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Multi-strategy hybrid whale optimization algorithms for complex constrained optimization problems①

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Abstract

A multi-strategy hybrid whale optimization algorithm (MSHWOA) for complex constrained optimization problems is proposed to overcome the drawbacks of easily trapping into local optimum, slow convergence speed and low optimization precision. Firstly, the population is initialized by introducing the theory of good point set, which increases the randomness and diversity of the population and lays the foundation for the global optimization of the algorithm. Then, a novel linearly update equation of convergence factor is designed to coordinate the abilities of exploration and exploitation. At the same time, the global exploration and local exploitation capabilities are improved through the siege mechanism of Harris Hawks optimization algorithm. Finally, the simulation experiments are conducted on the 6 benchmark functions and Wilcoxon rank sum test to evaluate the optimization performance of the improved algorithm. The experimental results show that the proposed algorithm has more significant improvement in optimization accuracy, convergence speed and robustness than the comparison algorithm.

Key words: whale optimization algorithm (WOA), good point set, nonlinear convergence factor, siege mechanism

0 Introduction

With the development of human society and economy, the complexity of optimization problems in practical applications is increasing and the traditional methods such as gradient descent method and Newton iteration method can not meet the actual demand. Therefore, numerous swarm intelligent optimization algorithms[1] based on the simulation of social phenomena and organizational behaviors have been advanced by many scholars. These swarm intelligent optimization algorithms have the advantage of strong robustness and self-organizing ability and are widely used in the fields of video defogging[2], satellite image segmentation[3], path planning[4], and so on.

Since Holland[5] proposed genetic algorithm in 1975 by studying adaptive survival mechanism and Darwinian evolution, many scholars have proposed a variety of swarm intelligent optimization algorithms. Among them, Mirjalili[6] proposed a new swarm intelligent optimization algorithm—whale optimization algorithm (WOA) in 2016, which simulated foraging behavior of humpback whales. WOA algorithm has the advantage of few adjustment parameters and high accuracy.

According to no free lunch theorem for optimization, no algorithm can solve all optimization problems. Therefore, similar to other swarm intelligent optimization algorithms, WOA still has some shortcomings such as easiness to fall into local optimal and slow algorithm convergence in the late stage. In order to overcome the shortcomings of WOA, many scholars have proposed some improved versions of the WOA algorithm and applied them to real world problems. Chakraborty et al. [7] designed a whale optimization algorithm based on hunger search, which combined the hunger-driven search concept with the hunting behavior of humpback whales and improved the performance of WOA with the help of the hunting characteristics of hungry whales. Chen et al. [8] introduced Levy flight and chaotic local search strategies into the whale optimization algorithm to enhance the optimization ability of the algorithm in complex environments. Hu et al. [9] improved the searching ability of whale optimization algorithm by in-

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troducing Cauchy mutation and simulated annealing strategy, and applied the algorithm to radar task scheduling problem.

Although the improvement of WOA in the above research has improved the performance of the algorithm, the algorithm still has some problems such as slow convergence speed and insufficient exploration. Therefore, a multi-strategy hybrid whale optimization algorithm (MSHWOA) is proposed to improve the performance and balance the exploitation and exploration capability of the algorithm. The main contributions of this paper are summarized as follows.

(1) The good point set is used to initialize the population, which enhance the diversity of the population and improve the convergence speed of the algorithm in the early stage.

(2) The non-linearly varying convergence factor is introduced to balance the exploration and exploitation ability of the algorithm.

(3) The population siege strategy of the Harris Hawk optimization algorithm is introduced to update the shrinkage siege mechanism of the algorithm and enhance the speed of search for the optimal position by all the whale individuals.

(4) The efficiency of the proposed MSHWOA is evaluated on a comprehensive set of well-known benchmark functions and compared with a variety of competitive swarm intelligent algorithms.

The remainder of this paper is structured as follows. The proposed MSHWOA algorithm is specified in detail in Section 1. The experimentation and verification of the proposed method on benchmark functions are performed in Section 2. The conclusion and potential future research directions are presented in Section 3.

1 Improved whale optimization algorithm

1.1 Position initialization using good point set

The distribution of the initial solution has an impact on the search accuracy of the algorithm. The standard WOA initializes the population in a random way, and this method produces a poor uniformity of the initial population, which reduces the search efficiency of the algorithm to a certain extent. The theory of good point set is applied to the population initialization stage of MSHWOA, and the method of sub-circular domain is adopted to construct the good point set in order to improve the search ability of the algorithm[10].

Let G_s be the unit cube in the s dimensional space, then:

$$P_n(k) = \{(\{r_1^{(n)}k\}, \{r_2^{(n)}k\}, \cdots, \{r_s^{(n)}k\}), k = 1, 2, \cdots, n\} \quad (1)$$

where, $r = \{2\cos(\frac{2\pi k}{p})\}, 1 \leqslant k \leqslant s$, and p is the smallest prime number satisfying $\frac{p-3}{2} \geqslant s$.

The deviation $\varphi(n)$ satisfies $\varphi(n) = C(r,\varepsilon)n^{\varepsilon-1}$, where $C(r,\varepsilon)n^{\varepsilon-1}$ is a constant only related to ε and $r(\varepsilon > 0)$, $\{r_s^{(n)}k\}$ represents the fractional part, n represents the number of points, and $P_n(k)$ is called the good point set, and r is the good point.

The essence of the good point set is to construct a uniform distribution of points in the unit cube G_s in the s dimensional space, and the distribution of points generated by using the good point set theory is more uniform than that generated by a random method.

The good point set is mapped to the search space as shown in Eq. (2).

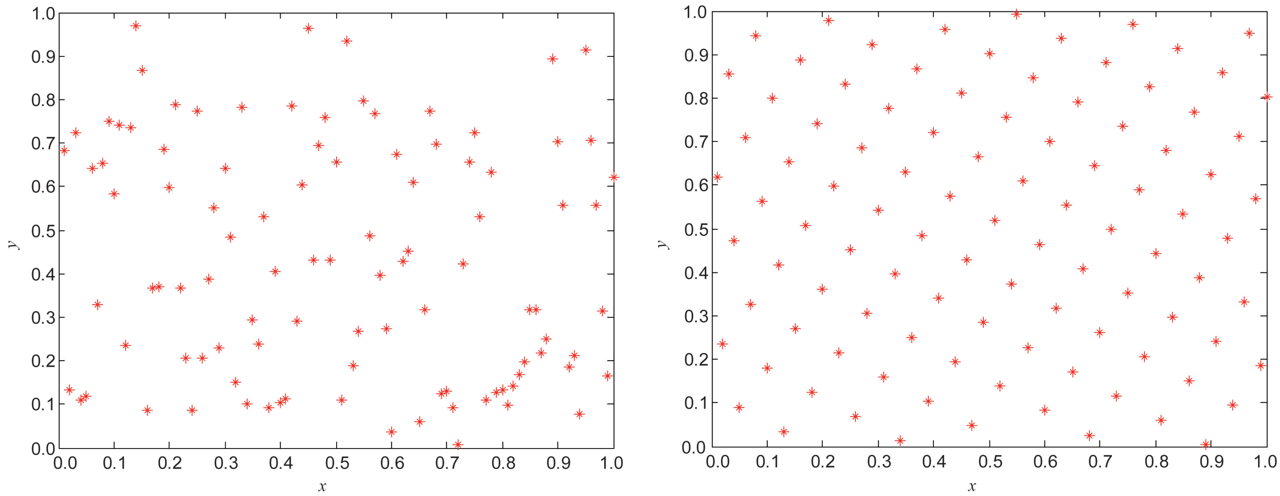
$$x_i(j) = (ub_j - lb_j) \cdot \{r_j^{(i)} \cdot k\} + lb_j \quad (2)$$

where, ub_j and lb_j denote the upper and lower bounds of the j dimension respectively.

The initial population distribution of 100 particles randomly generated in the two-dimensional search space is shown in Fig. 1(a), and the initial population distribution of 100 particles generated in the two-dimensional search space using the good point set theory is shown in Fig. 1(b). Compared with the randomly generated initial population in Fig. 1(a), the population generated by the good point set is uniformly distributed, which increases the diversity of the initial population and helps avoid falling into local optimum.

1.2 Nonlinear variation convergence factor

The exploration and exploitation ability of the WOA algorithm depends heavily on the variation of the convergence factor a. The larger the convergence factor, the better the global search ability of the algorithm and the better the ability to jump out of the local optimum; the smaller the convergence factor is, the better the local exploitation ability of the algorithm and the faster the convergence speed. However, in the basic WOA



(a) 100 random point set (b) 100 good point set

Fig. 1 Population distribution

algorithm, the convergence coefficient decreases linearly with the number of iterations from 2 to 0. This linearly decreasing strategy makes the algorithm have better global search ability but slow convergence speed in the early stage, and fast convergence speed but easiness to make the algorithm fall into the local optimum in the later stage. This phenomenon is more obvious when solving the optimal value of multi-peak function.

In order to make WOA have a strong global search ability and maintain a fast convergence rate in the early iteration, as well as a fast convergence rate and ability of jumping out of the local optimum in the late iteration, the value of a should be large in the early iteration, while be small when it enters the local search phase in the late iteration. To this end, a convergence coefficient update formula that varies nonlinearly with the number of iterations is proposed as shown in Eq. (3).

$$a = (a_{\text{initial}} - a_{\text{final}}) + \frac{0.45 \log_2\left(\dfrac{t}{Max\_iter}\right)}{1 - \mu \dfrac{t}{t_{\max}}} \quad (3)$$

where a_{initial} and a_{final} are the initial and final values of the convergence factor, t is the current number of iterations, Max_iter is the maximum number of iterations, and μ is the nonlinear adjustment coefficient.

The variation curves of the convergence factor with the number of iterations before and after the improvement is shown in Fig. 2. It can be seen that after the improvement, the decreasing degree of a is faster and then slower, and the proportion of $a > 1$ in the iterations is smaller, which improves the global search speed in the early stage of the algorithm and improves the local search accuracy in the later stage.

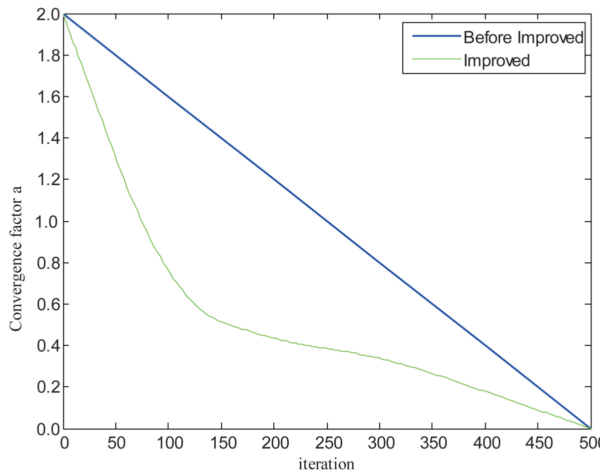


Fig. 2 Variation curve of convergence factor a

1.3 Harris Hawks siege mechanism

The Harris Hawks optimization algorithm[11] is an meta-heuristic algorithm which simulates the predatory action of Harris Hawks. The siege predation mechanism in the algorithm makes the algorithm have a strong global search capability.

In the process of WOA algorithm, the individual whale usually conducts random exploration, and the lack of communication between the individual and the group makes some individuals conduct multiple useless explorations at a distance from the prey, which affects the efficiency of the algorithm. The siege strategy in the Harris Hawks algorithm is introduced to update the whale location formula, which is shown as follows.

$$X(t+1) = \begin{cases} Y & f(Y) < f(X(t)) \\ Z & f(Z) < f(X(t)) \end{cases} \quad (4)$$

$$Y = X^*(t) - A \cdot D_1 \quad (5)$$

$$Z = Y + \boldsymbol{S} \cdot \boldsymbol{LF}(D) \quad (6)$$

where X, Y, Z represent the individuals in population; A represents the vector coefficient; $X^*(t)$ represents the new individuals, $f(x)$ represents the position adaptation value of x, which means the adaptation value is calculated by substituting a position into the adaptation function; $\boldsymbol{LF}(D)$ is the dimensional random vector generated by the Lévy flight, and the Lévy flight formula is shown in Eqs (7) – (8).

$$\boldsymbol{LF}(D) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}} \quad (7)$$

$$\sigma = \left[\frac{\Gamma(1+\beta)\sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1+\beta}{2}) \times \beta \times 2^{\frac{\beta-1}{2}}} \right]^{\frac{1}{\beta}} \quad (8)$$

where, u and v are random values between (0,1), β is set to 1.5, and $\Gamma(x)$ is the Gamma function.

1.4 Implementation steps of MSHWOA

Step 1 The parameters related to the algorithm are initialized: population size N, spatial dimension dim, searchable space of the population $[ub_j, lb_j]$, maximum number of iterations of the algorithm *Max_iter*.

Step 2 The initial population based on good point set theory is generated using Eq. (2).

Step 3 The fitness value of each whale individual is calculated and the optimal individual position is selected.

Step 4 The parameter a is updated according to Eq. (3).

Step 5 The Harris Hawk siege mechanism is introduced to update the optimal solution, and the whale individual positions are updated according to Eq. (5) and Eq. (6).

Step 6 If the maximum number of iterations is reached, return to Step 4. Otherwise, output the result.

1.5 Time complexity analysis of MSHWOA

The time complexity indirectly reflects the convergence speed of the algorithm. In the WOA algorithm, assuming that the time required to initialize the parameters (population size N, search space dimension n, coefficients $\vec{a}, \vec{A}, \vec{C}$, etc.) is α_1, the time required to update the other whale individuals in the population in each dimension according to Eq. (7) is α_2, and the time required to solve the target fitness function is $f(n)$, then the time complexity of the standard WOA is

$$T_1(n) = O(\alpha_1 + N(n\alpha_2 + f(n))) = O(n + f(n)) \quad (9)$$

In the MSHWOA algorithm, the time required to initialize the parameters remains the same as the standard WOA, and the time required to initialize the population using the good point set is α_3, and in the loop phase of the algorithm, let the time required to execute the Harris Hawk breakout strategy be α_4 and the time required to update the individual whale positions be α_5, then the time complexity of MSHWOA is

$$T_2(n) = O(\alpha_1 + N(n\alpha_3 + \alpha_4 + n\alpha_5 + f(n))) = O(n + f(n)) \quad (10)$$

The time complexity of the MSHWOA and the basic WOA algorithm is consistent:

$$T_1(n) = T_2(n) = O(n + f(n)) \quad (11)$$

In summary, the proposed improvement strategy for WOA defects does not increase the time complexity.

2 Simulation results and discussions

All algorithms are coded on Matlab R2020a, and all of the simulation experiments are performed on a computer with Intel (R) Core (TM) i7-4790 CPU (3.60 GHz) and 16.00 GB RAM. For fair comparisons, the population size N for all algorithms is set to 100, and the maximal number of FEs *Max_iter* is set to 1 000.

Due to the limitation of the paper space, the performance of MSHWOA is tested using six different types of benchmark test functions[12], and the information of the specific test functions is shown in Table 1. Among them, f_1, f_2 belong to single-peak function; f_3, f_4 belong to multi-peak functions; and f_5, f_6 belong to multi-peak function with fixed dimensions.

2.1 Efficiency analysis of the improvement strategy

To fully verify the effectiveness of different improvement strategies, the basic WOA is combined with these strategies separately to obtain the following algorithms: (1) WOA with the good point set sequence initialization strategy (SWOA); (2) WOA with the nonlinear convergence factor strategy (NWOA); (3) WOA with the Harris Hawks siege mechanism (HWOA); (4) WOA with all improved strategies (MSHWOA).

The effectiveness of the improved strategies is evaluated by comparing the best value (Best), mean value (Mean) and standard deviation (Std), and the experimental results are shown in Table 2.

Table 1 Benchmark test functions

Test function	Dimension	Range	Optimum value
$f_1(x) = \sum_{i=1}^{n} x_i^{\ 2}$	30	[−100,100]	0.000
$f_2(x) = \sum_{i=1}^{n} [100\ (x_{i+1} - x_i^{\ 2})^2 + (x_i - 1)^2]$	30	[−100,100]	0.000
$f_3(x) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^{\ 2}}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)) + 20 + e$	30	[−32,32]	0.000
$f_4(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^{\ 2} - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	30	[−600,600]	0.000
$f_5(x) = (\frac{1}{500} + \sum_{j=1}^{25} (j + \sum_{i=1}^{2} (x_i - a_{ij})^6)^{-1})^{-1}$	2	[−65,65]	0.998
$f_6(x) = 4x_1^{\ 2} - 2.1x_1^{\ 4} + \frac{1}{3}x_1^{\ 6} + x_1x_2 - 4x_2^{\ 2} + 4x_2^{\ 4}$	2	[−5, 5]	−1.030

Table 2 Comparison results of different improvement strategies

Function	Algorithm	*Best*	*Mean*	*Std*
f_1	WOA	6.85E-23	8.13E-18	4.56E-21
	SWOA	7.65E-36	3.96E-21	2.07E-35
	NWOA	3.56E-48	5.82E-41	5.68E-45
	HWOA	6.25E-95	8.86E-86	0.00E+00
	MSHWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**
f_2	WOA	3.94E+02	9.88E+03	2.34E-02
	SWOA	2.87E-02	3.88E+01	8.24E-02
	NWOA	2.65E-01	6.84E+03	8.57E-01
	HWOA	6.72E-02	6.90E-02	3.56E+01
	MSHWOA	**4.56E-02**	**1.03E+00**	**5.87E-01**
f_3	WOA	3.33E+00	1.31E+01	5.35E+01
	SWOA	1.72E-02	5.85E+02	3.02E+01
	NWOA	6.86E-04	7.86E-01	6.86E-02
	HWOA	3.68E-12	6.56E-08	1.35E-09
	MSHWOA	**9.62E-15**	**3.32E-14**	**9.69E-12**
f_4	WOA	2.32E+00	1.53E+02	2.87E-02
	SWOA	2.97E-04	5.09E-01	4.37E-03
	NWOA	2.85E-04	1.32E-01	3.54E-00
	HWOA	6.17E-07	5.57E-05	2.15E-04
	MSHWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**
f_5	WOA	5.32E+00	7.85E+00	2.58E+01
	SWOA	3.58E+01	2.25E+02	6.23E+01
	NWOA	5.22E+01	9.37E+03	6.85E-02
	HWOA	5.63E+02	7.86E+04	5.65E-02
	MSHWOA	**9.98E-01**	**7.85E+03**	**5.28E-01**
f_6	WOA	5.86E+02	7.55E+02	5.25E+01
	SWOA	1.73E+01	8.67E+03	6.84E-01
	NWOA	6.55E+01	8.98E+01	2.48E-02
	HWOA	−1.03E+00	8.53E+01	6.56E-04
	MSHWOA	**−1.03E+00**	**6.25E+00**	**8.84E-03**

First, SWOA has limited ability to improve the performance of the algorithm, and the good point set strategy cannot completely avoid the algorithm from falling into local optimum. HWOA can find the theoretical optimum of functions f_1 and f_4 due to the introduction of the Harris Hawks siege mechanism, which enhances the algorithm global exploration ability. Second, on the multi-peaked function f_6, most of the improved algorithms can find the optimal value of the test function, and the standard deviation of MSHWOA is the smallest, and the standard deviation performance of HWOA, NWOA, and SWOA are ranked second to fourth, which further verifies the stability of MSHWOA. Finally, MSHWOA has a great improvement in the solution accuracy and more stable performance in the search for the best solution compared with the original algorithm.

2.2 Comparative analysis of MSHWOA and other algorithms

In order to fully verify the performance of the algorithm, MSHWOA is compared with three improved swarm intelligence optimization algorithms——MSSSA[13], IChOA[14], MSCA[15] and three new swarm intelligence optimization algorithms——SCSO[16], SOA[17], and SMA[18]. Among them, MSSSA is an enhanced sparrow search algorithm incorporating adaptive parameter strategies, IChOA is an improved chimpanzee optimization algorithm based on somersault foraging strategy, MSCA is a sine cosine optimization algorithm for multilevel search through adaptive multi scale control factors, and SCSO, SOA and SMA are novel swarm intelligence optimization algorithms proposed in recent years that have been applied in different disciplines and engineering fields. The parameters of each algorithm is set up according to Refs [13 – 15]. The experimental result is shown in Table 3.

Table 3 Comparison results of solving functions by different algorithms

Function	Algorithm	*Best*	*Mean*	*Std*
f_1	MSSSA	4.26E-102	4.28E-96	9.65E-93
	IChOA	6.56E-115	6.84E-103	9.62E-100
	MSCA	8.24E-80	6.35E-75	6.26E-54
	SCSO	4.52E-45	2.25E-25	4.29E-46
	SOA	5.56E-42	1.58E-30	2.58E-25
	SMA	6.83E-25	9.84E-10	5.69E-06
	MSHWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**
f_2	MSSSA	6.42E-03	8.52E-02	9.35E-05
	IChOA	3.45E-02	6.82E-02	1.28E-01
	MSCA	1.92E-04	5.67E-02	6.88E+01
	SCSO	5.36E-03	2.97E-02	6.83E-01
	SOA	8.32E+00	1.58E+02	3.68E+02
	SMA	6.85E-03	5.26E-01	6.35E-01
	MSHWOA	**3.92E-02**	**1.03E+00**	**3.45E-01**
f_3	MSSSA	1.56E-12	6.37E-09	6.52E-05
	IChOA	8.86E-16	6.85E-14	2.58E-11
	MSCA	6.35E-06	8.67E-02	6.97E+01
	SCSO	3.25E-13	5.28E-10	9.54E-06
	SOA	9.58E-03	6.85E-01	8.34E+01
	SMA	1.21E-05	3.56E-03	5.85E-01
	MSHWOA	**5.86E-15**	**5.98E-14**	**6.62E-10**
f_4	MSSSA	2.56E-18	1.48E-14	5.24E-12
	IChOA	2.89E-05	5.64E-04	2.37E-06
	MSCA	3.27E-02	3.59E-01	5.36E+01
	SCSO	7.86E-05	6.24E-03	1.29E-06
	SOA	5.25E+00	6.36E+02	9.85E+01
	SMA	6.83E-01	7.85E+01	3.58E-01
	MSHWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**
f_5	MSSSA	7.83E+01	6.57E+03	8.87E-01
	IChOA	6.85E+01	9.52E+02	4.97E-02
	MSCA	8.95E+02	2.74E+03	6.54E+00
	SCSO	1.26E+01	5.82E+02	6.95E-02
	SOA	4.58E+00	2.58E+02	6.53E+01
	SMA	6.65E+01	5.82E+03	8.56E-01
	MSHWOA	**9.98E-01**	**3.57E+02**	**3.39E-01**

(Continued Table 3)

Function	Algorithm	*Best*	*Mean*	*Std*
f_6	MSSSA	6.85E+01	5.39E+03	3.65E+00
	IChOA	-1.03E+00	6.38E+01	2.56E-01
	MSCA	7.55E+02	8.21E+04	6.28E+01
	SCSO	-1.03E+00	6.58E+01	2.56E-01
	SOA	6.82E+01	8.56E+01	1.35E+00
	SMA	5.25E+00	4.52E+01	8.56E-01
	MSHWOA	**-1.03E+00**	**2.25E+00**	**9.62E-02**

The experimental results in Table 3 show that MSHWOA finds the theoretical optimal values of the functions f_1 and f_4. For the function f_3, there are a large number of local minima in its solution space, which makes some algorithms unable to find the global optimum of the function, and MSHWOA shows higher convergence accuracy in solving this function compared with other algorithms. For functions f_5 and f_6, MSHWOA can find its theoretical optimal value, and has higher stability than other algorithms. Therefore, MSHWOA has strong performance in finding the optimal value and shows competitive advantage in solving complex function optimization problems.

2.3 Comparison and analysis between MSHWOA and other improved WOA algorithms

MSHWOA is compared with the excellent improved whale optimization algorithm in Refs [19,20]. The parameters of the comparison algorithm are set according to the original paper, and the test function in Table 1 is used as the experimental function to compare the optimization performance. The experimental results are shown in Table 4.

As can be seen from Table 4, the optimization ability of MSHWOA based on 5 groups of test functions is obviously superior to other comparison algorithms under the same operating environment. For function f_1 –

Table 4 Comparison results of solving functions by different WOA algorithms

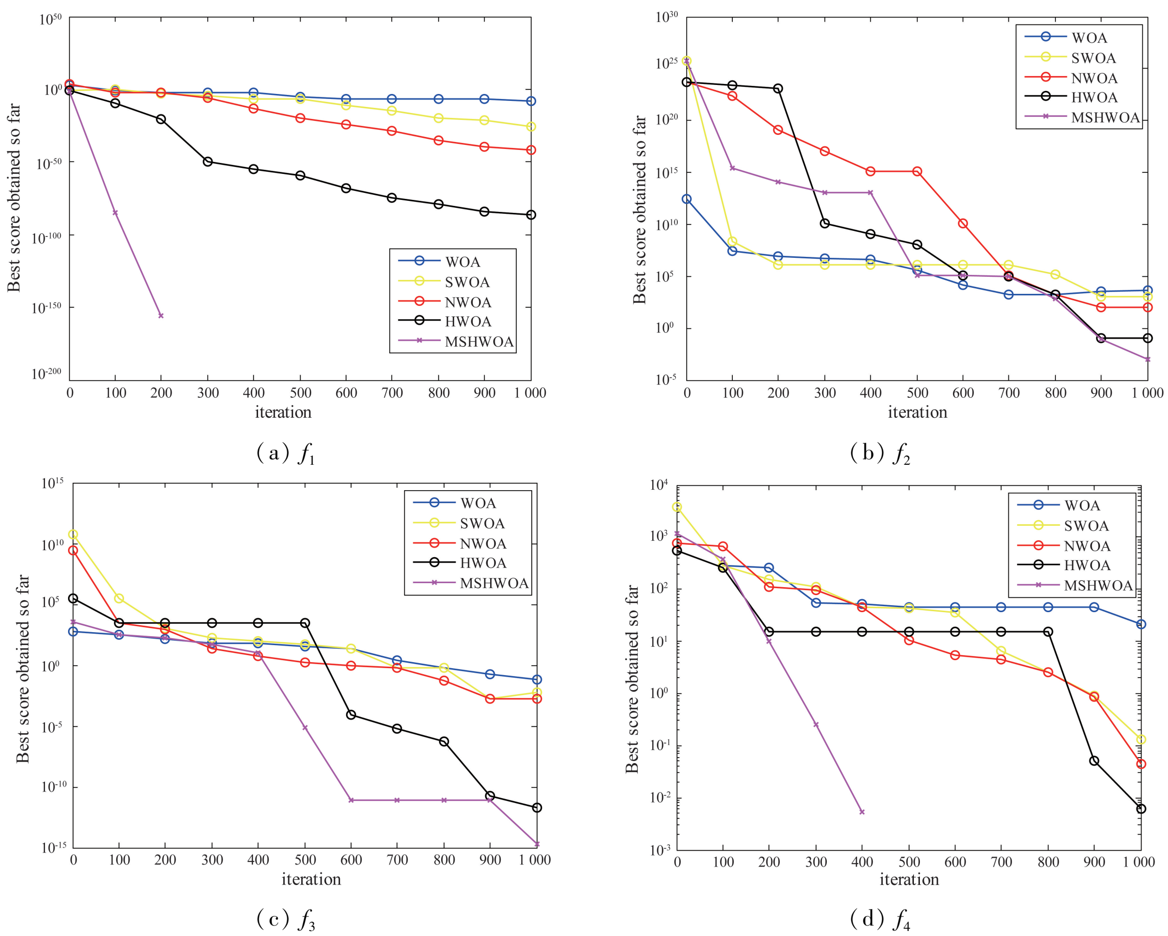
Function	Algorithm	*Best*	*Mean*	*Std*
f_1	WOA	3.69E-23	5.67E-14	1.08E-11
	DEWOA	5.83E-39	6.25E-31	2.58E-36
	EWOA	1.59E-42	9.86E-36	2.57E-38
	MSWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**
f_2	WOA	3.96E+01	6.45E+02	1.88E-01
	DEWOA	4.58E+01	5.83E+02	6.37E-00
	EWOA	8.59E-01	2.56E+01	6.83E+01
	MSWOA	**5.11E-03**	**2.54E-01**	**3.68E-01**
f_3	WOA	7.85E+01	6.36E+02	2.62E+01
	DEWOA	5.26E-08	6.35E-06	5.12E-05
	EWOA	2.67E-12	2.52E-09	6.59E-10
	MSWOA	**8.23E-16**	**6.85E-13**	**1.34E-15**
f_4	WOA	5.37E+01	1.56E+02	5.65E-01
	DEWOA	6.35E-32	5.21E-29	6.14E-24
	EWOA	7.84E-30	2.58E-23	4.13E-21
	MSWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**
f_5	WOA	9.03E+03	5.65E+05	2.86E+02
	DEWOA	3.68E+01	6.57E+01	6.12E+00
	EWOA	8.14E+01	6.39E+02	5.36E+02
	MSWOA	**9.98E-01**	**1.57E+00**	**5.87E-01**
f_6	WOA	5.86E+02	7.55E+02	5.25E+01
	DEWOA	-1.03E+00	3.58E+03	5.12E+00
	EWOA	-1.03E+00	9.35E+03	6.27E+01
	MSWOA	**-1.03E+00**	**3.83E+02**	**1.25E-02**

f_3, MSHWOA algorithm always maintains the first place in optimization performance compared with the comparison algorithm and has excellent stability. The convergence speed of MSHWOA algorithm is also improved compared with the original algorithm. For function f_5, MSHWOA algorithm can find the theoretical optimal value in the test function of multiple extreme value points. All in all, MSHWOA algorithm has excellent optimization ability and stability regardless of single-peak function or multi-peak function.

2.4 Convergence curve analysis

Convergence speed, convergence accuracy and ability to avoid local optimum are important indicators to test the optimization algorithm. To reflect the dynamic convergence characteristics of MSHWOA, the average convergence curves of the algorithm in subsection 3.1 for the four benchmark test functions f_1, f_2, f_3, and f_4 are given in Fig. 3(a) – (d).

According to Fig. 3, the convergence speed and accuracy of MSHWOA are significantly better than the rest of the comparison algorithms. For functions f_1 and f_4, the solution accuracy of MSHWOA and HWOA is much higher than the standard WOA; for the complex multi-peaked test functions, the convergence curves of some of the comparison algorithms tend to level off shortly after the beginning, making it difficult to jump out of the local optimum, while MSHWOA gradually approaches the global optimum as the number of iterations increases. Overall, MSHWOA converges faster and has a better ability to jump out of the local optimum.



(a) f_1 (b) f_2

(c) f_3 (d) f_4

Fig. 3 Average convergence curve

2.5 Rank sum test analysis

The Wilcoxon signed ranks test is performed at a significance level of 5% to examine the outcome of each run. MSHWOA and 6 comparison algorithms are subjected to 30 independent operations on five classical functions, and the experimental results are shown in Table 5, the symbols '+', '–' and '=' respectively indicate that the performance of MSHWOA is better than, worse than, and equivalent to the corresponding comparison algorithm. According to Table 5, most of the P-values on the classical test functions are less than

0.05, and the performance of the two algorithms is comparable only on individual functions, which indicates that MSHWOA is generally better than other comparison algorithms.

Table 5 Wilcoxon rank sum test analysis

Function	WOA	MSSSA	IChOA	MSCA	SCSO	SOA	SMA
f_1	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12
f_2	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12
f_3	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12
f_4	7.36E-10	7.36E-10	N/A	3.25E-12	3.25E-12	3.25E-12	3.25E-12
f_5	7.36E-10	7.36E-10	3.25E-12	3.25E-12	3.25E-12	N/A	3.25E-12
f_6	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12	3.25E-12
+/=/−	6/0/0	6/0/0	5/1/0	6/0/0	6/0/0	5/1/0	6/0/0

2.6 Friedman test

In order to further verify the significant differences between MSHWOA and other algorithms, Friedman test is adopted to conduct non-parametric tests on the test algorithms in subsection 2.2.

The basis of this test is from Table 4, and the test results are shown in Table 6. P-value represents progressive significance, which indicates that there is a significant difference between test data when its value is less than 0.01. According to Table 6, P-value is 8.35E−09 which is far less than 0.01, indicating that there are significant differences between MSHWOA and other comparison algorithms. From the average ranking value of each algorithm, MSHWOA also obtains the smallest result. Overall, the optimization ability of MSHWOA is significantly improved compared with other comparison algorithms in a statistical sense.

Table 6 Friedman test result

Algorithm	Average rank
MSSSA	4.17
IChOA	3.33
MSCA	3.50
SCSO	4.00
SOA	5.83
SMA	5.83
MSHWOA	1.17
P-value	8.35E-09

3 Conclusion

A novel algorithm named MSHWOA is proposed to address the issue of trade-off between exploration and exploitation in WOA. Firstly, the sequence of good point set is introduced to initialize the population to enhance the population diversity. Then, the nonlinear convergence factor is introduced to balance the exploration and exploitation ability of the algorithm. Finally, the siege mechanism of Harris Hawks algorithm is introduced to enhance the global search ability of the algorithm. The simulation experiments on six classical test functions prove that the improved algorithm has better search performance, and the algorithm is more stable and converges faster. In the future, using MSHWOA to solve classic engineering problems is a potential research direction.

References

[1] WANG P, WANG F. Overview of intelligent optimization algorithms from the perspective of quantum[J]. Journal of University of Electronic Science and Technology of China, 2022, 51(1): 2-15.
[2] ZHANG N, ZENG L X, HE X X, et al. Receding horizon particle swarm optimization based video defogging algorithm[J]. Control and Decision, 2021, 36(9): 2218-2224.
[3] JIA H M, LANG C B, OLIVA D, et al. Dynamic Harris hawks optimization with mutation mechanism for satellite image segmentation[J]. Remote Sensing, 2019, 11(12): 1421-1456.
[4] THARWAT A, ELHOSENY M, HASSANIEN A E, et al. Intelligent Bézier curve-based path planning model using chaotic particle swarm optimization algorithm[J]. Cluster Computing, 2019, 22(2): 4745-4766.
[5] GREFENSTETTE J J. Genetic algorithms and machine learning[J]. Machine Learning, 1988, 3(2): 95-99.
[6] MIRJALILI S, LEWIS A. The whale optimization algorithm[J]. Advances in Engineering Software, 2016(95): 51-67.
[7] CHAKRABORTY S, SAHA A K, CHAKRABORTY R, et al. HGWOA: an ensemble of hunger games search and whale optimization algorithm for global optimization[J]. International Journal of Intelligent Systems, 2022, 37(1): 52-104.
[8] CHEN H L, XU Y T, WANG M J, et al. A balanced whale optimization algorithm for constrained engineering design problems[J]. Applied Mathematical Modelling, 2019, 71: 45-59.
[9] HU B, ZHU Y H, ZHOU Y N. Simulated annealing whale

radar resource scheduling algorithm based on Cauchy mutation[J]. Journal of Northwestern Polytechnical University, 2022, 40(4): 796-803.

[10] XIAO C X, CAI Z X, WANG Y, et al. Constrained optimization evolutionary algorithm based on good lattice points principle[J]. Control and Decision, 2009, 24(2): 249-253, 258.

[11] AAHA B, SM C, HF D, et al. Harris hawks optimization: algorithm and applications[J]. Future Generation Computer Systems, 2019(97): 849-872.

[12] YAO X, LIU Y. Evolutionary programming made faster[J]. IEEE Transactions on Evolutionary Computation, 1999, 3(2): 82-102.

[13] MENG K, CHEN C, XIN B. MSSSA: a multi-strategy enhanced sparrow search algorithm for global optimization[J]. Frontiers of Information Technology & Electronic Engineering, 2022, 23(12): 1828-1847.

[14] DU N T, ZHOU Y Q, DENG W. Improved chimp optimization algorithm for three-dimensional path planning problem[J]. Multimedia Tools and Applications, 2022 (81): 27397-27422.

[15] SHEN Y X, ZHANG X F, FANG X, et al. A multi-scale sine cosine algorithm for optimization problems[J]. Control and Decision, 2022, 37(11): 2860-2868.

[16] SEYYEDABBASI A, KIANI F. Sand cat swarm optimization: a nature-inspired algorithm to solve global optimization problems[J]. Engineering with Computers, 2023, 39: 2627-2651.

[17] GAURAV D, VIJAY K. Seagull optimization algorithm: theory and its applications for large-scale industrial engineering problems[J]. Knowledge-Based Systems, 2019, 165(1): 169-196.

[18] LI S M, CHEN H L, WANG M J, et al. Slime mould algorithm: a new method for stochastic optimization[J]. Future Generation Computer Systems, 2020, 111(1): 300-323.

[19] ZENG N Y, SONG D D, LI H, et, al. Improved whale optimization algorithm and turbine disk structure optimization [J]. Journal of Mechanical Engineering, 2021, 57(20): 254-265.

[20] YAN Z P, ZHANG J Z, ZENG J, et, al. Nature-inspired approach: an enhanced whale optimization algorithm for global optimization[J]. Mathematics and Computers in Simulation, 2021, 185: 17-46.

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