that in n-dimensional hypercube, $|M| \leq 4$ with at most one faulty node, in n-dimensional star graph, $|M| \leq 8$ with at most 3 faulty nodes. A similar argument of Lemma 3 can prove the rightness of step 4) in Algorithm 3.

Theorem 11 The algorithm DSCD has a time complexity $O(N\log_2 N)$, where N is the number of the nodes of the system.

Proof In Algorithm 3, step 1) costs $O(kn)$ time. Step 2) costs $O(N\log_2 N) + O(N)$ time. Step 3) and 4) costs $O(1)$ time. Hence the total time is $O(N\log_2 N)$.

Now the performance of the algorithm by computer simulation is shown below. Run the algorithm 1 000 times and the faulty nodes are randomly distributed in the system. Table 2 and Table 3 show the performance of this algorithm applied to n- dimensional hypercubes and star graphs.

Table 2 The number of faulty nodes identified by the algorithm under the n-dimensional hypercube

Dimension	7	8	9	10	11	12
Faulty nodes number	21	25	29	33	37	41
Identified faulty nodes	21	25	29	33	37	41

Table 3 The number of faulty nodes identified by the algorithm under the n- dimensional star graph

Dimension	7	8	9	10	11	12
Faulty nodes number	35	43	51	59	67	75
Identified faulty nodes	35	43	51	59	67	75

6 Conclusions

Under PMC model, a new method is proposed, which is called Double-Syndrome diagnostic to diagnosis the faulty nodes by comparing the 2 syndromes. In general, the average number of faulty nodes which can be identified by Double-Syndrome diagnostic is much larger than other methods. Furthermore, for a given faulty node set F_c, in order to deal with the remaining faulty nodes in the system, two-step (k, t) -diagnosable strategy and two-step (k, t/t) -diagnosable strategy are proposed. For a given t' -diagnosable system, its two-step (k, t) -diagnosability has a minimum value which is equal to t'. Meanwhile, with the purpose of increasing the diagnosability, the concept of conditional two-step (k, t) -diagnosable system and the concept of conditional two-step (k, t/t) -diagnosable system are proposed. Similarly, for a given conditionally t' -diagnosable system, the conditional two-step (k, t/t) -diagnosability has a minimum value which is equal to t'.

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