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Mixing matrix estimation of underdetermined blind source separation based on the linear aggregation characteristic of observation signals①

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Abstract

Under the underdetermined blind sources separation (UBSS) circumstance, it is difficult to estimate the mixing matrix with high-precision because of unknown sparsity of signals. The mixing matrix estimation is proposed based on linear aggregation degree of signal scatter plot without knowing sparsity, and the linear aggregation degree evaluation of observed signals is presented which obeys generalized Gaussian distribution (GGD). Both the GGD shape parameter and the signals' correlation features affect the observation signals sparsity and further affected the directionality of time-frequency scatter plot. So a new mixing matrix estimation method is proposed for different sparsity degrees, which especially focuses on unclear directionality of scatter plot and weak linear aggregation degree. Firstly, the direction of coefficient scatter plot by time-frequency transform is improved and then the single source coefficients in the case of weak linear clustering is processed finally the improved K-means clustering is applied to achieve the estimation of mixing matrix. The proposed algorithm reduces the requirements of signals sparsity and independence, and the mixing matrix can be estimated with high accuracy. The simulation results show the feasibility and effectiveness of the algorithm.

Key words: underdetermined blind source separation (UBSS), sparse component analysis (SCA), mixing matrix estimation, generalized Gaussian distribution (GGD), linear aggregation

0 Introduction

Blind source separation (BSS) refers to a process of recovering source signals solely from observed signals in cases of unknown transmission channel and source signals, which has been widely applied in areas such as wireless communications, speech recognition, image processing and analysis and processing of biomedical signals, etc, with more potential application values in many other fields, therefore, BSS has always been one of the hot topics for signal processing research[1-5]. In some practical applications, the number of source signals is usually unknown and the number of observed signals is usually less than that of source signals, which is defined as the underdetermined blind source separation (UBSS). As sparse characteristics in practical applications exhibit in many signals, and such characteristics can be reflected either in time domain or in transform domain[6], most researchers have focused on sparse component analysis (SCA)-based methods for solving the problem of UBSS[7-9].

In 1999, Lee, et al[10] proposed a method of estimating a mixing matrix prior to the reconstruction of source signals, which is commonly known as the two-step method. The two-step method is a widely used method in solving sparse signal separation problems at present, and high-accuracy estimation of the mixing matrix lies at the core of this two-step method[11-13]. In recent years, many new SCA and two-step method-based algorithms have been proposed for solving UBSS, including K-means[14] degenerate unmixing estimation technique (DUET) method[15], time-frequency ratio of mixtures (TIFROM) algorithm[16], hyper-plane clustering algorithm[17] and non-linear projection column masking algorithm[18] etc. However, these algorithms assume that the signals are either sparse or weakly-sparse in most cases without further investigating the

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sparsity characteristic. In this regard, He, et al[19] proposed a method of measuring the sparsity of the signals using generalized Gaussian signals, whereby the properties of the generalized Gaussian signals and the equal probability density line were used to assess the sparsity of the signals and achieved certain results. However, this method requires source signals to be independent from one another, which is rarely true in practical applications, the accuracy of the assessment method is greatly impacted in practical applications.

To achieve precise reconstruction of source signals, it is necessary to accurately estimate the mixing matrix. Generally, the mixing matrix estimation mainly relies on the observation signals' linear aggregation characteristic in scatter plots. The higher linear aggregation degree is, the clearer the directivity it presents in the scatter plot and the more accurate the mixing matrix estimation is. Therefore, investigating the linear aggregation degree and enhancing it are particularly important for the mixing matrix estimation.

Considering usually unknown sparsity and independence of source signals in linear mixing UBSS problems, this work investigates properties of generalized Gaussian distribution, correlation coefficients and SCA, and proposes a new algorithm for mixing matrix estimation in UBSS. This algorithm first assessed the linear aggregations degree of observed signals with a combination of sparse degree and correlation coefficients. Then the specific estimation method was investigated according to different degrees of linear aggregations. The research also focuses on estimating mixing matrix with weak linear aggregation through choosing coefficients of time-frequency domain, further processing single source points and estimating the mixing. This method lowers the demands for the signal's sparsity and independence, as well as enables high accuracy estimation of the mixing matrix.

1 Problem formulation of UBSS SCA

Without considerations for noise, an instantaneous linear mixing BSS model can be expressed as

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{A} \cdot \boldsymbol{S}(\boldsymbol{t}) \quad t = 1,2,\cdots,T \tag{1}$$

where $\boldsymbol{X}(\boldsymbol{t}) = [\boldsymbol{x}_1(\boldsymbol{t}), \boldsymbol{x}_2(\boldsymbol{t}), \cdots, \boldsymbol{x}_M(\boldsymbol{t})]^{\mathrm{T}}$ represents the observed signals obtained by M sensors; $A = R^{M \times N}$ is the unknown mixing matrix; and $\boldsymbol{S}(\boldsymbol{t}) = [\boldsymbol{s}_1(\boldsymbol{t}), \boldsymbol{s}_2(\boldsymbol{t}), \cdots, \boldsymbol{s}_N(\boldsymbol{t})]^{\mathrm{T}}$ is N unknown source signals. The problem is defined as underdetermined BSS when the number of observed signals in BSS is smaller than that of source signals, i. e. when $M < N$. Sparse component analysis (SCA) methods are mainly on the basis of sparse characteristics of signals in scatter plot to estimate mixing matrix A and then solve the UBSS problem. Eq. (1) can also be written as

$$\begin{bmatrix} \boldsymbol{x}_1(\boldsymbol{t}) \\ \vdots \\ \boldsymbol{x}_M(\boldsymbol{t}) \end{bmatrix} = \begin{bmatrix} a_{11} \\ \vdots \\ a_{M1} \end{bmatrix} \boldsymbol{s}_1(\boldsymbol{t}) + \begin{bmatrix} a_{12} \\ \vdots \\ a_{M2} \end{bmatrix} \boldsymbol{s}_2(\boldsymbol{t}) \cdots + \begin{bmatrix} a_{1N} \\ \vdots \\ a_{MN} \end{bmatrix} \boldsymbol{s}_N(\boldsymbol{t}) \tag{2}$$

In UBSS, the source signals are not only required to be independent of each other[20] but also generally assumed to be strong sparse signals, which means at any time t_k there exists only one source, and then Eq. (2) can be formulated as

$$\begin{bmatrix} \boldsymbol{x}_1(\boldsymbol{t}_k) \\ \vdots \\ \boldsymbol{x}_M(\boldsymbol{t}_k) \end{bmatrix} = \begin{bmatrix} a_{11} \\ \vdots \\ a_{M1} \end{bmatrix} \boldsymbol{s}_j(\boldsymbol{t}_k) \tag{3}$$

And Eq. (3) can be deduced as

$$\frac{\boldsymbol{x}_1(\boldsymbol{t}_k)}{a_{1j}} = \frac{\boldsymbol{x}_2(\boldsymbol{t}_k)}{a_{2j}} = \cdots = \frac{\boldsymbol{x}_M(\boldsymbol{t}_k)}{a_{Mj}} \tag{4}$$

It can be seen from Eq. (4) that the source signals are independent and sparse, the direction of the straight line is determined by the j^{th} column of the mixing matrix A. If mixing matrix A is column full rank, i. e. column vector is not correlated, the column number is equal to the number of source signals and it will present distinct direction in scatter plot. Nowadays, researchers have investigated some approaches to estimate the mixing matrix, such as potential function, K-means clusters, fuzzy clusters which are mainly according to the linear aggregation in scatter plot. If the degree of linear aggregation is strong which results in distinct direction and then a high-accuracy estimation of mixing matrix is achieved by directly using clustering algorithm. In reality, the independence and the source signal sparsity are unknown, so is the degree of the linear aggregation. Most of the existing algorithms are assumed that the signals are strong or weak sparse, however they do not discuss the aggregation degree and its effect to the estimation accuracy of mixing matrix.

This study focuses on two aspects: one is how to measure the sparsity degree for given signals, the other is how to improve the sparsity to estimate the mixing matrix. So a method to evaluate the observation signals sparsity and further estimate the mixing matrix is proposed. The related theories and processing steps now are introduced as follows.

2 Linear aggregation degree measure based on GGD and correlation coefficient

It's obvious that the linear aggregation clarity directly influences the estimation accuracy of mixing matrix; therefore it's necessary to study the linear aggregation characteristics and the factors.

Many physical signals statistically obey or approximately obey generalized Gaussian distribution (GGD), which makes GGD signals have attracted much attention in signal processing field. This paper investigates the important parameters of GGD and the correlation coefficients, whereby it puts forward to a new measurement of signal linear aggregation characteristic which is helpful to estimate mixing matrix accurately.

Generalized Gaussian distribution is a kind of symmetric distribution whose special cases are normal distribution and Laplacian distribution, and its limit forms are δ function and uniform distribution. The probability density function (PDF) of the GGD family is shown in[21]

$$f(x;\mu,\alpha,\beta) = [\frac{\partial}{2\beta\Gamma(1/\alpha)}]\exp\{-[\frac{|x-\mu|}{\beta}]^{\alpha}\},\quad -\infty < x < \infty \qquad (5)$$

where x, u, $\alpha > 0$ and $\beta > 0$ represent variable, mean, shape parameter and scale parameter respectively. $\Gamma(m) = \int_0^{+\infty} e^{-t} t^{m-1} \mathrm{d}t$ is the gamma function. The PDF of GGD family is determined by shape parameter α and scale parameter β: α determines the PDF's contour and is relevant with PDF's attenuation speed whereas β determines PDF's wave crest width of GGD.

Parameter α is calculated as

$$\alpha = f^{-1}(\frac{\Gamma^2(2/\alpha)}{\Gamma(3/\alpha)\Gamma(1/\alpha)}) \qquad (6)$$

Generally there is certain relationship between α and β in the following:

$$\beta = \sigma[\Gamma(1/\alpha)/\Gamma(3/\alpha)]^{\frac{1}{2}} \qquad (7)$$

where σ is the standard deviation. From Eq.(7) it concludes that the GGD probability density function actually is determined only by shape parameter α. Studies show when α infinitely approximates zero, the GGD probability density function is approximate to δ function which indicates a strong sparse signal; when α is equal to 1, GGD is degraded into Laplacian distribution, which is a sparse signal; when α is equal to 2, GGD is further degraded into Gaussian distribution and signal's sparsity becomes unapparent; and when α approaches $+\infty$, GGD tends to uniform distribution infinitely, which is a non-sparse signal. Therefore, the smaller the shape parameter α is, the sparser the signal is and the stronger the linear aggregation is.

One of requirement of UBSS is independent of each other, actually it is difficult to ensure the signals are completely independent in practical situations. The signal linearity degree is influenced not only by sparse linear extent but also by the signal's independence. The traditional independence between signals is defined by the probability density function. However this work studies the signals correlation combined with the entropy of information theory, and defines center entropy on the basis of the center of the signal related to the independence and the entropy measure[22]. Center entropy is defined as the center of the correlation:

$$u_{\sigma}(X,Y) = \frac{1}{N}\sum_{i=1}^{N} G_{\sigma}(x_i - y_i) - \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} G_{\sigma}(x_i - y_i) \qquad (8)$$

where X, Y represent any two signals; N is the signal length; G_{σ} Gaussian kernel $\sigma = \sigma_x[4N^{-1}(2d+1)^{-1}]^{\frac{1}{d+4}}$, ($d$ is dimension signal; σ_x the signal standard deviation). Further combined with signal's correlation, the center corr-entropy is defined as the measure of independence as

$$\gamma(X,Y) = \max(|u(X,Y)|, |u(-X,Y)|) \qquad (9)$$

The center corr-entropy γ is a large value, which means that the relevance of the signals is weak and the signal is of stronger independence; similarly γ is a small value, which means that the signals relevance is stronger and the signal is of weaker independence. If the signals have stronger independence, its center corr-entropy is near to 1; and on the contrary weak independence means center corr-entropy near to 0.

In summary, the center corr-entropy of signals is bigger and the shape parameter is smaller, which means that the linear aggregation degree of signals is stronger and there will have clear directional lines in scatter plot. The center corr-entropy of signals is smaller and the shape parameter is bigger, which means that the linear aggregation degree of signals is weaker and the scatter plot showing directional blur even without showing directionality. Therefore, shape parameter α and center corr-entropy γ can be used to measure signals linear aggregation degree.

3 Mixing matrix estimation under weak linear aggregation

Mixing matrix $\boldsymbol{A}$ can be estimated directly using the clustering method with the clear line direction of observation signals. On the contrary, $\boldsymbol{A}$ is estimated with large errors even can not be estimated with weak direction. Generally, many actual signals are not sparse in time domain and there appear certain but not enough sparsity in the transformation domain which results in obscurely linear aggregation. This study proposes to detect and choose the transformation coefficients of time-frequency domain and it results in strong linear aggregation.

3.1 Enhance linear aggregation based on linear coefficients

Eq. (1) is processed by short-time Fourier transform (STFT) and it obtains the following:

$$\boldsymbol{X}(\boldsymbol{t},\boldsymbol{k}) = \sum_{i=1}^{n} \boldsymbol{a}_i \boldsymbol{S}_i(\boldsymbol{t},\boldsymbol{k}) \quad (10)$$

Assume that only source signal $s_1(t_1, k_1)$ is non-zero on a certain (t_1, k_1) in time-frequency domain while the other signals are all zero, then there exists:

$$\boldsymbol{X}(\boldsymbol{t}_1,\boldsymbol{k}_1) = \begin{bmatrix} x_1(t_1,k_1) \\ \vdots \\ x_M(t_1,k_1) \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{M1} \\ \vdots & \ddots & \vdots \\ a_{1N} & & \\ \cdots & & \\ a_{MN} & & \end{bmatrix} \begin{bmatrix} s_1(t_1,k_1) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11}s_1(t_1,k_1) \\ \vdots \\ a_{M1}s_1(t_1,k_1) \end{bmatrix} \quad (11)$$

Respectively dividing the real and imaginary components of any two observed signals in Eq. (11) and the following ratio is obtained:

$$\frac{\mathrm{Re}[X_i(t_1,k_1)]}{\mathrm{Re}[X_j(t_1,k_1)]} = \frac{\mathrm{lm}[X_i(t_1,k_1)]}{\mathrm{lm}[X_j(t_1,k_1)]} = \frac{a_{i1}}{a_{j1}} \quad (12)$$

where $i \in M$, $j \in M$, and $i \neq j$. It can conclude that the ratios a_{i1}/a_{j1} between the real and imaginary components of any two observed signals are equal and at the same direction with column vector of mixing matrix. Similarly in other case of only single source signal, the ratios between the real and imaginary components of any two observed signals would also be equal.

More generally, it is assumed that the ratio of the i^{th} column of the mixing matrix A is equal to the ratio of coefficients which aggregates on the i^{th} column of mixing matrix in time frequency point (t_Z, k_Z). So Eq. (13) is got:

$$\frac{X_2(t_Z,k_Z)}{X_1(t_Z,k_Z)} = \frac{a_{2i}S_i(t_Z,k_Z)}{a_{1i}S_i(t_Z,k_Z)} \quad (13)$$

The simplest case of Eq. (13) is that only one signal source presents while other source signals are inactive or offsetting each other at point (t_Z, k_Z) Eq. (13) can be written as

$$\frac{\mathrm{Re}[X_2(t_Z,k_Z)]}{\mathrm{Re}[X_1(t_Z,k_Z)]} = \frac{\mathrm{lm}[X_2(t_Z,k_Z)]}{\mathrm{lm}[X_1(t_Z,k_Z)]} = \frac{a_{2i}}{a_{1i}} \quad (14)$$

It can conclude from Eq. (14) that the ratio of coefficient real component is equal to the imaginary component radio as well as the ratio of mixing matrix corresponds column. Consequently, it can define these coefficients as linear coefficients which have strong linear aggregation in the time-frequency domain and can exhibits distinct directionality in the scatter plot. This work enhanced linear aggregation degree based on the linear coefficients.

However, actually the values of $\mathrm{Re}[X_i(t,k)]/\mathrm{Re}[X_j(t,k)]$ and $\mathrm{lm}[X_i(t,k)]/\mathrm{lm}[X_j(t,k)]$ in Eq. (12) are not completely equal and certain discrepancies exist between these values. Considering the actual feasibility, threshold value ω is set and Eq. (13) can be expressed as

$$\left| \frac{\mathrm{Re}[X_i(t,k)]/\mathrm{Re}[X_j(t,k)] - \mathrm{lm}[X_i(t,k)]/\mathrm{lm}[X_j(t,k)]}{\mathrm{Re}[X_i(t,k)]/\mathrm{Re}[X_j(t,k)] + \mathrm{lm}[X_i(t,k)]/\mathrm{lm}[X_j(t,k)]} \right| < \omega \quad (15)$$

where ω is an empirical value with a range of $(0, 0.1)$ which is modified according to the signals. After linear coefficients process, the linear aggregation of observation signals is enhanced.

3.2 Mixing matrix estimation

Assume that $M = 2$, these linear coefficients are subject to unitization processed by Eq. (16) and a set U is obtained:

$$U(t_Z,k_Z) = \frac{X(t_Z,k_Z)}{\| X(t_Z,k_Z) \|_2} \quad (t_Z,k_Z) \in \Omega_Z \quad (16)$$

where $X(t_Z, k_Z)$ is the linear coefficient selected from time-frequency domain, and Ω_Z is a set of all selected linear coefficients. These vectors in Ω_Z are all unit vectors, and by further calculating the ratios among the vectors it obtains a set H:

$$h_s = \frac{u_2(t_S,k_S)}{u_1(t_S,k_S)}, \quad S \in Z,\ h \in H^{L\times Z} \quad (17)$$

The set H is classified by clustering method and if any two h_m, h_n in H are equal or approximately equal, they are classified into same category.

The conventional K-means algorithm is sensitive to initial clustering centers, whereby differences in clustering centers would greatly affect the clustering results. Therefore, the optimization of initial clustering centers plays a very important role in K-means algorithm. The initial clustering centers are determined by the centers of selected areas which have high data density of set H. The conventional K-means clustering algorithm is then used to classify these data in set H. Denote the positions of the same category as $\{C_n \mid n \in N\}$, it's correct that the positions of the same category in set H correspond to the same category positions in set U. The mean value $\bar{h}$ of each category, i. e. the mean slope, is calculated by

$$\bar{h}_n = \frac{1}{sum(C_n)} \sum h_{C_n} \quad (18)$$

where $sum(C_n)$ is the number of all vectors in the n^{th} category, $\sum h_{C_n}$ is the sum of all elements in the n^{th} category. The estimation of mixing matrix column vectors is achieved by

$$\hat{a}_n = [\cos(\arctan \bar{h}_n), \sin(\arctan \bar{h}_n)]^T n \in N \quad (19)$$

4 Algorithm simulation and result analysis

The proposed method is used to process three different cases which belong to UBSS and the simulation results and analysis are presented. The normalized mean square error (NMSE) is applied to evaluate the mixing matrix accuracy and the comparisons with other methods are also presented.

The normalized mean square error is defined as

$$\text{NMSE} = \text{E}\left(\frac{\| \tilde{\boldsymbol{A}} - \boldsymbol{A} \|_2^2}{\| \boldsymbol{A} \|_2^2}\right) \quad (20)$$

where $\tilde{\boldsymbol{A}}$ is the estimated mixing matrix, and $\boldsymbol{A}$ is the original mixing matrix.

(1) Strong linear aggregation, i. e. shape parameter α and correlation coefficient r are small:

Consider the case: $M = 2$ and $N = 6$, which means six source signals and two observation signals. The six source signals are intercepted from flute acoustical signals of Bofill's study[23] and sampling points are 32768. Mixing matrix $\boldsymbol{A}$ is generated randomly, and its column vectors are normalized as:

$$\boldsymbol{A} = \begin{bmatrix} 0.1016 & 0.5045 & 0.4313 & 0.7845 & 0.0594 & 0.9121 \\ -0.9948 & -0.8634 & 0.9022 & 0.6202 & 0.9982 & -0.4099 \end{bmatrix}$$

The two observation signals are generated in accordance with Eq. (1) and shown in Fig. 1.

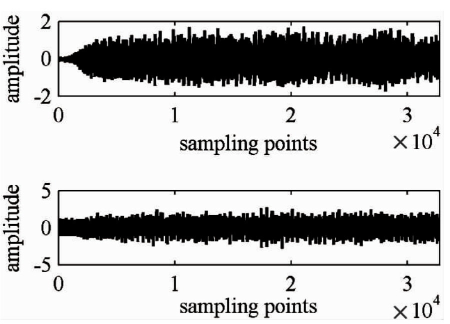


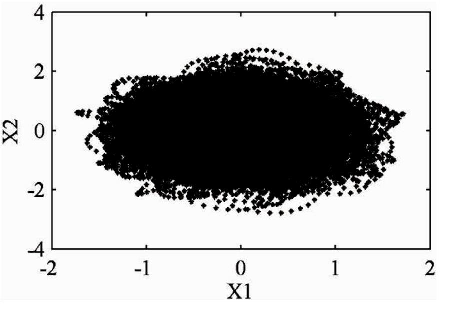
Fig. 1 Waveform of observation signals in the time domain

Obeying the generalized Gaussian distribution, shape parameter α and correlation coefficient r of two observation signals are calculated in the time domain (TD) and time-frequency domain (TFD), respectively, and the result is shown in Table 1.

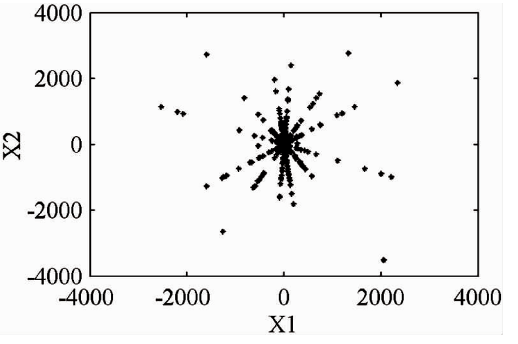
Table 1 Shape parameter and correlation coefficient

shape parameters α in TD	shape parameters α in TFD	center corr-entropy r
2.2150	0.0820	0.000912

Table 1 shows that shape parameter α is greater than two in TD and becomes small in TFD, and correlation coefficient of the two observed signals is small. Therefore it concludes that observation signals will present strong linear aggregation and distinct directionality in time-frequency domain. Fig. 2 illustrates the conclusion: (1) is scatter plot of transformation coefficients in TD and (2) is scatter plot of linear coefficients in TFD.



(a) Scatter plot in TD



(b) Scatter plot in TFD

Fig. 2 Scatter plots of source points in TD and TFD

Fig. 2 verifies the conclusions and therefore mixing matrix $\boldsymbol{A}$ can be estimated directly using improved K-means clustering algorithm and it can obtain the estimation result:

$$\hat{\boldsymbol{A}} = \begin{bmatrix} 0.1026 & 0.5050 & 0.4309 & 0.7840 & 0.0584 & 0.9118 \\ -0.9947 & -0.8631 & 0.9024 & 0.6207 & 0.9982 & -0.4106 \end{bmatrix}$$

The estimation error was $\min \| \hat{\boldsymbol{A}}p - \boldsymbol{A} \|_2 = 0.00044$, where $p \in \boldsymbol{P}$ and $\boldsymbol{P}$ is a unit substitution mean value set[24]. In order to assess the matrix estimation accuracy by the proposed algorithm, the mixing matrix is estimated by the method mentioned in

Ref. [23] and the result is shown as follows:

$$\bar{A} = \begin{bmatrix} 0.0882 & 0.5225 & 0.4205 & 0.7934 & 0.0724 & 0.9026 \\ -0.9961 & -0.8526 & 0.9073 & 0.6088 & 0.9974 & -0.4305 \end{bmatrix}$$

Calculate normalized mean square errors of $\hat{A}$ by the proposed algorithm which can be named as Lcluster algorithm and $\bar{A}$ by Bofill's algorithm[23], respectively, the comparison is shown in Fig. 3.

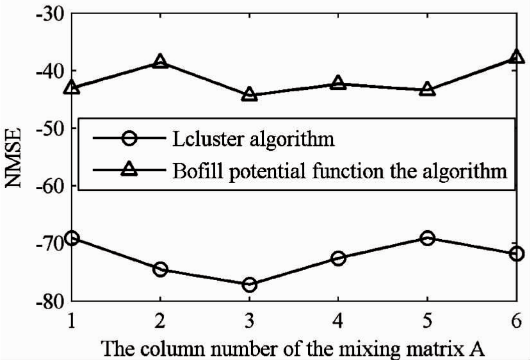


Fig. 3 NMSE comparison of Bofill's and Lcluster algorithm

Fig. 3 shows that NMSE of estimation by the proposed algorithm is significantly smaller than that by Bofill potential function algorithm.

(2) Weak linear aggregation, i. e. large shape parameter α and small correlation coefficient r:

Consider the case: $M = 2$ and $N = 4$, i. e. four source signals and two observed signals. The four source signals are actual speech signals from http://www.speech.cs.cmu.edu/cmu_arctic/. The sampling points of each source signals are 35920, and the mixing matrix is

$$A = \begin{bmatrix} 0.7635 & 0.5029 & -0.1667 & -0.9039 \\ 0.6458 & 0.8644 & 0.9860 & 0.4277 \end{bmatrix}$$

The two observation signals are generated according to Eq. (1) and shown in Fig. 4.

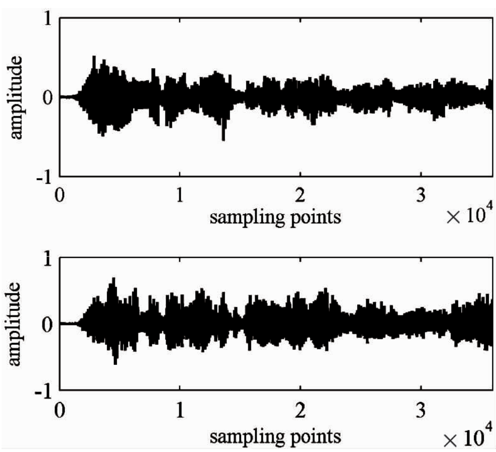


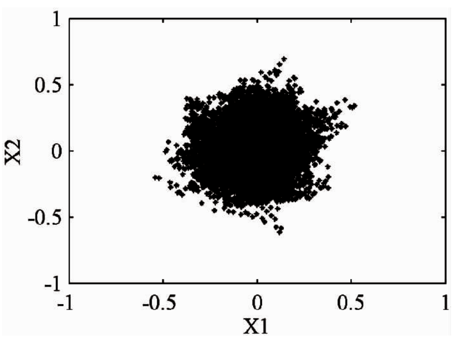
Fig. 4 Waveform of observation signals in time domain

Obeying the generalized Gaussian distribution, shape parameter α and correlation coefficient r of two observation signals are calculated in TD and TFD, respectively, and the result was shown in Table 2.

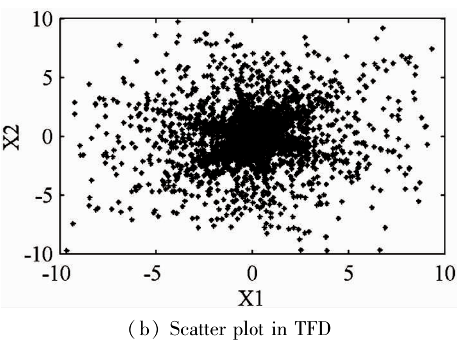
Table 2 Shape parameter and correlation coefficient

shape parameters α in TD	shape parameter α in TFD	correlation coefficient r
1.9200	0.4530	0.0973

It can be seen from Table 2 that the correlation coefficient is small, while the shape parameter is not sufficiently small both in TD and in TFD, So it is deduced that the signals will present weak linear aggregation in the TFD. Fig. 5 includes scatter plots of two observation signals in TD and TFD, respectively.



(a) Scatter plot in TD



(b) Scatter plot in TFD

Fig. 5 Scatter plots of source points in TD and TFD

Fig. 5(b) shows that the scatter plot of observation signals presented obscure directivity in TFD and the conclusion is verified by Fig. 5. The linear coefficients in TFD are selected according to Eq. (16), wherein ω was selected as 0.025. The scatter plot of linear coefficients is shown in Fig. 6.

These linear coefficients are used to estimate mixing matrix in accordance with the proposed method in Section 4.2 and obtain the estimated value of mixing matrix:

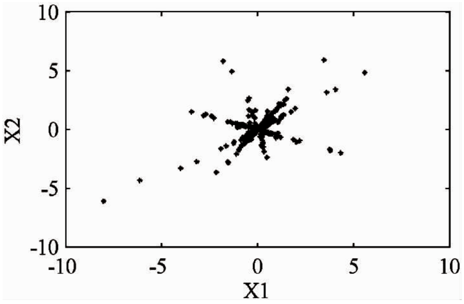


Fig. 6 Scatter plot of linear coefficients in TFD

$$\hat{A} = \begin{bmatrix} 0.7628 & 0.5019 & -0.1656 & -0.9034 \\ 0.6466 & 0.8649 & 0.9862 & 0.4288 \end{bmatrix}$$

The estimation error is $\min \| \hat{\boldsymbol{A}}p - \boldsymbol{A} \|_2 = 0.0011$, which indicates that estimation result is relatively accurate using the proposed algorithm. Under this circumstance, the mixing matrix can not be estimated by Bofill's algorithm. In order to assess the estimation accuracy Lcluster is compared with another method in Ref. [25] called V. G. Reju algorithm. The NMSE comparison result is shown in Fig. 7.

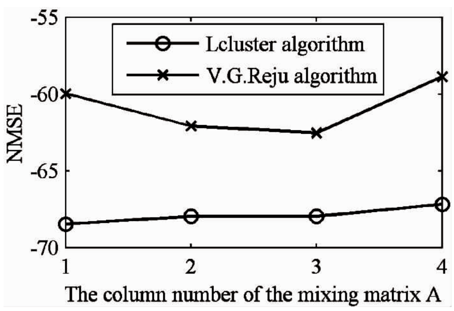


Fig. 7 NMSE comparison with Lcluster and VG. Reju algorithm

Simulation shows that NMSE of estimated mixing matrix by proposed algorithm is significantly smaller than that by V. G. Reju algorithm.

5 Conclusions

This study investigates the mixing matrix estimation under UBSS by studying the linear aggregation characteristic and correlation effect of observation signals. In order to accurately estimate the mixing matrix with unknown sparsity, the relationship between shape parameter and sparsity of observation signals following the generalized Gaussian distribution is studied. The effects of shape parameter and correlation coefficient of the observed signals on the linear aggregation character is focussed, and then the specific processing algorithms aimed at different linear aggregations are adopted. The mixing matrix can be estimated directly using the improved K-means algorithm if the linear aggregation is strong; on the contrary, mixing matrix is estimated by selecting the appropriate single source coefficients based on linear coefficients before applying the improved K-means. Experiment results show that the proposed method can effectively estimate the mixing matrix with different sparsity while retaining the characteristics of low computational complexity and high estimation accuracy.

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