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Analysis of quaternary digital chaotic sequence performance based on chaotic matrix①

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Abstract

With good randomness and high sensitivity to initial values, chaotic sequences have been extensively used in secure communication. Real chaotic sequences are highly sensitive to initial values. It is an analog quantity in the domain of attraction, which is not conducive to the transmission of digital signals. In order to improve the stability, real chaotic sequences can be quantized into digital chaotic sequences. According to the relationship between the information rate and the symbol rate, the symbol rate of binary sequence is the same as the information rate. The information rate can be doubled by quantizing a real-valued sequence into a quaternary sequence. The chaotic sequence has weak periodicity. Moreover, the periodicity of binary digital chaotic sequences is much weaker than that of quaternary chaotic sequences. Compared with the multi-dimensional chaotic map, the one-dimensional chaotic map has small key space and low security. In this paper, a new real-valued chaotic sequence is generated based on the chaotic matrix method constructed by Logistic map and Kent map. Two quantization methods are used to digitize the real-valued chaotic sequence to obtain the quaternary digital chaotic sequence. Moreover, the randomness, the time series complexity and the correlation of the new quaternary chaotic sequence are compared and studied. The simulation results demonstrate that the quaternary digital chaotic sequence obtained by the chaotic matrix has good randomness and correlation.

Key words: chaotic matrix, digital chaotic sequence, randomness

0 Introduction

The study of chaos theory is considered to be the third major scientific discovery after quantum mechanics and relativity. Chaos is one of the main manifestations of the complex dynamics of non-columnar dynamic mapping. It can also be identified and widely concerned by the researchers in the fields of information security, spread spectrum communication and industrial control[1-3]. A lot of researches have been done on the generation and optimization of chaotic sequences at home and abroad[4-6], but there are still some problems. The real-valued chaotic sequence is an analog quantity in the attraction domain, which is not convenient for the transmission of digital signals. Therefore, it is necessary to quantify the real-valued chaotic sequence. Commonly used methods include the gate limit method[7], bit extraction quantization method[8] and median quantization method[9].

For sequences generated by a single chaotic map, non-columnar inverse method can be used to quickly estimate chaotic parameters, which has potential safety hazards. A sequence construction method in combination of combined mapping is proposed, which increased the complexity and security of chaotic sequences[10-12]. And anti-attack ability has the randomness, balance and correlation comparable to the original chaotic model sequence. The encryption algorithm proposed in Ref. [13] is based on chaos theory and DNA sequence operations. Three different fractional logistic mappings are selected, corresponding to three different chaotic sequences. Through experimental testing and security analysis, the algorithm can achieve good encryption results, has a larger key space and a higher key sensitivity, and has a higher resistance to attack.

For the combination of Logistic and Kent mapping, this paper studies the chaotic matrix, proposes a new way to obtain the combined chaotic sequence, and studies the confidentiality, balance and non-periodicity of different quantization methods. The experimental results show that the chaotic sequence of the last row of

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the combined mapping of Logistic and Kent has good confidentiality, balance and non-periodicity.

1 Quaternary digital chaotic sequence

1.1 Logistic and Kent mapping equations

Logistic is a simple and widely used dynamic system, and the mapping equation is

$$x_{n+1} = \mu x_n(1 - x_n) \tag{1}$$

where, x_n is the initial value of the Logistic mapping equation, x_{n+1} is the value obtained by the Logistic Eq. (1) after parameter μ transformation. x_n, $x_{n+1} \in (0, 1)$ and control parameter $\mu \in (0, 4]$.

Kent is another discrete chaotic mapping equation, and the mapping equation is

$$k_{n+1} = \begin{cases} \dfrac{k_n}{a} & 0 < x_n \leqslant a \\ \dfrac{(1 - k_n)}{(1 - a)} & a < x_n < 1 \end{cases} \tag{2}$$

where, k_n is the initial value of the Kent mapping equation, k_{n+1} is the value obtained by the Kent mapping Eq. (2) after parameter a transformation. k_n, $k_{n+1} \in (0, 1)$ and control parameter $a \in (0, 4)$.

1.2 Chaotic matrix

Traditional chaotic sequences have certain limitations, such as low sequence complexity and simple mapping equations, therefore, improving chaotic sequences performance has become a research hotspot in recent years. Ref. [14] used two Logistic chaotic maps with different initial values to generate composite chaotic sequences, and effectively expanded the initial space, making the generated sequences more complex and unpredictable. It proposed a secure data transmission scheme based on chaotic compressed sensing and designed a Bernoulli chaotic sensing matrix, which was proved to have good anti-noise capability and anti-attack capability[15]. In order to obtain sequences with better random performance, this paper implements multi-dimensional mapping based on the chaotic matrix idea. First, the real-valued chaotic row vector $(x_{11}, x_{12}, x_{13}, \cdots, x_{1n})$ of length N is generated by Logistic mapping equation, and each column of logistic real-valued chaotic sequences is conducted with Kent mapping. Each column is a new compound real-valued chaotic sequence, and chaotic matrix X is generated by Eq. (3).

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \tag{3}$$

In order to eliminate the chaotic transient effect, the composite real-valued chaotic sequence of the Nth column $m_{n1} = (x_{1n}, x_{2n}, x_{3n}, \cdots, x_{mn})$ can be taken. Meanwhile, the last value of each sequence in the last row can be conducted with two chaotic mappings to generate a new chaotic real-valued sequence $m_{n2} = (x_{m1}, x_{m2}, x_{m3}, \cdots, x_{mn})$. Different chaotic real-valued sequences obtained by these two methods are compared and analyzed.

1.3 Digital chaotic sequences

Since the chaotic sequences generated by chaotic mapping are real-valued sequences, they should be converted into digital chaotic sequences in practice while maintaining good pseudo-randomness of the generated digital chaotic sequences. In this paper, the two sequences are quantified in two ways for further analysis.

First, the N-nary digital chaotic sequence is obtained by N-nary of the sequence through probability density.

The probability density function $\rho 1(x)$ of Logistic at $\mu = 4$ full mapping is calculated as

$$\rho 1(x) = \begin{cases} \dfrac{1}{\pi \sqrt{x(1 - x)}} & 0 < x < 1 \\ 0 & \text{others} \end{cases} \tag{4}$$

where, x is the real value in the Logistic sequence.

The probability density function $\rho 2(x)$ of Kent mapping equation at parameter $a = 0.4$ obeys a uniform distribution within $(0,1)$:

$$\rho 2(x) = 1 \tag{5}$$

After the second Kent mapping of the real-valued chaotic sequence matrix is obtained by Eqs (1) and (2), the sequence obtained by each column can realize N-nary by setting the dynamic sub-intervals $(0, a]$, $(a, b]$, $(b, c]$, $\cdots$, $(k, 1]$ when the parameter values required by logistic and Kent are satisfied. $(0, a]$, $(a, b]$, $(b, c]$, $\cdots$, $(k, 1]$ are N sub-intervals, namely Eqs (6) and (7).

$$\int_0^a \rho 1_1(x)\mathrm{d}x = \int_a^b \rho 1_2(x)\mathrm{d}x = \int_b^c \rho 1_3(x)\mathrm{d}x = \cdots = \int_k^1 \rho 1_n(x)dx \tag{6}$$

$$\rho 2_1(x) = \rho 2_2(x) = \rho 2_3(x) = \rho 2_4(x) = \cdots = \rho 2_n(x) = \frac{1}{n} \tag{7}$$

Through Eq. (3), m_{n1}, as the real-valued chaotic sequence in the last column of the chaotic matrix, is the second Kent mapping with the value of the first Logistic mapping as the initial value, and m_{n2} as new real-valued chaotic sequence is obtained from the last value of each secondary mapping sequence. The N-nary method based on probability density is not applicable for m_{n2}. Therefore, the binary digital chaotic sequence m_{n2} is obtained through uniform quantization by Eq. (8), and it is converted into an N-nary digital chaotic sequence after binary conversion.

$$m_{n2} = \begin{cases} 0 & 0 < m_{n2} < 0.5 \\ 1 & 0.5 \leqslant m_{n2} < 1 \end{cases} \tag{8}$$

2 Sequence complexity

2.1 Approximate entropy

Approximate entropy (ApEn) is a noncolumnar dynamic parameter that can quantify the regularity and unpredictability of time series fluctuation. A non-negative number represents the complexity of a time sequence, reflecting the possibility of new information in time sequence. Complex time series corresponds to large approximate entropy.

There is an N-dimensional time sequence $u(1)$, $u(2), \cdots, u(N)$. m-dimensional vector $\boldsymbol{X}(1)$, $\boldsymbol{X}(2)$, $\cdots$, $\boldsymbol{X}(N-m+1)$ is constructed.

$$\boldsymbol{X}(i) = [u(i), u(i+1), \cdots, u(i+m-1)] \tag{9}$$

where, m is an integer representing the length of the comparison vector, and r is a real number representing the measured value of 'similarity'.

For $1 \leqslant i \leqslant N+m-1$, the number of vectors satisfying Eq. (10) is as follows.

$$C_i^m(r) = \frac{(number\ of\ \boldsymbol{X}(j)\ such\ that\ d[\boldsymbol{X}(i), \boldsymbol{X}(j)] \leqslant r])}{(N-m+1)} \tag{10}$$

where

$$d[\boldsymbol{X}, \boldsymbol{X}^*] = \max | u(a) - u^*(a) | \tag{11}$$

$u(a)$ is an element of vector $\boldsymbol{X}$, d is the distance between vector $\boldsymbol{X}(i)$ and $\boldsymbol{X}(j)$, which is determined by the maximum difference of the corresponding elements. The value of j is in the range of $[1, N-m+1]$, including $i = j$.

Define

$$\phi^m(r) = (N-m+1)^{-1} \sum_{i=1}^{N-m+1} \log(C_i^m(r)) \tag{12}$$

The approximate entropy is defined as

$$ApEn = \phi^m(r) - \phi^{m+1}(r) \tag{13}$$

2.2 Sample entropy

Sample entropy (SampEn) is an improved method measuring the complexity of time sequence based on approximate entropy.

There is an N-dimensional time sequence $u(1)$, $u(2), \cdots, u(N)$. m-dimensional vector $\boldsymbol{X}(1)$, $\boldsymbol{X}(2)$, $\cdots$, $\boldsymbol{X}(N-m+1)$ is reconstructed as

$$\boldsymbol{X}(i) = [u(i), u(i+1), \cdots, u(i+m-1)] \tag{14}$$

where, m is an integer representing the length of the comparison vector, and r is a real number representing the measured value of 'similarity'.

For $1 \leqslant i \leqslant N+m-1$, the number of vectors satisfying Eq. (15) is

$$B_i^m(r) = \frac{(number\ of\ \boldsymbol{X}(j)\ such\ that\ d[\boldsymbol{X}(i), \boldsymbol{X}(j)] \leqslant r])}{(N-m)} \tag{15}$$

where, $i \neq j$ and

$$d[\boldsymbol{X}, \boldsymbol{X}^*] = \max | u(a) - u^*(a) |, \boldsymbol{X} \neq \boldsymbol{X}^* \tag{16}$$

$u(a)$ is an element of vector $\boldsymbol{X}$, d is the distance between $\boldsymbol{X}(i)$ and $\boldsymbol{X}(j)$, which is determined by the maximum difference between the corresponding elements. The value of j is in the range of $[1, N-m+1]$, but $i \neq j$.

$$B^m(r) = (N-m+1)^{-1} \sum_{i=1}^{(N-m+1)} B_i^m(r) \tag{17}$$

Assume $k = m+1$, and repeat Eqs (14) and (15) to obtain Eq. (18).

$$A^k(r) = (N-k+1)^{-1} \sum_{i=1}^{(N-k+1)} A_i^k(r) \tag{18}$$

where

$$A_i^m(r) = \frac{(number\ of\ \boldsymbol{X}(j)\ such\ that\ d[\boldsymbol{X}(i), \boldsymbol{X}(j)] \leqslant r])}{(N-k)} \tag{19}$$

where $i \neq j$.

The sample entropy is defined as Eq. (20).

$$sampEn = \lim_{N\to\infty} - \ln[A^k(r)/B^m(r)] \tag{20}$$

3 Simulation and analysis

In the experiment, the chaotic matrices of Logistic and Kent, Logistic and Logistic, Kent and Kent combination mapping are compared.

In order to better analyze randomness, weak periodicity and sequence complexity, through Logistic mapping at $\mu = 4$, the initial value is 0.6, and the Kent mapping parameter is $a = 0.4$. Based on the last row or the last column of Eq. (3), N is set to 4. In the case of full mapping, calculated by Eq. (6), the values of a, b, and c are 0.1464, 0.5 and 0.8536 respectively. For the parameters in ApEn and SampEn, m is set to 2, and r is taken as 0.1 times the standard deviation of the sequence. The performance of quaternary digital chaotic sequences with different quantization methods is compared.

3.1 Analysis of the performance of the last column in the chaotic matrix (quantified by probability density)

A 10 000 × 10 000 chaotic matrix is generated by Eq. (3), and the last column $m_{n1} = (\boldsymbol{x}_{1n}, \boldsymbol{x}_{2n}, \boldsymbol{x}_{3n}, \cdots, \boldsymbol{x}_{mn})$ is quantified into quaternary digital chaotic sequence according to probability density to simulate sample entropy, approximate entropy, weak periodicity and frequency.

First, the last column of the chaotic matrix formed by Logistic and Kent mapping is quantized to quaternary according to the probability density for further simulation.

Table 1 shows the approximate entropy and sample entropy of the last column in the chaotic matrix (quantified by probability density) when the length of the quaternary digital chaotic sequence is 10 000.

Table 1 Approximate entropy and sample entropy of the last column through probability density quantization

	Logistic + Kent (probability density quantification)	Logistic + Logistic (probability density quantification)	Kent + Kent (probability density quantification)
ApEn	0.6972	0.6933	0.6948
SampEn	0.6256	0.6932	0.6243

Fig. 1 describes the chaotic matrices formed by Logistic mapping and Kent mapping. The chaotic matrices formed by Logistic and Logistic mapping and Kent and Kent mapping are the simulation diagrams with the number and weight of sub-blocks. The quantification subinterval is determined by the probability density. From the simulation data, it can be concluded that the chaotic matrix formed by Logistic and Kent mapping has stable block weight when there are 14 sub-blocks. The block weight of the chaotic matrix formed by Logistic and Logistic mapping is stable when there are 12 sub-blocks. The chaotic matrix formed by Kent and Kent mapping has stable block weight in the case of 13 sub-blocks. Meanwhile, the last column of the chaotic matrix is quadrupled according to the probability density. The approximate entropy and the sample entropy of the three combinations are basically the same.

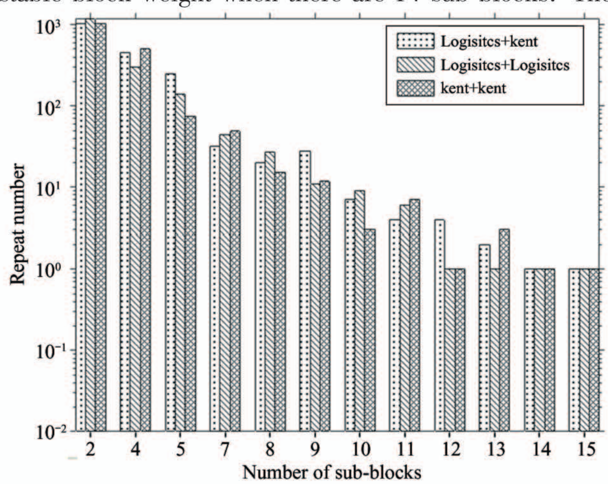


Fig. 1 Chaotic matrix last column sequence (probability density quantification)

3.2 Performance analysis of the last column of the chaotic matrix (binary to quaternary)

A 20 000 × 20 000 chaotic matrix is generated by Eq. (3), and the last column $m_{n1} = (\boldsymbol{x}_{1n}, \boldsymbol{x}_{2n}, \boldsymbol{x}_{3n}, \cdots, \boldsymbol{x}_{mn})$ is quantized into a binary digital chaotic sequence according to Eq. (8). The binary digital chaotic sequence is converted into a quaternary digital chaotic sequence to simulate sample entropy, approximate entropy, weak periodicity and frequency. The length of the quaternary sequence is 10 000.

Table 2 shows the approximate entropy and the sample entropy of the last column (taking two bits into binary to quaternary) of the chaotic matrix mapped by three combinations when the length of the quaternary digital chaotic sequence is 10 000.

Table 2 Approximate entropy and sample entropy of the last column through probability density quantization

	Logistic + Kent (take two bits into binary to quaternary)	Logistic + Logistic (take two bits into binary to quaternary)	Kent + Kent (take two bits into binary to quaternary)
ApEn	1.1271	1.3842	1.2690
SampEn	1.1015	1.3868	1.1049

Fig. 2 shows the simulation diagrams of the chaotic matrix formed by Logistic + Kent mapping, the chaotic matrix formed by Logistic + Logistic mapping, the chaotic matrix formed by Kent + Kent mapping taking the number of subblocks and the complex number of block weight. The two-bit binary number of the sequence in the last column is converted to a one-bit quaternary number. The simulation data demonstrate that the chaotic matrix formed by Logistic and Kent mapping has stable block weight when there are 8 sub-blocks, the block weight is stable when there are 9 sub-blocks in the chaotic matrix formed by Logistic and Logistic mapping, and the block weight of the chaotic matrix formed by Kent and Kent mapping is stable when there are 9 sub-blocks. The last column of the chaotic matrix is taken from a two-bit binary number into a one-bit quaternary number for quaternary. The sequence obtained by the chaotic matrix of Logistic and Logistic mapping is better than the other two mapping combinations, and the quaternary sequence from two-bit binary into one-bit quaternary is better than that of probability density quaternization.

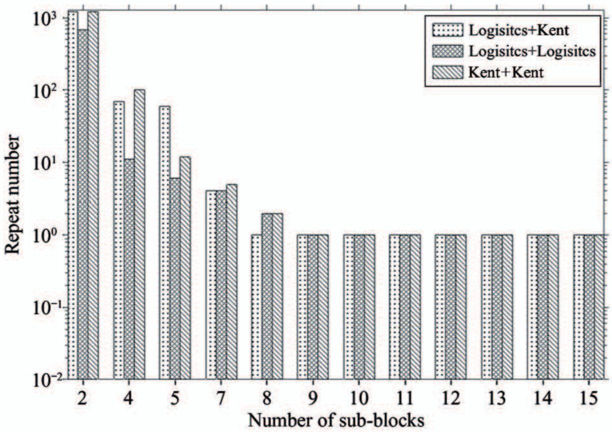


Fig. 2 Chaotic matrix last column sequence (take two bits into binary to quaternary)

3.3 Performance analysis of the last column of the chaotic matrix (taking two-bit binary into quaternary)

A 20 000 × 20 000 chaotic matrix is generated by Eq. (3), and the last row is $m_{n2} = (\boldsymbol{x}_{m1}, \boldsymbol{x}_{m2}, \boldsymbol{x}_{m3}, \cdots, \boldsymbol{x}_{mn})$. First, it is quantized into a binary digital chaotic sequence according to Eq. (8). Second, the binary digital chaotic sequence is converted into a quaternary digital chaotic sequence by two bits to simulate sample entropy, approximate entropy, weak periodicity and frequency. The quaternary chaotic sequence is 10 000-bit.

Table 3 shows the approximate entropy and the sample entropy of the last row (two-bit binary to one-bit quaternary) of the chaotic matrix mapped by the three combinations when the length of the quaternary digital chaotic sequence is 10 000.

Table 3 Approximate entropy and sample entropy of the last row after probability density quantization

	Logistic + Kent (take two bits into binary to quaternary)	Logistic + Logistic (take two bits into binary to quaternary)	Kent + Kent (take two bits into binary to quaternary)
ApEn	1.3843	1.3842	1.1862
SampEn	1.3871	1.3868	1.0756

Fig. 3 presents the simulation diagrams of the chaotic matrices formed by Logistic + Kent mapping, Logistic + Logistic mapping and Kent + Kent mapping by taking the number of sub-blocks and the complex number of block weight. The two-bit binary number is converted into a one-digit quaternary number. From the simulation data, it can be found that the chaotic matrix formed by Logistic + Kent mapping has stable block weight when there are 7 sub-blocks, the block weight is stable when there are 8 sub-blocks in the chaotic matrix formed by Logistic + Logistic mapping, and the block weight of the chaotic matrix formed by Kent + Kent mapping is stable when there are 7 sub-blocks. The last column of the chaotic matrix is taken from a two-bit binary number into a one-bit quaternary number for quaternary. The sequence obtained by the chaotic matrix of Logistic + Kent mapping is better than the other two mapping combinations, and its performance is the best among the sequences obtained by three quantification methods and different methods of taking chaotic matrix.

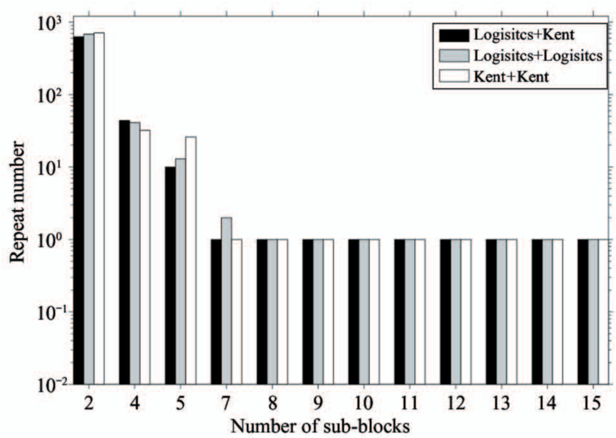


Fig. 3 Chaotic matrix last row sequence (take two bits into binary to quaternary)

Since the new sequence generated by the last bit value of every sequence in the last column of Eq. (3) does not conform to the probability quantization rule, the performance of the last column quantized by probability density is not discussed here.

3.4 Frequency test

The test examines the proportions of '0', '1', '2', and '3' in the entire sequence. Here, the above combination mapping methods and quantization method obtain chaotic matrix in different ways, and the proportion of each frequency is tested.

It can be seen from Tables 4 and 5 that the last row or the last column of the chaotic matrix mapping formed by Kent + Kent mapping has poor frequency, but the frequency of the last column of the chaotic matrix formed by Logistic + Kent mapping is much better than that formed by Kent + Kent mapping. The probability of each code is similar and basically stable.

Table 4 Frequency test (two-bit binary to one-bit quaternary)

	Frequency of 0	Frequency of 1	Frequency of 2	Frequency of 3
Logistic + Logistic (take chaotic matrix with another initial value)	2476	2525	2480	2519
Logistic + Logistic (chaotic matrix, sequence formed by the last value of each initial value)	2476	2525	2480	2519
Kent + Kent (take chaotic matrix with another initial values)	2019	3038	2979	1964
Kent + Kent (chaotic matrix, sequence formed by the last value of each initial value)	4715	2292	2310	683
Logistic + Kent (take chaotic matrix with another initial value)	2041	3078	2892	1989
Logistic + Kent (chaotic matrix, sequence formed by the last value of each initial value)	2497	2530	2488	2485

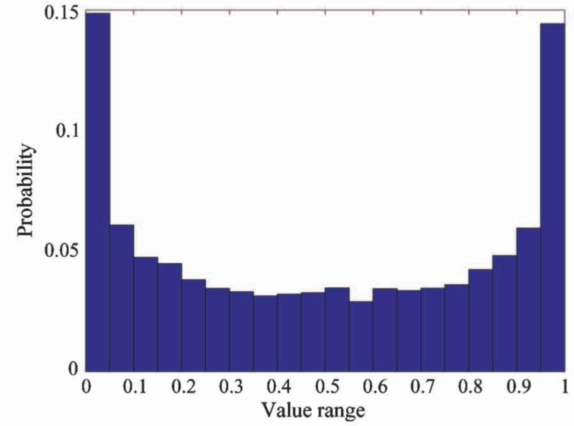
Table 5 Frequency test for 4 quantization according to probability density

	Frequency of 0	Frequency of 1	Frequency of 2	Frequency of 3
Logistic + Logistic (take chaotic matrix with another initial values)	2451	2515	2520	2514
Kent + Kent (take chaotic matrix with another initial values)	2437	2525	2534	2504
Logistic + Kent (take chaotic matrix with another initial values)	2599	2453	2483	2465

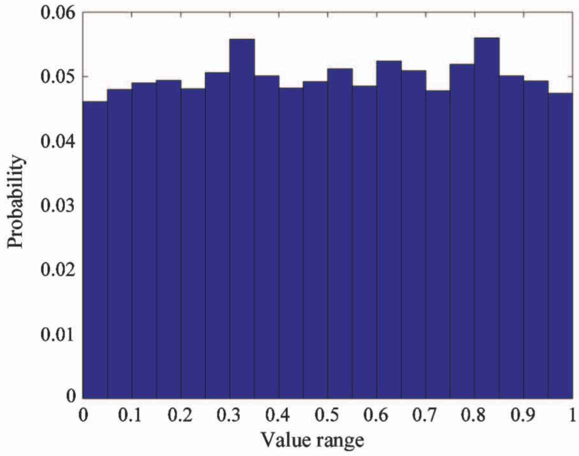
Fig. 4 shows the probability density of the one-dimensional Logistic mapping, one-dimensional Kent mapping, and new sequence after transformation according to Eq. (3). It can be seen from the figure that the one-dimensional Kent mapping and the new sequence after the transformation by Eq. (3) is more uniform than the one-dimensional Logistic mapping distribution. Table 3 shows that the new sequence by the Eq. (3) has uniform distribution characteristics under full mapping conditions, and higher entropy and sample entropy, indicating that the sequence has a higher degree of irregularity in determining the time series, and the complexity is higher, which can make the system more secure.

4 Conclusions

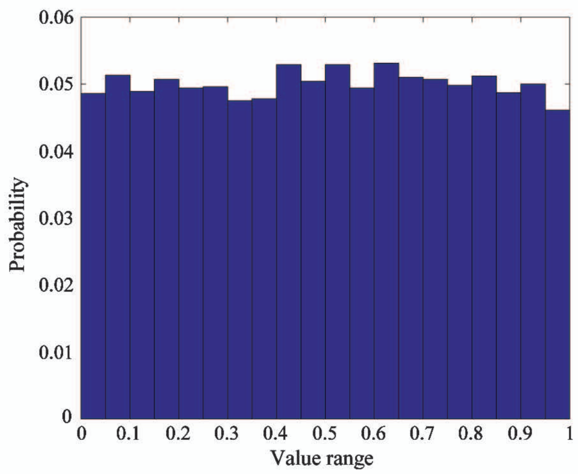
Based on the chaotic matrix formed by different mapping combinations, the performance of the nine chaotic sequences is verified by different quantization methods and different methods of taking chaotic matrix sequences. The simulation results demonstrate that the sequence in the last row of the chaotic matrix formed by Logistic mapping and Kent mapping is first binary, and a quaternary sequence is obtained by converting from two-bit binary to one-bit quaternary. By comparing the approximate entropy, sample entropy, block repetition, namely, weak periodicity and frequency test, performance of quaternary sequence is superior to that obtained by the other eight quantization methods and different methods of taking chaotic matrix. The characteristics of the quaternary digital chaotic sequence are verified. The multi-user communication only needs to slightly change the initial condition of generating the address code, so that the users can have different address codes, which provides a large number of address codes.



(a) Logistic probability density



(b) Kent probability density



(c) Logistic + Kent probability density

Fig. 4 Probability density of three sequences

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