doi:10.3772/j.issn.1006-6748.2021.03.003

Analysis of asymptotically tight approximation SER for cooperative NOMA systems[①]

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Abstract

The closed-form formula derivation of the power domain cooperative non-orthogonal multiple access (NOMA) system is of great significance for further improving the performance of the system. However, the system performance formulas of the channel capacity and the paired bit error rate pairwise error probability (PEP) are too complicated, which have increased the difficulty in system performance optimization. Therefore, based on the amplify forward (AF) relay cooperative NOMA model, the signal interference noise ratio (SINR) formulas of the two user nodes are constructed. Through the assumption of that, the symbol error rate (SER) of each user is fair, the simplification condition of moment generating function (MGF) with the harmonic mean form is satisfied. Combined with the SER calculation formula of MGF, the system SER asymptotically tight approximation formula with simple structure is derived at high signal-to-noise ratio (SNR). The Monte Carlo simulation results show that, the formula can accurately describe the SER performance of the power domain cooperative NOMA system with the non-ideal successive interference cancellation (SIC) system when SNR is high. Under the condition of certain total power, the optimal power allocation factor is solved in order to minimize the total system SER.

Key words: non-orthogonal multiple access (NOMA), amplify forward (AF), symbol error rate (SER), moment generating function (MGF)

0 Introduction

With the rapid increase in demand for high-speed data services, further improvement of spectrum efficiency has become a research hotspot in the field of communications. As a new type of multiple access technology, non-orthogonal multiple access (NOMA) can greatly improve the spectrum efficiency of the system. To this end, NOMA has become one of the key technologies in 5G[1]. In the Internet of Things (IoT) scenario, the limitation of low power consumption requires large-scale user nodes to adopt uplink access, and has higher requirements on the reliability and rate of data transmission, so it is practical to study the uplink communication of NOMA to guarantee the quality of service for user nodes[2]. Symbol error rate (SER) fairness is an important requirement to ensure the communication reliability of communication nodes in the IoT scenario, and the goal of improving the system performance under the premise of ensuring SER fairness of each communication nodes should be realized. User nodes are on the edge of the cell. Cooperative diversity technology is one of the effective means to significantly improve the quality of service of user nodes[3]. In NOMA system, continuous self-interference cancellation technology makes strong users have prior information of other user data, so basically strong users can be used as relay nodes. Therefore, the performance of the system can be improved by cooperative NOMA system.

In order to compare the influence of power allocation on NOMA system and orthogonal multiple access (OMA) system, the relationship between the power allocation coefficients of two user nodes when the two-user nodes NOMA system is superior to the OMA system is studied from the perspective of system ergodic sum rate and outage probability in Ref. [4]. Ref. [5] studied the problem of minimizing the total power of user quality of service limitation in the multi-cell NOMA systems. A distributed power control algorithm was proposed by using the characteristics of Perron-Frobenius eigenvalues and standard interference functions in Ref. [6]. In addition, considering the application of NOMA in the uplink IoT, the influence of circuit ener-

① Supported by the National Natural Science Foundation of China (No. 62001001).
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Received on July 27, 2020

gy consumption limitation on the frequency efficiency and energy efficiency of NOMA system was studied in Ref. [7]. In order to make further use of the potential gain of NOMA in power domain, the fair rate of system user nodes with total throughput limit and total transmission power limit were studied in Refs [8,9]. In Ref. [10], the bit error rate (BER) formula using orthogonal phase shift keying in uplink NOMA system was derived, and the accuracy of the expression is verified by simulation. Different from the previous research, the pairwise error probability (PEP) performance of traditional NOMA systems was studied in Ref. [11]. The closed-form of PEP for different users is derived, and the expressions are simplified in the case of high signal-to-noise ratio (SNR). On this basis, the overall BER performance of the system is improved by power optimization. At present, the research on cooperative NOMA system is mainly in channel capacity analysis. The research on BER performance analysis is insufficient, the analysis and optimization process of existing user quality of service is more complex, which makes it difficult to apply in practice. However, in the actual IoT scenario, BER fairness index is more important. Therefore, the SER formula with simple structure is convenient to improve the quality of service of user nodes through low complexity power optimization. The research in this paper provides theoretical support and implementation means for the practical application of user nodes quality of service improvement scheme in uplink NOMA system.

In summary, for the problem of system performance analysis and optimization of the uplink coordinated NOMA system, under the condition of high SNR and user node fairness, the simplified idea of the harmonic mean form is used to derive the moment generating function (MGF). The SINR formulas of two user nodes, which satisfy the harmonic mean form, are constructed. Then the asymptotically tight approximation SER formula with simple structure is obtained. In order to minimize the tight approximation formula of the SER, the optimal power allocation relationship between the total power of two user nodes and the amplify forward (AF) power of the relay node is determined under the condition of a certain total power.

1 System model

In this paper, the uplink NOMA communication system is composed of two user nodes, one relay node and the base station (BS) at the center of the cell. In the system model, the two user nodes are weak user nodes at the edge of the cell, and the relay nodes are strong user nodes close to the central BS of the cell, and both of them are single antenna nodes. The two user nodes share the same time domain and frequency domain resources at the transmitter. The single antenna relay node acts as the half duplex relay node (R, relay) to AF the linear superposition signal sent by the two user nodes.

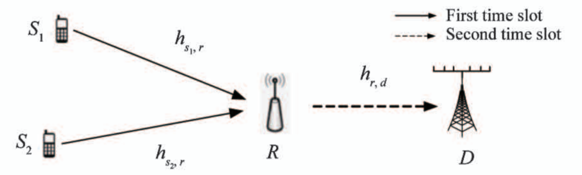

Fig. 1 The AF uplink cooperative NOMA communication model

In the first time slot, the relay node R receives the signal sent by the user group as

$$y_r = x_r + n_{s,r} \tag{1}$$

where $n_{s,r}$ denotes the Gaussian white noise whose mean is zero and variance is N_0. The linear superposition signals x_r are sent by the two user nodes.

$$x_r = \sqrt{P_1}(\sqrt{\alpha_1}h_{s_1,r}x_1 + \sqrt{\alpha_2}h_{s_2,r}x_2) \tag{2}$$

In this formula, the power normalized signal x_i denotes sent by the i-th user node, $E[|x_i|^2] = 1$. $h_{s_1,r}$ and $h_{s_2,r}$ denote the channel coefficients between the two user node s_1 and s_2 to relay node R respectively. The variance is $\delta^2_{s_1,r}$ and $\delta^2_{s_2,r}$ respectively. The total power of two user nodes is P_1. α_1 and α_2 denote the power allocation coefficients of two user nodes respectively, which are subject to $\alpha_1 + \alpha_2 = 1$.

Before the signal at the relay node is forwarded, a linear noise reduction process is performed on the signal. The noise reduction factor is

$$\frac{1}{\sqrt{P_1(\alpha_1|h_{s_1,r}|^2 + \alpha_2|h_{s_2,r}|^2) + N_0}} \tag{3}$$

The signal amplified by the relay node is

$$\frac{\sqrt{P_2}}{\sqrt{P_1(\alpha_1|h_{s_1,r}|^2 + \alpha_2|h_{s_2,r}|^2) + N_0}}y_r \tag{4}$$

where P_2 is the transmission power of the relay node R.

In the second time slot, the destination node D receives the superimposed signal from the relay node R as

$$\begin{aligned} y_d &= \frac{\sqrt{P_2}}{\sqrt{P_1(\alpha_1|h_{s_1,r}|^2 + \alpha_2|h_{s_2,r}|^2) + N_0}}h_{r,d}y_r + n_{r,d} \\ &= \frac{\sqrt{P_1P_2\alpha_1}}{\sqrt{P_1(\alpha_1|h_{s_1,r}|^2 + \alpha_2|h_{s_2,r}|^2) + N_0}}h_{s_1,r}h_{r,d}s_1 \\ &\quad + \frac{\sqrt{P_1P_2\alpha_2}}{\sqrt{P_1(\alpha_1|h_{s_1,r}|^2 + \alpha_2|h_{s_2,r}|^2) + N_0}}h_{s_2,r}h_{r,d}s_2 \\ &\quad + \tilde{n}_{r,d} \end{aligned} \tag{5}$$

where $h_{r,d}$ is the channel coefficient from relay node R to destination node D, and the variance is $\delta^2_{r,d}$. The mean value of the Gaussian white noise $n_{r,d}$ is zero and the variance is N_0.

$$\tilde{n}_{r,d} = \frac{\sqrt{P_2}}{\sqrt{P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0}} h_{r,d} n_{s,r} + n_{r,d} \quad (6)$$

Assuming that the $n_{s_i,r}$ and $n_{r,d}$ are independent noise variables, the mean value of the noise $\tilde{n}_{r,d}$ is zero and the variance is

$$\left(\frac{P_2 |h_{r,d}|^2}{P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0} + 1\right) N_0 \quad (7)$$

The decoding process at the destination node D is performed as follows: the destination node D uses the non-ideal successive interference cancellation (SIC) method to decode the received signal of two user nodes. For the sake of generality, suppose that $|h_{s_1,r}|^2 > |h_{s_2,r}|^2$. Since the decoding order in the uplink NOMA system is sequentially performed in descending order of channel quality, the first signal is detected as s_1 and the signal s_2 is treated as noise. The SINR of the signal s_1 is Eq. (8).

$$SINR_{s_1} = \frac{\dfrac{P_1 P_2 \alpha_1 |h_{s_1,r}|^2 |h_{r,d}|^2}{P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0}}{\dfrac{P_1 P_2 \alpha_2 |h_{s_2,r}|^2 |h_{r,d}|^2}{P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0} + \left(\dfrac{P_2 |h_{r,d}|^2}{P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0} + 1\right) N_0}$$
$$= \frac{P_1 P_2 \alpha_1 |h_{s_1,r}|^2 |h_{r,d}|^2}{P_1 P_2 \alpha_2 |h_{s_2,r}|^2 |h_{r,d}|^2 + P_2 |h_{r,d}|^2 + [P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0] N_0} \quad (8)$$

Assuming that the signal s_1 is completely decoded and can be completely removed in it, the SINR of the signal s_2 is

$$SINR_{s_2} = \frac{\dfrac{P_1 P_2 \alpha_2 |h_{s_2,r}|^2 |h_{r,d}|^2}{P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0}}{\left(\dfrac{P_2 |h_{r,d}|^2}{P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0} + 1\right) N_0}$$
$$= \frac{1}{N_0} \frac{P_1 P_2 \alpha_2 |h_{s_2,r}|^2 |h_{r,d}|^2}{P_2 |h_{r,d}|^2 + P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2) + N_0} \quad (9)$$

2 Theoretical analysis of SER performance

2.1 MGF simplification theorem for harmonic mean form

Theorem The hypothesis of X_1 and X_2 is two independent exponential random variables with the parameters β_1 and β_2[12]. Then, the MGF of $Z = \frac{X_1 X_2}{X_1 + X_2}$ is

$$M_z(s) = \frac{(\beta_1 - \beta_2)^2 + (\beta_1 + \beta_2)s}{\Delta^2} + \frac{2\beta_1\beta_2 s}{\Delta^3} \ln \frac{(\beta_1 + \beta_2 + s + \Delta)^2}{4\beta_1\beta_2} \quad (10)$$

where $s > 0$, and $\Delta = \sqrt{(\beta_1 - \beta_2)^2 + 2(\beta_1 + \beta_2)s + s^2}$.

Based on the further simplification of the closed boundary formula of SER, when β_1 and β_2 tends to zero, $\Delta = s$. The Eq. (10) can be simplified as

$$M_z(s) \approx \frac{\beta_1 + \beta_2}{s} + \frac{2\beta_1\beta_2}{s^2} \ln \frac{s^2}{\beta_1\beta_2} \quad (11)$$

Note that in Eq. (11), the second term on the right side of the equation tends to zero faster than the first term. Therefore, the MGF in Eq. (11) can be simplified as follows.

$$M_z(s) \approx \frac{\beta_1 + \beta_2}{s} \quad (12)$$

According to Ref. [13], assuming that the user node signal adopts M-PSK modulation mode, in this formula, $s = b_{psk}/\sin^2\theta$, $b_{psk} = \sin^2(\pi/M)$.

2.2 SINR harmonic form construction

According to the above-mentioned theorem, the SINR of the two user nodes is constructed in the form of the harmonic mean value of two independent random variables, and the formula of the closed boundary of the SER is derived through the calculation formula of the SER of the MGF.

Under the condition of high SNR, Eq. (8) can be approximated as follows.

$$\frac{\dfrac{P_1 P_2 \alpha_1 |h_{s_1,r}|^2 |h_{r,d}|^2}{N_0}}{\dfrac{P_1 P_2 \alpha_2 |h_{s_2,r}|^2 |h_{r,d}|^2}{N_0} + \dfrac{P_2 |h_{r,d}|^2}{N_0} + P_1(\alpha_1 |h_{s_1,r}|^2 + \alpha_2 |h_{s_2,r}|^2)} \quad (13)$$

Under the condition of high SNR, Eq. (9) can be approximated as

$$\frac{1}{N_0}\frac{P_1P_2\alpha_2 \mid h_{s_2,r}\mid^2 \mid h_{r,d}\mid^2}{P_2 \mid h_{r,d}\mid^2 + P_1(\alpha_1 \mid h_{s_1,r}\mid^2 + \alpha_2 \mid h_{s_2,r}\mid^2)} \quad (14)$$

To guarantee the SER fairness between the two user nodes and to facilitate the derivation of the formula, the following is supposed.

$$\alpha_1 \mid h_{s_1,r}\mid^2 \approx \alpha_2 \mid h_{s_2,r}\mid^2 \quad (15)$$

The upper boundary of Eq. (13) is

$$\frac{P_1P_2\alpha_2 \mid h_{s_2,r}\mid^2 \mid h_{r,d}\mid^2}{P_2 \mid h_{r,d}\mid^2 + 2P_1\alpha_2 \mid h_{s_2,r}\mid^2} = \frac{1}{2}\frac{2P_1\alpha_2 \mid h_{s_2,r}\mid^2 \times P_2 \mid h_{r,d}\mid^2}{2P_1\alpha_2 \mid h_{s_2,r}\mid^2 + P_2 \mid h_{r,d}\mid^2} = \frac{1}{2}\frac{X_3X_4}{X_3+X_4} \quad (16)$$

where $N_0 = 1$, $X_1 = 2P_1(\alpha_1 \mid h_{s_1,r}\mid^2)$ obeys the exponential allocation of parameter $\beta_1 = 1/2P_1\alpha_1\delta^2_{s_1,r}$, $X_2 = P_2 \mid h_{r,d}\mid^2/N_0$ obeys the exponential allocation of parameter $\beta_2 = 1/P_2\delta^2_{r,d}$. The upper boundary of Eq. (14) is

$$\frac{P_1P_2\alpha_2 \mid h_{s_2,r}\mid^2 \mid h_{r,d}\mid^2}{P_2 \mid h_{r,d}\mid^2 + 2P_1\alpha_2 \mid h_{s_2,r}\mid^2} = \frac{1}{2}\frac{2P_1\alpha_2 \mid h_{s_2,r}\mid^2 \times P_2 \mid h_{r,d}\mid^2}{2P_1\alpha_2 \mid h_{s_2,r}\mid^2 + P_2 \mid h_{r,d}\mid^2} = \frac{1}{2}\frac{X_3X_4}{X_3+X_4} \quad (17)$$

where $N_0 = 1$, $X_3 = 2P_1\alpha_2 \mid h_{s_2,r}\mid^2$ obeys the exponential allocation of parameter $\beta_3 = 1/2P_1\alpha_2\delta^2_{s_2,r}$, $X_4 = P_2 \mid h_{r,d}\mid^2$ obeys the exponential allocation of parameter $\beta_4 = 1/P_2\delta^2_{r,d}$. It's easy to see that Eqs (16) and (17) conform to the mean of the harmonic mean of two independent random variables, so the corresponding MGF of Eq. (16) is as follows.

$$M_{SINR_{s_1}} = \frac{\beta_1+\beta_2}{2s} = \frac{\frac{1}{2P_1\alpha_1\delta^2_{s_1,r}} + \frac{1}{P_2\delta^2_{r,d}}}{2s} \quad (18)$$

The corresponding MGF of Eq. (17) is

$$M_{SINR_{s_2}} = \frac{\beta_3+\beta_4}{2s} = \frac{\frac{1}{2P_1\alpha_2\delta^2_{s_2,r}} + \frac{1}{P_2\delta^2_{r,d}}}{2s} \quad (19)$$

2.3 SER calculation of MGF

Assuming that the user node signal s_i adopts the M-PSK modulation method, the average SER is calculated by the MGF method, and the asymptotically tight approximation formula is

$$SER_{psk_{s_i}} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_{SINR_{s_i}}\left(\frac{b_{psk}}{\sin^2\theta}\right)\mathrm{d}\theta \quad (20)$$

It is assumed that QPSK modulation is used, where $M=4$, $b_{psk} = \sin^2(\pi/M)$, $M_{SINR_{s_i}}(b_{psk}/\sin^2\theta)$ is the MGF corresponding to $SINR_{s_i}$.

According to Eq. (12), the corresponding simple form MGF is obtained , and then from the closed boundary Eq. (20) of SER, the closed solution of SER of two user nodes is obtained as follows, respectively.

$$\begin{aligned} SER_{psk_{s_1}} &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_{SINR_{s_1}}\left(\frac{b_{psk}}{\sin^2\theta}\right)\mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{\beta_1+\beta_2}{2s}\mathrm{d}\theta \\ &= \frac{1}{2\pi}(\beta_1+\beta_2)\int_0^{\frac{(M-1)\pi}{M}} \frac{1}{s}\mathrm{d}\theta \end{aligned} \quad (21)$$

$$\begin{aligned} SER_{psk_{s_2}} &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_{SINR_{s_2}}\left(\frac{b_{psk}}{\sin^2\theta}\right)\mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{\beta_3+\beta_4}{2s}\mathrm{d}\theta \\ &= \frac{1}{2\pi}(\beta_3+\beta_4)\int_0^{\frac{(M-1)\pi}{M}} \frac{1}{s}\mathrm{d}\theta \end{aligned} \quad (22)$$

where $\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{1}{s}\mathrm{d}\theta$ is a constant and expressed by N, then:

$$SER_{psk_{s_1}} = \frac{1}{2}N\left(\frac{1}{2P_1\alpha_1\delta^2_{s_1,r}} + \frac{1}{P_2\delta^2_{r,d}}\right) \quad (23)$$

$$SER_{psk_{s_2}} = \frac{1}{2}N\left(\frac{1}{2P_1\alpha_2\delta^2_{s_2,r}} + \frac{1}{P_2\delta^2_{r,d}}\right) \quad (24)$$

so the total SER of the system is

$$\begin{aligned} SER_{total} &= SER_{psk_{s_1}} + SER_{psk_{s_2}} \\ &= \frac{1}{2}N\left(\frac{1}{2P_1\alpha_1\delta^2_{s_1,r}} + \frac{1}{P_2\delta^2_{r,d}}\right) + \frac{1}{2}N\left(\frac{1}{2P_1\alpha_2\delta^2_{s_2,r}} + \frac{1}{P_2\delta^2_{r,d}}\right) \\ &= \left(\frac{1}{4P_1\alpha_1\delta^2_{s_1,r}} + \frac{1}{4P_1\alpha_2\delta^2_{s_2,r}} + \frac{1}{P_2\delta^2_{r,d}}\right)N \end{aligned} \quad (25)$$

3 The power allocation optimization

Set to the total power of the system is P, based on the power constraint condition $P_1 + P_2 = P$, the power allocation coefficients of two user nodes α_1 and α_2 can be obtained by calculating the partial derivation of P_1 or P_2 in the total SER Eq. (25) of the system. The relationship among the power allocation factors of two user nodes and their total power and the AF power of the relay node are as follows, respectively.

$$P_1 = \frac{\delta_{r,d}}{\delta_{r,d} + 2\delta_{s,r}\delta_{s_2,r}\sqrt{\frac{\alpha_1\alpha_2}{\alpha_1\delta^2_{s_1,r} + \alpha_2\delta^2_{s_2,r}}}}P \quad (26)$$

$$P_2 = \frac{2\delta_{s_1,r}\delta_{s_2,r}\sqrt{\dfrac{\alpha_1\alpha_2}{\alpha_1\delta_{s_1,r}^2 + \alpha_2\delta_{s_2,r}^2}}}{\delta_{r,d} + 2\delta_{s_1,r}\delta_{s_2,r}\sqrt{\dfrac{\alpha_1\alpha_2}{\alpha_1\delta_{s_1,r}^2 + \alpha_2\delta_{s_2,r}^2}}}P \quad (27)$$

Based on the condition of $\alpha_1 | h_{s_1,r} |^2 \approx \alpha_2 | h_{s_2,r} |^2$, Eqs(26) and (27) can be simplified as follows.

$$P_1 = \frac{\delta_{r,d}}{\delta_{r,d} + \sqrt{2\alpha_1\delta_{s_1,r}^2}}P \quad (28)$$

$$P_2 = \frac{\sqrt{2\alpha_1\delta_{s_1,r}^2}}{\delta_{r,d} + \sqrt{2\alpha_1\delta_{s_1,r}^2}}P \quad (29)$$

In order to solve the optimal power allocation factors of the two user nodes when the total SER performance is optimal, it can be assumed that $Z = \frac{1}{4P_1\alpha_1\delta_{s_1,r}^2} + \frac{1}{4P_1\alpha_2\delta_{s_2,r}^2} + \frac{1}{P_2\delta_{r,d}^2}$, so the minimal value of the SER_{total} is equal to solve the minimal value of Z. In order to solve the minimal value of Z easily, the intermediate variables are assumed as $x = P_1\alpha_1$, $y = P_1\alpha_2$. Then, Z can be further expressed as

$$Z = \frac{1}{4x\delta_{s_1,r}^2} + \frac{1}{4y\delta_{s_2,r}^2} + \frac{1}{[P - (x + y)]\delta_{r,d}^2} \quad (30)$$

By calculating the partial derivation of x and the minimal value in Eq. (29), the optimal power allocation factor of the user node s_1 is obtained as

$$\alpha_1 = \frac{\delta_{r,d}P_2}{2\delta_{s_1,r}P_1} \quad (31)$$

By calculating the partial derivation of y and the minimal value in Eq. (30), the optimal power allocation factor of the user node s_2 is obtained as

$$\alpha_2 = \frac{\delta_{r,d}P_2}{2\delta_{s_2,r}P_1} \quad (32)$$

According to the variance of the actual channel coefficients of the cooperative NOMA system, the optimal power allocation factors of the two user nodes α_1 and α_2 when the SER performance of the system is optimal can be solved by the joint vertical Eq. (31) and Eq. (32).

4 Simulation results and analysis

In order to verify the accuracy of the total SER asymptotically tight approximation formula and the optimal power allocation scheme derived in this paper, the Monte Carlo simulation method is adopted. The BS receives the signal and decodes it using a SIC technique based on maximum likelihood (ML) detection. The decoding order of SIC decoding is determined by the channel gain in descending order. Therefore, the signal s_1 is firstly detected, and the signal s_2 is treated as noise. Because the signal s_1 cannot be completely decoded, there is an error residual for decoding the signal s_2. The system simulation parameters are set as follows: PSK modulation mode is used to analyze the SER performance of the single-relay AF cooperative uplink NOMA system. According to the two user nodes channel gain assumption $| h_{s_1,r} |^2 > | h_{s_2,r} |^2$, the channel coefficient variance of the user node 1 to the relay node R in the simulation $\delta_{s_1,r}^2 = 2$, the channel coefficient variance of the user node 2 to the relay node R is $\delta_{s_2,r}^2 = 1$, and the channel coefficient variance of the relay node R to the destination node D is $\delta_{r,d}^2 = 1$.

The comparison of system SER performance between the asymptotically tight approximation SER formula and the simulation is shown in Fig. 2. The SNR is represented by P/N_0 in the figure. According to the actual channel gains and user nodes fairness conditions, the power allocation coefficient of the two user nodes are $\alpha_1 = 1/3$, $\alpha_2 = 2/3$ respectively. By Eq. (28) and Eq. (29), the power allocation coefficients $P_1 = 0.464P$, $P_2 = 0.536P$ are calculated. It can be seen from Fig. 2 that when the SNR is larger than 15 dB (that is, under the condition of high SNR) the fitting effect between the actual simulation value of the system and the theoretical value of the deduced SER formula is good, which verifies the accuracy of the system asymptotically tight approximation SER formula derived by MGF.

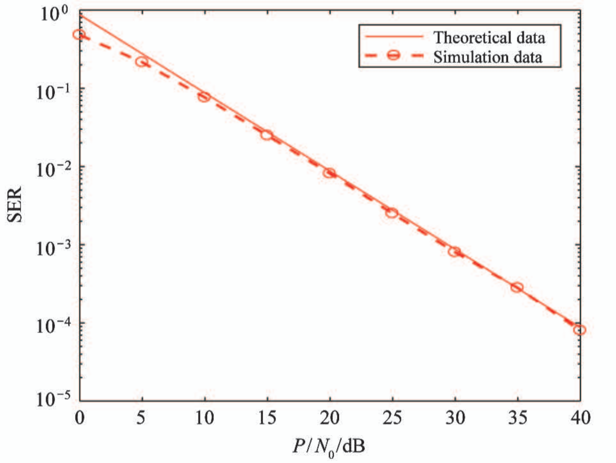

Fig. 2 The comparison of system SER performance between the asymptotically tight approximation SER formula and the simulation

Fig. 3 shows the comparison of system SER performance among three power allocation cases. Power allocation factor α_1 and α_2 for two user nodes are selected from the following three cases: (1) $\alpha_1 = 0.1$, $\alpha_2 = 0.9$; (2) $\alpha_1 = 1/3$, $\alpha_2 = 2/3$; (3) $\alpha_1 = 0.3$, $\alpha_2 = 0.7$. It can be seen from Fig. 3 that the system SER performance of the case (1) is the worst, the case (2) is the best, and the case (3) is between case (1) and case (2). So the case (2) is the optimal power allocation among three power allocation cases of two user nodes under the constraint of SER performance fairness. System SER performance is obviously improved compared with other power allocation cases, which proves that fairness constraint Eq. (15) is reasonable.

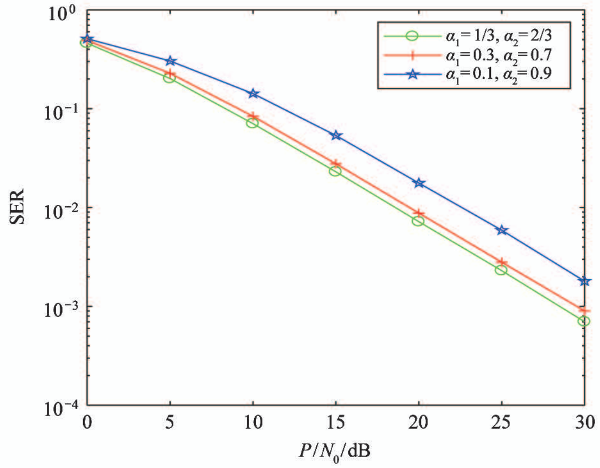

Fig. 3 The comparison of system SER performance among three power allocation cases

Fig. 4 shows a comparative analysis of SER performance fairness between two user nodes under the condition of SER fairness. Since SER is affected by the decoding order and power allocation, the constraint condition of Eq. (15) are used to control power allocation so that the user's SER is similar to guarantee SER

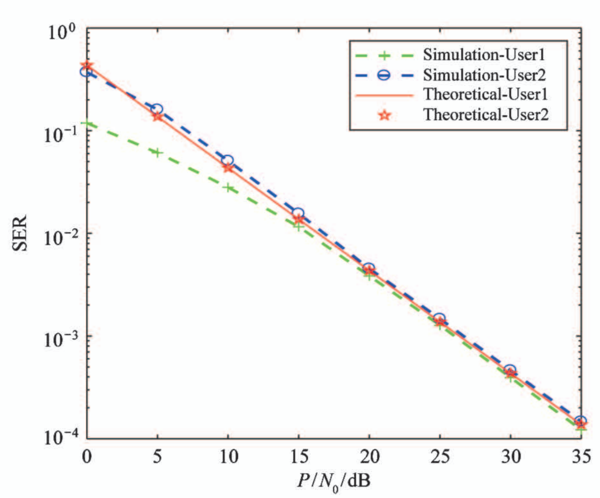

Fig. 4 The analysis of performance fairness of two user nodes SER

fairness. It can be seen from Fig. 4, with the increase of SNR, the theoretical curve and the simulation curve are gradually close to each other, and the approach effect becomes better. In the case of high SNR, although the SER performance of the two user nodes is not completely consistent, the difference between them is kept in a very small range, which indicates that the fairness of the SER performance of the two user nodes is achieved in the process of power allocation optimization in this paper.

5 Conclusion

The performance of the power domain NOMA system SER is studied in the cooperative uplink NOMA communication system. In order to solve the problem that the SER performance analysis in the cooperative NOMA system is complex, under the condition of high SNR and user fairness, the SINR formula of user node is proposed. The harmonic mean form is derived to obtain a simple MGF, and then the system asymptotically tight approximation to the SER formula is obtained, and the system is optimized for power allocation to achieve optimal system performance. The simulation results show that when the SNR is high, the theoretical value of the asymptotically tight approximation SER formula is fitted well with the simulation value of the actual system, which verifies the accuracy of the formula and the constraint conditions of the performance fairness under the optimal power allocation scheme. By minimizing the asymptotically tight approximation SER formula, the optimal power allocation relationship between the total power of two user nodes and the AF power of the relay node is determined.

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