

# The relationship between extra connectivity and $t/k$ -diagnosability under the PMC model<sup>①</sup>

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## Abstract

It is well-known that connectivity is closely related to diagnosability. If the relationships between them can be established, many kinds of diagnosability will be determined directly. So far, some notable relationships between connectivity and diagnosability had been revealed. This paper intends to find out the relationship between extra connectivity and  $t/k$ -diagnosability under the PMC (Preparata, Metze, and Chien) model. Then, applying this relationship, the  $t/k$ -diagnosability of bijective connection (BC) networks are determined conveniently.

**Key words:** extra connectivity,  $t/k$ -diagnosability, the PMC model

## 0 Introduction

Connectivity and diagnosability are generally considered as two important indicators that are used to evaluate the reliability of multiprocessor computer systems. They are also considered as two closely related parameters. So far, some important results had been achieved in the study of connectivity and diagnosability. But there are still some outstanding diagnosability measurement problems, especially for interconnection networks with insufficient regularity and symmetry.

Research shows that the various diagnosability of the interconnection network will increase with the improvement of the relevant connectivity, and show an obvious linear relationship. If it can be found out the relationship between diagnosability and related connectivity, it can be greatly simplified the measurement process of diagnosabilities and quickly calculate various diagnosabilities of a series of interconnection networks. Therefore, it is a very important and valuable scientific issue to study the relationship between connectivity and diagnosability.

## 1 Preliminaries

In general, a multiprocessor system can be modeled by  $G(V, E)$ , where  $V(G)$  and  $E(G)$  are the node set and the edge set, respectively. Let  $x \in V(G)$  and

$A, B \subset V(G)$ ,  $N(x)$  is the set of all the neighbors of  $x$ ,  $N(A) = \bigcup_{a \in A} N(a) - A$  and  $N_B(A) = N(A) \cap B$ .

The connectivity  $k(G)$  of  $G$  is an important measure for fault tolerance of  $G$ . However, connectivity underestimates the resilience of large networks<sup>[1]</sup>. To compensate for this shortcoming, many kinds of connectivity are introduced, such as conditional connectivity<sup>[2]</sup>, restricted connectivity<sup>[3]</sup>, super connectivity<sup>[4]</sup>, extra connectivity<sup>[5]</sup>, et al. Among them,  $g$ -extra connectivity of  $G$ , written as  $k_g(G)$ , is the minimum size over all the  $g$ -extra cuts of  $G$ . Any subset  $F \subset V(G)$  is a  $g$ -extra cut of  $G$  if  $G - F$  is disconnected and each component of  $G - F$  has size at least  $g + 1$ . Clearly,  $k_0(G) = k(G)$ .

In the operation of multiprocessor systems, identifying faulty processors is an important problem. In the process of identifying faulty processors, a fault diagnosis model and a diagnosis strategy are indispensable. At present, one of the widely adopted fault diagnosis model is PMC (Preparata, Metze, and Chien) model<sup>[6]</sup>. Under the PMC model, each pair of adjacent nodes can be allowed to test each other. If the tester is fault-free (faulty), its outcomes are correct (unreliable, respectively). For any edge  $(u, v) \in E(G)$ ,  $u \xrightarrow{0} v$  ( $u \xrightarrow{1} v$ ) represents the outcome of test  $u \rightarrow v$  is fault-free (faulty, respectively). In addition,  $u \xrightarrow{00} v$  represents  $u \xrightarrow{0} v$  and  $v \xrightarrow{0} u$ . A collection of all the test results is called a

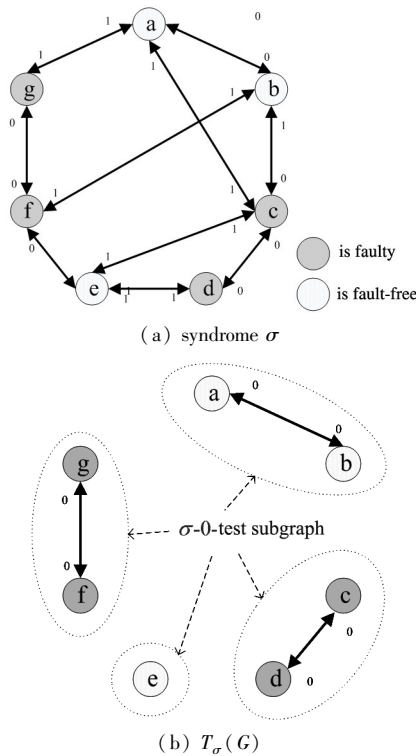
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syndrome  $\sigma$ . Ref. [7] introduced the  $t/k$ -diagnosis strategy, which requires that when the number of fault nodes does not exceed  $t$ , all fault nodes can be isolated in a set of node and at most  $k$  nodes might be misdiagnosed. The  $t/k$ -diagnosability of  $G$ , is the maximum of  $t$  such that  $G$  is  $t/k$ -diagnosable<sup>[7]</sup>. The  $t/k$ -diagnosability of a series of regular networks under PMC model is determined<sup>[8-15]</sup>. It is well-known that the constraints of  $t/k$ -diagnosability and  $k$ -extra connectivity are basically the same. Therefore, Refs[11] and [12] believe that it is an interesting direction to analyze the relationship between extra connectivity and  $t/k$ -diagnosability. In this paper, the relationship between  $g$ -extra connectivity and  $t/k$ -diagnosability under the PMC model are revealed.

## 2 $\sigma$ -0-test subgraph

Under the PMC model, given a graph  $G$  and a syndrome  $\sigma$ , each connected subgraphs or isolated points is called a  $\sigma$ -0-test subgraph of  $G$  by removing all those edges whose outcomes are '1'<sup>[16]</sup>. The set of all the  $\sigma$ -0-test subgraph of  $G$  is denoted by  $T_\sigma(G)$ . Then,  $V(T_\sigma(G)) = V(G)$  and  $E(T_\sigma(G)) = \{(u, v) \in E(G), u \leftrightarrow v\}$  (see Fig. 1).



**Fig. 1** The illustration of  $\sigma$ -0-test subgraph

Given a syndrome  $\sigma$ , for any  $\sigma$ -0-test subgraph  $S \in T_\sigma(G)$ , all the nodes in  $S$  have the same status

(fault-free or faulty). Then, under the PMC model, the following properties are shown as follows.

**Property 1** Given a syndrome  $\sigma$ , let  $F$  be a fault set of  $G$ . Then any component  $C$  of  $G-F$  is a  $\sigma$ -0-test subgraph of  $G$  and each node in  $C$  is fault-free.

**Property 2** Given a syndrome  $\sigma$ , let  $F$  be a fault set of  $G$ . Then  $F$  will be divided into one or several  $\sigma$ -0-test subgraphs of  $G$ .

## 3 The relationship between extra connectivity and $t/k$ -diagnosability under the PMC model

Let  $X_n$  be an  $n$ -dimensional interconnection network and  $X_n$  can be divided into to copies of  $X_{n-1}$ , written as  $L$  and  $R$ . Then, the following four conditions will be used in the rest of this paper.

(1) Let  $S \subset V(R)$  ( or  $S \subset V(L)$  ) with  $|S| = g \geq 1$ ,  $|N_R(S)| + |N_L(S)| \geq k_{g-1}(X_n)$  for  $n \geq 8$  and  $1 \leq g \leq n - 4$ .

(2) For any positive integers  $g, g_0$  and  $g_1$  with  $g, g_0, g_1 \geq 1$ . If  $g = g_0 + g_1$ , then  $k_{g_0-1}(X_{n-1}) + k_{g_1-1}(X_{n-1}) \geq k_{g-1}(X_n)$  for  $n \geq 8$  and  $1 \leq g \leq n - 4$ .

(3) The function  $f(g) = k_g(X_n)$  increases with increasing  $g$  for  $n \geq 8$  and  $1 \leq g \leq n - 4$ .

(4)  $k_{g+1}(X_n) \geq k_g(X_n) + n - g - 4$  for  $n \geq 8$  and  $1 \leq g \leq n - 5$ .

**Theorem 1** Let  $S \subset V(X_n)$  with  $|S| = g$ . If  $X_n$  satisfies the conditions (1) and (2),  $|N(S)| \geq k_{g-1}(X_n)$  for  $n \geq 8$  and  $1 \leq g \leq n - 4$ .

**Proof** The proof is by induction on  $g$ . If  $g = 1$ ,  $|S| = 1$ . Then  $|N(S)| \geq k(X_n) = k_0(X_n)$ . Hence, the theorem is true for  $g = 1$ . Assume that  $|N(S)| \geq k_{h-1}(X_n)$  with  $|S| = h$  and  $2 \leq h \leq g - 1$ .

Decompose  $X_n$  into two copies of  $X_{n-1}$ , denoted by  $L$  and  $R$ . Let  $S_0 = S \cap V(L)$  and  $S_1 = S \cap V(R)$ . Let  $|S_0| = g_0$  and  $|S_1| = g_1$ . Then  $g_0 + g_1 = g$ . Without loss of generality, let  $|S_0| \leq |S_1|$ .

**Case 1**  $|S_0| = 0$ .

Since  $|S_0| = 0$ ,  $|S_1| = g$  and  $|N(S)| \geq |N_R(S_1)| + |N_L(S_1)|$ . By condition (1),  $|N_R(S)| + |N_L(S)| \geq k_{g-1}(X_n)$ . So,  $|N(S)| \geq k_{g-1}(X_n)$ .

**Case 2**  $|S_0| \geq 1$ .

Since  $|S_0| \geq 1$ ,  $|N(S)| \geq |N_L(S_0)| + |N_R(S_1)|$  such that  $N_L(S_0) = N(S_0) \cap V(L)$  and  $N_R(S_1) = N(S_1) \cap V(R)$ . By the induction hypothesis,  $|N_L(S_0)| \geq k_{g_0-1}(X_{n-1})$  and  $|N_R(S_1)| \geq k_{g_1-1}(X_{n-1})$ . By condition (2),  $|N_L(S_0)| + |N_R(S_1)| \geq k_{g_0-1}(X_{n-1}) + k_{g_1-1}(X_{n-1}) \geq k_{g-1}(X_n)$ . So,  $|N(S)| \geq k_{g-1}(X_n)$ .

The theorem holds.

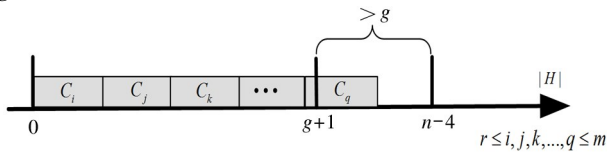
**Theorem 2** If  $X_n (n \geq 8)$  satisfies the conditions (1) – (3), let  $S \subset V(X_n)$  and  $2 \leq g + 1 \leq |S| \leq n - 4$ . Then  $|N(S)| \geq k_g(X_n)$ .

**Proof** Let  $|S| = h$ . By Theorem 1,  $|N(S)| \geq k_{h-1}(X_n)$ . By condition (3),  $k_{h-1}(X_n) \geq k_g(X_n)$ . Hence,  $|N(S)| \geq k_g(X_n)$ .

**Theorem 3** If  $X_n (n \geq 8)$  satisfies the conditions (1) – (4) and  $|V(X_n)| > 2k_g(X_n) + g - 1$ , let  $F \subset V(X_n)$  with  $|F| \leq k_g(X_n) - 1$  and  $1 \leq g \leq n/2 - 3$ . If  $X_n - F$  is disconnected,  $X_n - F$  has a largest component  $C_1 (|C_1| \geq g + 1)$  and the union of the remaining components has at most  $g$  nodes.

**Proof** Let all the components of  $X_n - F$  be  $C_1, C_2, \dots, C_m$  with  $|C_1| \geq |C_2| \geq \dots \geq |C_m|$ . Suppose that  $|C_1|, |C_2|, \dots, |C_{r-1}| \geq g + 1$  and  $|C_r|, |C_{r+1}|, \dots, |C_m| \leq g$  for  $r \geq 1$ .

Thus,  $|F| \geq |N(C_r \cup C_{r+1} \cup \dots \cup C_m)|$ . Suppose that  $g + 1 \leq |C_r| + |C_{r+1}| + \dots + |C_m| \leq n - 4$ . By Theorem 2,  $|F| \geq |N(C_r \cup C_{r+1} \cup \dots \cup C_m)| \geq k_g(X_n)$ , which contradicts  $|F| \leq k_g(X_n) - 1$ . Therefore, either  $|C_r| + |C_{r+1}| + \dots + |C_m| \geq n - 3$  or  $|C_r| + |C_{r+1}| + \dots + |C_m| \leq g$ . Suppose that  $|C_r| + |C_{r+1}| + \dots + |C_m| \geq n - 3$ . Since  $|C_r|, |C_2|, \dots, |C_{r-1}| \leq g \leq n/2 - 3, (n - 4) - (g + 1) = n - g - 5 \geq (2g + 6) - g - 5 > g$ . Therefore, there exists a union  $H$  of some components of  $C_r, C_{r+1}, \dots, C_m$ , such that  $n - 4 \geq |H| \geq g + 1$  (see Fig. 2). By Theorem 2,  $|N(H)| \geq k_g(X_n)$ , which contradicts  $|F| \leq k_g(X_n) - 1$ . Therefore,  $|C_r| + |C_{r+1}| + \dots + |C_m| \leq g$ .



**Fig. 2** The illustration of  $|H|$

Since  $|C_r| + |C_{r+1}| + \dots + |C_m| \leq g$  and  $|F| \leq k_g(X_n) - 1$  and  $|V(X_n)| > 2k_g(X_n) + g - 1, |C_1| + |C_2| + \dots + |C_{r-1}| = |V(X_n)| - |F| - (|C_r| + |C_{r+1}| + \dots + |C_m|) > 2k_g(X_n) + g - 1 - (k_g(X_n) - 1) - g > k_g(X_n) > 0$ . Therefore, there exists at least a component  $C_1$  of  $X_n - F$  such that  $|C_1| \geq g + 1$  nodes.

If  $X_n - F$  has at least two components, where each component has at least  $g + 1$  nodes. That is  $|C_1|, |C_2|, \dots, |C_{r-1}| \geq g + 1$  with  $r \geq 3$ . Suppose that there exists a component  $C_i$  with  $n - 4 \geq |C_i| \geq g + 1 \geq 2$  and  $1 \leq i \leq r - 1$ . By Theorem 2,  $|N(C_i)| \geq k_g(X_n)$ . Then  $|F| \geq |N(C_i)| \geq k_g(X_n)$ , which con-

tradicts  $|F| \leq k_g(X_n) - 1$ . Therefore,  $|C_1|, |C_2|, \dots, |C_{r-1}| \geq n - 3 \geq g + 2$ , which shows that  $F \cup C_r \cup C_{r+1} \cup \dots \cup C_m$  is a  $(g + 1)$ -extra cut of  $X_n$ . Thus,  $|F \cup C_r \cup C_{r+1} \cup \dots \cup C_m| = |F| + |C_r| + |C_{r+1}| + \dots + |C_m| \geq k_{g+1}(X_n)$ . By condition (4),  $k_{g+1}(X_n) \geq k_g(X_n) + n - g - 4$ . Since  $|C_r| + |C_{r+1}| + \dots + |C_m| \leq g$  and  $1 \leq g \leq n/2 - 3,$   
 $|F| + |C_r| + |C_{r+1}| + \dots + |C_m| \geq k_{g+1}(X_n)$   
 $\Rightarrow |F| \geq k_{g+1}(X_n) - (|C_r| + |C_{r+1}| + \dots + |C_m|)$   
 $\Rightarrow |F| \geq (k_g(X_n) + n - g - 4) - g$   
 $= k_g(X_n) + n - 2g - 4$   
 $\geq k_g(X_n) + (2g + 6) - 2g - 4$   
 $\geq k_g(X_n) + 2,$

which contradicts  $|F| \leq k_g(X_n) - 1$ . So,  $X_n - F$  has exactly one component which have at least  $g + 1$  nodes.

**Theorem 4** If  $X_n (n \geq 8)$  satisfies the conditions (1) – (4) with  $|V(X_n)| > 2k_g(X_n) + g - 1$  and  $1 \leq g \leq n/2 - 3$ , let  $F$  be a fault set with  $|F| \leq k_g(X_n) - 1$ . Then, under any syndrome  $\delta$  produced by  $F$ , the maximal  $\sigma$ -0-test subgraph of  $X_n$  is fault-free.

**Proof** If  $X_n - F$  is connected,  $|V(X_n) - F| = |V(X_n)| - |F| \geq 2k_g(X_n) + g - 1 - (k_g(X_n) - 1) = k_g(X_n) + g$ . Since  $|F| \leq k_g(X_n) - 1, |X_n - F| > |F|$ . By Property 1,  $X_n - F$  is a the maximal  $\sigma$ -0-test subgraph of  $X_n$  and each node of  $X_n - F$  is fault-free. If  $X_n - F$  is disconnected, by Theorem 3,  $X_n - F$  has a largest component  $C (|C| \geq g + 1)$  and the union of the remaining components has at most  $g$  nodes.

By Property 1,  $C$  is a  $\sigma$ -0-test subgraph. Since  $|C| \geq |V(X_n)| - |F| - g$   
 $> 2k_g(X_n) + g - 1 - (k_g(X_n) - 1) - g$   
 $= k_g(X_n),$

$|C| > |F|$ . Therefore, by Properties 1 and 2,  $C$  is the maximal  $\sigma$ -0-test subgraph and each node of  $C$  is fault-free.

**Theorem 5** If  $X_n (n \geq 8)$  satisfies the conditions (1) – (4) with  $|V(X_n)| > 2k_g(X_n) + g - 1$ , then  $X_n$  is  $k_g(X_n) - 1/g$ -diagnosable for  $1 \leq g \leq n/2 - 3$ .

**Proof** Let  $F$  be a fault set of  $X_n$  with  $|F| \leq k_g(X_n) - 1$ . By Theorem 3,  $X_n - F$  has a largest component  $C (|C| \geq g + 1)$  and the union of the remaining components has at most  $g$  nodes. By Theorem 4,  $C$  is the maximal  $\sigma$ -0-test subgraph and every node in  $C$  is fault-free. Therefore, there are  $|F| + g$  nodes undiagnosed. Then, all the faulty nodes can be isolated to within a set of at most  $|F| + g$  nodes.

There are at most  $g$  nodes might be misdiagnosed. Therefore,  $X_n$  is  $k_g(X_n) - 1/g$ -diagnosable.

#### 4 Application to BC networks

An  $n$ -dimensional bijective connection (BC) net-

work is denoted by  $B_n$  with  $|V(B_n)| = 2^n$ .  $B_n$  can be divided into two copies of  $B_{n-1}$ , written as  $L$  and  $R$ , and there exists a perfect matching between  $L$  and  $R$  (see Fig. 3). Then  $B_n$  has the following lemmas.

**Lemma 1** <sup>[8]</sup> Let  $S \subset V(B_n)$  with  $|S| = g$  and  $1 \leq g \leq n + 1$ . Then,  $|N(S)| \geq \frac{1}{2}g^2 + (n - \frac{1}{2})g + 1$  for  $n \geq 5$ .

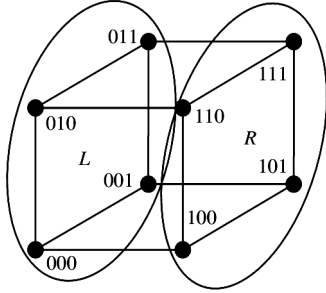


Fig. 3 The topology of  $B_3$

**Lemma 2** <sup>[17]</sup>  $k_g(B_n) = n(g + 1) - \frac{1}{2}g(g + 3)$  for  $n \geq 4$  and  $1 \leq g \leq n - 4$ .

**Lemma 3** For any three positive integers  $g, g_0$  and  $g_1$  with  $g, g_0, g_1 \geq 1$ . If  $g = g_0 + g_1$ , where  $n \geq 5$  and  $1 \leq g \leq n - 3$ , then  $k_{g_0-1}(B_{n-1}) + k_{g_1-1}(B_{n-1}) \geq k_{g-1}(B_n)$ .

**Proof** By Lemma 2,

$$\begin{aligned} &k_{g_0-1}(B_{n-1}) + k_{g_1-1}(B_{n-1}) \\ &= (n - 1)g_0 - \frac{1}{2}(g_0 - 1)(g_0 + 2) \\ &\quad + (n - 1)(g_1) - \frac{1}{2}(g_1 - 1)(g_1 + 2) \\ &= (n - 1)(g_0 + g_1) \\ &\quad - \frac{1}{2}(g_0^2 + g_0 - 2 + g_1^2 + g_1 - 2) \\ &= (n - 1)g - \frac{1}{2}(g^2 + g - 4) + g_0g_1 \end{aligned}$$

and  $k_{g-1}(B_n) = ng - \frac{1}{2}(g - 1)(g + 2)$ . Then,

$$\begin{aligned} &k_{g_0-1}(B_{n-1}) + k_{g_1-1}(B_{n-1}) - k_{g-1}(B_n) \\ &= (n - 1)g - \frac{1}{2}(g^2 + g - 4) + g_0g_1 \\ &\quad - ng + \frac{1}{2}(g - 1)(g + 2) \\ &= -g + g_0g_1 + 1 \\ &= -(g_0 + g_1) + g_0g_1 + 1 \\ &= (g_0 - 1)(g_1 - 1) \geq 0. \end{aligned}$$

Therefore,  $k_{g_0-1}(B_{n-1}) + k_{g_1-1}(B_{n-1}) \geq k_{g-1}(B_n)$ .

**Lemma 4** Decompose a  $B_n (n \geq 5)$  into two  $B_{n-1}$ , written as  $L$  and  $R$ . Let  $S \subset V(R)$  (or  $S \subset V(L)$ ) with  $|S| = g \geq 1$ . Then,  $|N_R(S)| + |N_L(S)| \geq$

$k_{g-1}(B_n)$  for  $1 \leq g \leq n - 3$ .

**Proof** By Lemma 1,  $|N_R(S)| \geq \frac{1}{2}g^2 + (n - \frac{3}{2})g + 1$ . By Lemma 2,  $k_{g-1}(B_n) = ng - \frac{1}{2}(g - 1)(g + 2)$ . Then,

$$\begin{aligned} &|N_R(S)| + |N_L(S)| - k_{g-1}(B_n) \\ &= |N_R(S)| + g - k_{g-1}(B_n) \\ &\geq \frac{1}{2}g^2 + (n - \frac{3}{2})g + 1 + g - ng \\ &\quad + \frac{1}{2}(g - 1)(g + 2) \\ &= \frac{1}{2}g^2 - \frac{3}{2}g + 1 + g + \frac{1}{2}g^2 + \frac{1}{2}g - 1 \\ &= g^2 \geq 0. \end{aligned}$$

Therefore,  $|N_R(S)| + |N_L(S)| \geq k_{g-1}(B_n)$ .

**Lemma 5** Function  $f(g) = k_g(B_n) = n(g + 1) - \frac{1}{2}g(g + 3)$  is monotonically increasing with increasing  $g$  when  $n \geq 4$  and  $1 \leq g \leq n - 4$ .

**Proof** The derivative of  $f(g)$  is  $f'(g) = n - g + \frac{3}{2}$ . Since  $1 \leq g \leq n - 4$ ,  $f'(g) > 0$ . Therefore,  $f(g)$  is monotonically increasing with increasing  $g$ .

**Lemma 6**  $k_{g+1}(B_n) \geq k_g(B_n) + n - g - 4$ , for  $n \geq 8$  and  $1 \leq g \leq n - 5$ .

**Proof** Since  $1 \leq g \leq n - 5$ ,  $1 \leq g + 1 \leq n - 4$ . By Lemma 2,

$$\begin{aligned} &k_{g+1}(B_n) - k_g(B_n) - n + g + 4 \\ &= n(g + 2) - \frac{1}{2}(g + 1)(g + 4) - n(g + 1) \\ &\quad + \frac{1}{2}g(g + 3) - n + g + 4 = 2 \geq 0. \end{aligned}$$

**Lemma 7**  $B_n$  (is  $k_g(B_n) - 1)/g$ -diagnosable, for  $n \geq 8$  and  $1 \leq g \leq n/2 - 3$ .

**Proof** Since  $|V(B_n)| = 2^n$  and  $k_g(B_n) = n(g + 1) - \frac{1}{2}g(g + 3)$ ,  $|V(B_n)| > 2k_g(B_n) + g - 1$ . By the definition of  $B_n$  and Lemmas 3 - 6,  $B_n$  satisfies the conditions (1) - (4). Therefore, by Theorem 5,  $B_n$  is  $(k_g(B_n) - 1)/g$ -diagnosable.

## 5 Conclusion

In the design and operation of large-scale multi-processor systems, reliability is a key issue to be considered. It is well-known that connectivity and diagnosability are two crucial subjects for reliability and fault tolerability and they are closely related to each other. This paper establishes a relationship between extra connectivity and  $t/k$ -diagnosability under the PMC model. Then, using this relationship, it is proved that  $B_n$  is

$(k_g(B_n) - 1)/g$ -diagnosable.

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