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Federal extended Kalman filter based on reconstructed observation in incomplete observations①

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Abstract

In the estimation and identification of nonlinear system state, aiming at the adverse effect of observation missing randomly caused by detection probability of used sensor which is less than 1, a novel federal extended Kalman filter (FEKF) based on reconstructed observation in incomplete observations (ROIO) is proposed in this paper. On the basis of multi-sensor observation sets, the observation is exchanged at different times to construct a new observation set. Based on each observation set, an extended Kalman filter algorithm is used to estimate the state of the target, and then the federal filtering algorithm is used to solve the state estimation based on the multi-sensor observation data. The effect of the sensor probing probability on the filtering result and the effect of the number of sensors on the filtering result are obtained by the simulation experiment, respectively. The simulation results demonstrate effectiveness of the proposed algorithm.

Key words: multi-sensor observation, incomplete observations (IO), federal extended Kalman filter (FEKF), reconstructed observation

0 Introduction

In the application of military army and industry, due to obstacle occlusion, sensor failure, external environment mutation and other uncertain factors, result in the detection probability of sensors is less than 1, so a corresponding estimation problem becomes an estimated problem with incomplete observation[1]. How to create a model for incomplete observation, Nahi, et al. proposed a linearly optimal filter in the mean square sense, the observation data is in the case where the loss probability a priori information is known at each sampling time and describes the loss of randomness of the measured data by adding a sequence of independent identically distributed random variables subject to the Bernoulli distribution to the state observation equation. 0 and 1 were used to describe the random loss of the observation data[2], where 1 indicated that the system had received the observation data and 0 indicated that the observation data was missing at any moment. Haddi[3], et al. further promoted the approach proposed by Nahi, by using the 0 and 1 random variable sequences that were not distributed independently of each other to describe the random loss of the measured data and got a corresponding filtering method. NaNacara[4], et al. used random sequence 0 and 1 to build a model, which was independent of each other, but had different distributions at different moments.

Aiming at realizing system state estimation with incomplete observation, some scholars have conducted in-depth researches and discussions, and then achieved certain results. For example, Sinopoli[5], et al. gave the correlated results of Kalman filter with incomplete observation, and discussed the threshold of the detection probability when the filter reached convergence, and the upper and lower bounds of the estimated covariance. Xu[6], et al. studied the relationship between the modified Riccati equation and the data loss position of the discrete system, and the expected convergence problem of the filter variance with incomplete observation, which provided a theoretical basis for the use of incomplete observation information in state estimation. Craig[7] studied the problem of state estimation in the irregular data loss. Boers[8], et al. discussed the optimal estimation problem with incomplete observation. Wang[9] gave the next recursive estimator based on the minimum mean square error. Gao[10], et al. studied

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the fuzzy filtering problem of nonlinear systems with incomplete observation. The above literatures summarized the state estimation model with incomplete observation and proposed some filtering methods to solve the problem of incomplete observation. However, the above methods are only applied to the single sensor. If the data disturbance arises, the filter accuracy will still be affected. Therefore, Carlson[11], et al. proposed a federated filter algorithm, where observation information was filtered by multi-sensor respectively and then used the weighted fusion method for multi-sensor data fusion. Qiu[12], et al. applied the traditional adaptive filtering to the federal filter algorithm, by compensating the information distribution coefficients, correcting the process noise and covariance of the unknown system. The above methods are applicable to linear problems, while in the military and engineering, non-linear problems are prevalent. Aiming at the nonlinear system problem, Zhao[13], et al. proposed cubature Kalman filter algorithm with incomplete observation Gao[14], et al. proposed UCMKF filter design for multi-channel probing probabilistic coupling with incomplete observation. Xu[15], et al. proposed the federal extended Kalman filter method, in which each sensor weighting the state estimate of the target is to obtain the final target estimate. Federated Kalman filter with high precision and good stability is widely used[16,17].

With the development of military and industry, the accuracy of the multi-sensor system under incomplete observation is put forward for higher requirement. On one hand, though using a more accurate sensor can make the observation error small, yet it will increase the hardware cost. On the other hand, though increasing the number of sensors and obtaining more abundant state information, this approach also increases hardware cost. Aiming at the above problem, this paper proposes a novel federal extend Kalman filtering algorithm based on reconstructed observation in incomplete observations (FEKF-ROIO). On the basis of the multi-sensor observation set, a new observation set is reconstructed to realize the reuse of the observation information, and then through the extended Kalman filter algorithm, to achieve system state estimation. This algorithm can improve the state estimation accuracy of the system by making full use of the redundant and complementary information of incomplete observations without increasing hardware cost.

1 Federal extended Kalman filter based on incomplete observation (FEKF-IO)

The typical performance of incomplete observations is the loss of randomness of the observations data, in order to build the model, a variable sequence satisfying the Bernoulli distribution is used to indicate whether the measured data is lost at moment. The system model is shown below.

$$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}) + \boldsymbol{\Gamma}_{k-1}\boldsymbol{w}_{k-1} \tag{1}$$

$$\boldsymbol{z}_k = \gamma_k \boldsymbol{h}_k(\boldsymbol{x}_k) + \boldsymbol{v}_k \tag{2}$$

Among them, $k \in \mathbb{N}$ is the time indicator, $\boldsymbol{x}_k$ and $\boldsymbol{z}_k$ indicate the state vectors and observation vectors at time k, respectively. $\boldsymbol{f}$ and $\boldsymbol{h}_k$ indicate system state evolution mapping and observation mapping, respectively. $\boldsymbol{\Gamma}_{k-1}$ indicates process noise distribution matrix. $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ indicate process evolution noise and observation noise, respectively. $\boldsymbol{w}_k$ satisfies Gaussian white noise characteristics. $\boldsymbol{w}_k \sim (0, \boldsymbol{Q}_k)$. $\boldsymbol{v}_k$ no longer meets the distribution $\boldsymbol{v}_k \sim (0, \boldsymbol{R}_k)$ at any k moment, and uses the following model to reflect the loss of random observation data. $\boldsymbol{v}_k \sim \begin{cases} N(0, \boldsymbol{R}_k) & \gamma_k = 1 \\ N(0, \sigma^2 \boldsymbol{I}) & \gamma_k = 0 \end{cases}$. $\boldsymbol{w}_k$, $\boldsymbol{v}_k$ and the initial state $\boldsymbol{x}_0 \sim (\bar{\boldsymbol{x}}_0, \boldsymbol{P}_0)$ are independent of each other. The addition of γ_k in the observation equation indicates the loss of randomness of the measured data. When $\gamma_k = 1$, reception of observation data is normal, the condition of the standard extended Kalman filter algorithm is satisfied. When $\gamma_k = 0$, observation data is lost, at this time $\sigma \to \infty$, the observation noise variance of the system is infinitely large, and data is extremely inaccuracy. γ_k has a probability distribution of $p(\gamma_k = 1) = \lambda_k$ at k moment. $0 < \lambda_k \leqslant 1$. $p(\cdot)$ indicates the probabilistic operation. λ_k indicates probing probability.

The federated extended Kalman filter algorithm uses a weighted fusion method in multi-source information fusion thought to provide a typical multi-sensor filtering fusion method. After the observation information of the multiple sensors is decentralized processing, then it focus on integration processing. Each sensor corresponds to a filter (extended Kalman filter), the filters work in parallel, the outputs (local state estimation) are processed by the main filter fusion, getting the global state estimate. Each filter is independent of each other, and the global estimation accuracy is higher than the local estimation. If the multi-sensor system has M sensors, the federated extended Kalman filter equation for the m sensor with incomplete observation is given below.

$$\hat{\boldsymbol{x}}_{k|k-1,m} = \boldsymbol{f}(\hat{\boldsymbol{x}}_{k-1|k-1,m}) \tag{3}$$

$$\boldsymbol{P}_{k|k-1,m} = \boldsymbol{f}_{k-1,}P_{k-1|k-1,m}\boldsymbol{f}_{k-1,}^{\mathrm{T}} + \boldsymbol{\Gamma}_{k-1,m}Q_{k-1,m}\boldsymbol{\Gamma}_{k-1,m}^{\mathrm{T}} \tag{4}$$

$$\tilde{\boldsymbol{z}}_{k|k-1,m} = \boldsymbol{z}_{k,m} - \boldsymbol{h}_{k,m}(\boldsymbol{x}_{k,m} - \hat{\boldsymbol{x}}_{k|k-1,m}) \tag{5}$$

$$\boldsymbol{K}_{k,m} = \boldsymbol{P}_{k|k-1,m}\boldsymbol{h}_{k,m}^{\mathrm{T}}[\boldsymbol{h}_{k,m}\boldsymbol{P}_{k|k-1,m}\boldsymbol{h}_{k,m}^{\mathrm{T}} + \boldsymbol{R}_{k,m}]^{-1} \tag{6}$$

$$\hat{\boldsymbol{x}}_{k|k,m} = \hat{\boldsymbol{x}}_{k|k-1,m} + \gamma_{k,m}\boldsymbol{K}_{k,m}(z_{k,m} - \boldsymbol{h}_{k,m}\hat{\boldsymbol{x}}_{k|k-1}) \tag{7}$$

$$\boldsymbol{P}_{k|k,m} = \boldsymbol{P}_{k|k-1,m} - \gamma_{k,m}\boldsymbol{K}_{k,m}\boldsymbol{h}_k\boldsymbol{P}_{k|k-1,m} \tag{8}$$

Among them

$$\boldsymbol{f}_{k-1} = \left.\frac{\partial \boldsymbol{f}_x(k-1)}{\partial \boldsymbol{x}_m}\right|_{x_m = \hat{x}_{k-1,m}}, \quad \boldsymbol{h}_{k,m} = \left.\frac{\partial \boldsymbol{h}_{x_m}(k)}{\partial \boldsymbol{x}_m}\right|_{x_m = \hat{x}_{k|k-1,m}}$$

$\boldsymbol{K}_{k,m}$ indicates the local state filter gain matrix of the m filter at k moment, and is used to measure the degree of utilization of the latest observation information in the current time state estimation.

For M local state estimates $\hat{\boldsymbol{x}}_{1|1,m}$, $\hat{\boldsymbol{x}}_{2|2,m}, \cdots, \hat{\boldsymbol{x}}_{k|k,M}$ and corresponding estimation error covariance matrixs $\boldsymbol{P}_{1|1,m}$, $\boldsymbol{P}_{2|2,m}, \cdots, \boldsymbol{P}_{k|k,M}$, the local estimates are independent of each other. The global optimal estimate can be expressed as

$$\hat{\boldsymbol{x}}_{k|k} = \left(\sum_{m=1}^{M}\boldsymbol{P}_{k|k,m}^{-1}\right)^{-1}\sum_{m=1}^{M}\boldsymbol{P}_{k|k,m}^{-1}\hat{\boldsymbol{x}}_{k|k,m} \tag{9}$$

2 Federal extended Kalman filtering based on reconstructed observation in incomplete observations

In the system state estimation, in order to make full use of incomplete observation information and improve the accuracy of state estimation, this paper proposes reconstructed observation of federal extended Kalman filtering based on incomplete observations. For the estimated system, in the context of incomplete observation, a plurality of sensors with probing probability less than 1 are used. The sensor observation sets are used to exchange the observation and build the new observation sets, and use the federal extended Kalman filter algorithm to achieve the system state estimation, and obtain a higher accuracy of the state estimates by weighted fusion algorithm.

2.1 Reconstructed observation strategy in incomplete observations

In the federal extended Kalman filter, the observation information provides the innovation for the filter to achieve the correction of the one-step prediction estimation result, so the observation information has a direct impact on the filter accuracy. The observation set of the sensor is $Z_{k,m} \triangleq \{z_{1,m}, z_{2,m}, \cdots, z_{k,m}\}$, $(k = 1, 2, \cdots, K;\ m = 1, 2, \cdots, M;\ M \geqslant 2)$, where k indicates the sampling time of each sensor, m indicates the number of sensors. When the number of sensors m is odd, sensors can be randomly composed of $(m-1)/2$ pairs of the sensor groups, one more sensor observation set does not participate in the reconstructed observation set. When the number of sensors m is even, sensors can be randomly composed of $m/2$ pairs of sensor groups. By randomly selecting a sensor group consisting of two sensors, recorded as sensor 1 and sensor 2, the observation information of sensor 1 and sensor 2 are exchanged once for the interval of a sampling time. For simplicity, observation information is interchanged at the even moment, the observation information remains unchanged at the odd moment, and the new observation set is reconstructed, which consists of sensor 3 and sensor 4, respectively. $Z_{k,3} = \begin{cases} z_{k,1} & k \text{ is odd} \\ z_{k,2} & k \text{ is even}\end{cases}$, $Z_{k,4} = \begin{cases} z_{k,2} & k \text{ is odd} \\ z_{k,1} & k \text{ is even}\end{cases}$.

At this time to measure noise

$$\boldsymbol{v}_{k,3} = \begin{cases} \boldsymbol{v}_{k,1} & k \text{ is odd} \\ \boldsymbol{v}_{k,2} & k \text{ is even}\end{cases}, \quad \boldsymbol{v}_{k,4} = \begin{cases} \boldsymbol{v}_{k,2} & k \text{ is odd} \\ \boldsymbol{v}_{k,1} & k \text{ is even}\end{cases}$$

The figure of reconstructed observation is shown in Fig. 1.

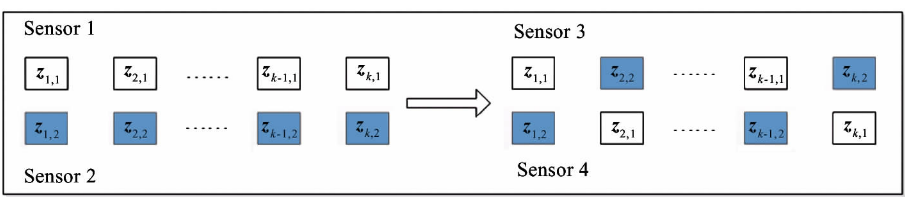


Fig. 1 The figure of reconstructed observation

The method of reconstructed observation in the filtering process can improve utilization of observation of the smaller observation noise variance sensor, and reduce utilization of observation of the larger observation noise variance sensor. In the following, the principle of FEKF-ROIO is described in detail by taking the randomly selected sensor group as an example, where the observation sets are $Z_{k,1}$ and $Z_{k,2}$, respectively. The new observation sets are Z_k^3 and Z_k^4, respectively. The extended Kalman filters after processing are denoted as filter 1, filter 2, filter 3, and filter 4, respectively.

Using Eq. (4) and Eq. (6) the one step prediction covariance is obtained and matrix is got, respectively, denoting $\boldsymbol{P}_{k|k-1,m}$ and $\boldsymbol{K}_{k,m}$, and $m = 1, 2, 3, 4$, indicate the sensor number. $\boldsymbol{K}_{k,m}$ indicates filter gain showing filter updating process. The greater of $\boldsymbol{K}_{k,m}$ shows the higher the utilization of the observation, the smaller of $\boldsymbol{K}_{k,m}$ shows the lower the utilization of the observation. This paper proves the relationship between $\boldsymbol{P}_{k|k,m}$, and obtains the relationship between $\boldsymbol{K}_{k,m}$, so as to illustrate the advantage of the reconstructed observation method.

Assume that the sensor is fully probed by the ideal situation, $\boldsymbol{v}_{k,1} \sim (0, \boldsymbol{R}_{k,1})$, $\boldsymbol{v}_{k,2} \sim (0, \boldsymbol{R}_{k,2})$, The observation noise variance of the two sensors after the reconstructed observation is satisfied

$$\boldsymbol{R}_{k,3} = \begin{cases} \boldsymbol{R}_{k,1} & k \text{ is odd} \\ \boldsymbol{R}_{k,2} & k \text{ is even} \end{cases}, \quad \boldsymbol{R}_{k,4} = \begin{cases} \boldsymbol{R}_{k,2} & k \text{ is odd} \\ \boldsymbol{R}_{k,1} & k \text{ is even} \end{cases}$$

Prove the relationship between $\boldsymbol{P}_{k|k,m}$.

It is assumed that the accuracy of sensor 1 is higher than that of sensor 2, that is $\boldsymbol{R}_{k,1} < \boldsymbol{R}_{k,2}$, set the same initial estimation error covariance $\boldsymbol{P}_{0|0}$.

1) When $k = 1$, substituting $\boldsymbol{P}_{0|0,m}$ into Eq. (4) to $\boldsymbol{P}_{1|0,m}$, which is $\boldsymbol{P}_{1|0,1} = \boldsymbol{P}_{1|0,2} = \boldsymbol{P}_{1|0,3} = \boldsymbol{P}_{1|0,4}$, substituting $\boldsymbol{P}_{1|0,m}$ and $\boldsymbol{R}_{1,m}$ into Eq. (8) to $\boldsymbol{P}_{1|1,m}$, at this time $\boldsymbol{P}_{1|1,1} = \boldsymbol{P}_{1|1,3} < \boldsymbol{P}_{1|1,2}$, $\boldsymbol{P}_{1|1,1} < \boldsymbol{P}_{1|1,4} = \boldsymbol{P}_{1|1,2}$.

2) When $k = 2$, substituting $\boldsymbol{P}_{1|1,m}$ into Eq. (4) to $\boldsymbol{P}_{2|1,m}$, which is $\boldsymbol{P}_{2|1,1} = \boldsymbol{P}_{2|1,3} < \boldsymbol{P}_{2|1,2}$, $\boldsymbol{P}_{2|1,1} < \boldsymbol{P}_{2|1,4} = \boldsymbol{P}_{2|1,2}$, substituting $\boldsymbol{P}_{2|1,m}$ and $\boldsymbol{R}_{2,m}$ into formula (8) to $\boldsymbol{P}_{2|2,m}$, at this time $\boldsymbol{P}_{2|2,1} < \boldsymbol{P}_{2|2,3} < \boldsymbol{P}_{2|2,2}$, $\boldsymbol{P}_{2|2,1} < \boldsymbol{P}_{2|2,4} < \boldsymbol{P}_{2|2,2}$.

3) When $k > 2$, which is $\boldsymbol{P}_{k|k,1} < \boldsymbol{P}_{k|k,3} < \boldsymbol{P}_{k|k,2}$, $\boldsymbol{P}_{k|k,1} < \boldsymbol{P}_{k|k,4} < \boldsymbol{P}_{k|k,2}$, for any moment of $k > 1$ is satisfied, the error covariance $\boldsymbol{P}_{k|k,1}$ of filter 1 is the smallest, the error covariance $\boldsymbol{P}_{k|k,2}$ of the filter 2 is maximum. The error covariance $\boldsymbol{P}_{k|k,3}$ of filter 3and $\boldsymbol{P}_{k|k,4}$ of filter 4 are between them.

Prove the relationship between $\boldsymbol{K}_{k,m}$

1) Assume that k is odd moment ($k \neq 1$), at this time, filter 1 and filter 3 use the same observation $z_{k,1}$, filter 2 and filter 4 use the same observation $z_{k,2}$, which is $\boldsymbol{R}_{k,1} = \boldsymbol{R}_{k,3}$, $\boldsymbol{R}_{k,2} = \boldsymbol{R}_{k,4}$, $\boldsymbol{P}_{k|k,i}$ and $\boldsymbol{R}_{k,i}$ into Eq. (6), which is

$$\begin{aligned} \boldsymbol{K}_{k,1} &= \frac{\boldsymbol{P}_{k|k,1}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k-1,1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,1}} < \frac{\boldsymbol{P}_{k|k,3}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k,3}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,3}} \\ &= \boldsymbol{K}_{k,3} \\ \boldsymbol{K}_{k,4} &= \frac{\boldsymbol{P}_{k|k,4}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k-1,4}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,4}} < \frac{\boldsymbol{P}_{k|k,2}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k,2}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,2}} \\ &= \boldsymbol{K}_{k,2} \end{aligned} \tag{10}$$

As can be seen from the above equation, for the utilization of the same observation $z_{k,1}$, filter 3 is higher than filter 1. For the utilization of the same observation $z_{k,2}$, filter 4 is lower than filter 2. It should be noted that when $k = 1$, through the above method $\boldsymbol{K}_{k,1} = \boldsymbol{K}_{k,3}$, $\boldsymbol{K}_{k,4} = \boldsymbol{K}_{k,2}$ is solved. Filter 3 and filter 1 are equal to the utilization of $z_{k,1}$, filter 4 and filter 2 are equal to the utilization of $z_{k,2}$

2) Assume that k is even moment, filter 1 and filter 4 use the same observation $z_{k,1}$, filter 2 and filter 3 use the same observation $z_{k,2}$, which is $\boldsymbol{R}_{k,1} = \boldsymbol{R}_{k,4}$, $\boldsymbol{R}_{k,2} = \boldsymbol{R}_{k,3}$. Substitute $\boldsymbol{P}_{k|k,i}$ and $\boldsymbol{R}_{k,i}$ into Eq. (6) to get

$$\begin{aligned} \boldsymbol{K}_{k,3} &= \frac{\boldsymbol{P}_{k|k-1,3}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k-1,3}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,2}} < \frac{\boldsymbol{P}_{k|k-1,2}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k-1,2}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,2}} \\ &= \boldsymbol{K}_{k,2} \\ \boldsymbol{K}_{k,1} &= \frac{\boldsymbol{P}_{k|k-1,1}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k-1,1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,1}} < \frac{\boldsymbol{P}_{k|k-1,4}\boldsymbol{H}_k^{\mathrm{T}}}{\boldsymbol{H}_k\boldsymbol{P}_{k|k-1,4}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{k,1}} \\ &= \boldsymbol{K}_{k,4} \end{aligned} \tag{11}$$

For the utilization of the same observation value $z_{k,1}$, filter 4 is higher than filter 1; for the utilization of the same observation value $z_{k,2}$, filter 3 is lower than filter 2.

From the above process, it can be seen that when $k > 1$ in any time, a filter (filter 3 or filter 4) compared with filter 1 improves the utilization of observation $z_{k,1}$ of the smaller observation noise variance. Simultaneously, another filter (filter 3 or filter 4) compared with filter 2 reduces the use of observation $z_{k,2}$ of the larger observation noise variance. As the result the fusion of filter 3 and filter 4 compared with the fusion of filter 1 and filter 2 improve the utilization of observation $z_{k,1}$ of the smaller observation noise variance and reduce the adverse interference of the observation $z_{k,2}$ of the larger observation noise variance. The process achieves better utilization of observation information, the overall filter accuracy has a certain role in upgrading.

The above proof process is based on the ideal observation complete detection scenario. Sensor 1 and sensor 2 are normally detected observation information at k moment. Assuming at k moment, only sensor 1 detects the observation information. Filter 3 is compared with filter 1, its utilization of detectable observation $z_{k,1}$ is higher, by improving the utilization of trusted observation (observation noise variance is small) to achieve the improvement of filtering accuracy. Assuming at k moment, only sensor 2 detects the observation information. Filter 4 is compared with filter 2, its utilization of detectable observation $z_{k,1}$ is lower, by reducing the utilization of uncertain observation (observation noise variance is larger) to achieve the improvement of filtering accuracy. Assuming at k moment, sensor 1 or sensor 2 does not detect observation information, then the utilization of observation information is 0 at this moment.

2.2 Federal extended Kalman filter based on reconstructed observation in incomplete observations

When multi-source information fusion is being processed, the observation information of each sensor is processed independently and the local state is sent to the central node. The fusion processing is performed, higher reliability is obtained with a lower cost, and the data processing delay is small. When a sensor fails, the entire multi-sensor system is less affected. In order to realize the full utilization of all the information in the reconstructed observation sets and real observation sets, FEKF-ROIO handles each observation set by an extended Kalman filter algorithm, respectively, and then obtain the local state estimation result by weighted fusion method , get the state optimal estimate.

The existing sensor observation sets are combined with reconstructed observation sets to calculate the local state estimated value $\hat{\boldsymbol{x}}_{k|k,m}$ and the local state estimated error covariance $\boldsymbol{P}_{k|k,m}$, according to Eq. (9) to obtain the system state optimal estimated value.

The FEKF-ROIO is implemented as follows:

(1) When $k = 0$, the filter is initialized.

Initialize the state estimated value and the estimate error covariance matrix.

$$\hat{\boldsymbol{x}}_{0|0} = \boldsymbol{x}_0, \boldsymbol{P}_{0|0} = \boldsymbol{P}_0$$

(2) The recursive estimate of the state is achieved at $k \geqslant 1$ moment.

1) reconstructed observation:

The observation information is reconstructed based on the parity of the sampling time, then obtain the new observation sets.

2) The solution of local estimation and estimation error covariance matrix:

The local estimated value $\hat{\boldsymbol{x}}_{k|k,m}$ of the filter and the corresponding estimated error covariance matrix $\boldsymbol{P}_{k|k,m}$ are solved by combining Eq. (5) – Eq. (8).

3) Weighted fusion:

The local estimated value $\hat{\boldsymbol{x}}_{k|k,m}$ are weighted fusion combined with the Eq. (9) to obtain the final state estimated value $\hat{\boldsymbol{x}}_{k|k}$.

(3) Order $k = k + 1$, go to step (1)

3 Simulation results and analysis

The simulation experiment uses RMSE as the performance index of the accuracy of the observation algorithm under Monte Carlo simulation conditions and Monte Carlo simulation number is 200. The experimental platform uses PC, Intel(R) Core i7-2600 CPU, frequency 3.40GHZ, RAM4G, Win10, the programming language is Matlab2012b. Use the sensor observation data to achieve the system state estimation that under horizontal and vertical directions a plane is formed. The equation of motion and the observation equation are as follows in cartesian coordinates.

$$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}_{k-1}\boldsymbol{w}_{k-1}$$

$$z_{k,m} = \gamma_k \boldsymbol{h}_{k,m}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k,m} \cdots m = 1,2,\cdots M$$

Among them, $\boldsymbol{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^{\mathrm{T}}$, $x_k, \dot{x}_k$, y_k and $\dot{y}_k$ indicate the position and velocity components of the target state in the horizontal and vertical directions, respectively. Observation matrix is

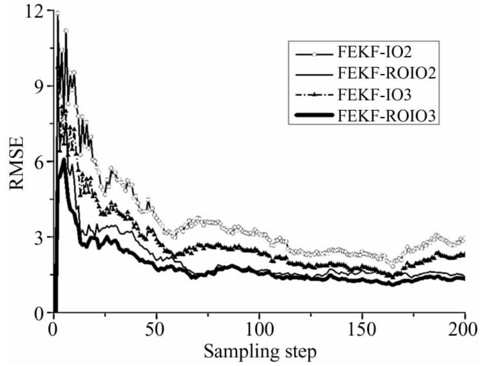
$$\boldsymbol{h}_k(\boldsymbol{x}_k) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \tan^{-1}(y_k/x_k) \end{bmatrix},$$

$$\boldsymbol{F} = \begin{bmatrix} 1 & \dfrac{\sin(\omega T)}{\omega} & 0 \\ 0 & \cos(\omega T) & 0 \\ 0 & \dfrac{(1 - \cos(\omega T))}{\omega} & 1 \\ 0 & \sin(\omega T) & 0 \end{bmatrix},$$ sampling interval $T = 1$. $\boldsymbol{\Gamma}_k = \begin{bmatrix} T^2/2 & 1 & 0 & 0 \\ 0 & 0 & T^2/2 & 1 \end{bmatrix}^{\mathrm{T}}$. System process noise w_k satisfies Gaussian white noise whose variance is Q_k distribution. $\boldsymbol{Q}_k = diag[0.1 \quad 0.1]$, $\boldsymbol{R}_{k,1} = diag[10 \quad 5]$, v_n is observation noise vector in Gaussian. $\boldsymbol{R}_{k,2} = 2\boldsymbol{R}_{k,1}, \boldsymbol{R}_{k,3} = 4\boldsymbol{R}_{k,1}$, system initial state $\boldsymbol{x}_0 = [500 \quad 50\mathrm{m/s} \quad 600 \quad 50\mathrm{m/s}]^{\mathrm{T}}$. The simulation step is 40.

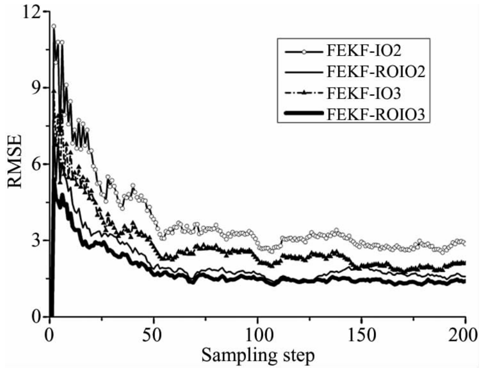
In order to verify the feasibility and validity of the algorithm, this algorithm is compared with the federal extended Kalman filter based on incomplete observations under different probing probabilities of the three sensors. Detection probability is 0.8, and this algorithm is compared with the federal extended Kalman filter algorithm based on incomplete observations by changing the number of sensors. Detection probability is 0.8, and this algorithm is compared with the outlier fusion EKF algorithm by changing the number of sensors. Detection probability is 0.8, this algorithm is compared with sensor without feedback fusion EKF algorithm by changing the number of sensors.

When the probing probability is 0.8, FEKF-IO2 indicates the federal extended Kalman filtering based on incomplete observations of two sensors, FEKF-ROIO2 indicates the reconstructed observation federal extended Kalman filter based on incomplete observation of two sensors, FEKF-IO3 indicates the federal extended Kalman filtering based on incomplete observation of three sensors, FEKF-ROIO3 indicates the reconstructed observation federal extended Kalman filtering based on incomplete observations of three sensors.

As can be seen from Fig. 2, the observation set is reconstructed, using observation redundancy and complementary information, making the estimation of FEKF-ROIO2 closer to the true value with the incomplete observation. By increasing the number of sensors, more observation information can be obtained by the estimated system so that the estimated value of the system is closer to the true value. Therefore, the RMSE of FEKF-ROIO3 is lower than the RMSE of FEKF-ROIO2, the RMSE of FEKF-IO3 is lower than the RMSE of FEKF-IO2. Secondly, Due to the introduction of the reconstructed observation method, the utilization of the observation of the smaller observation noise variance is improved. The utilization of the observation of the larger observation noise variance is reduced. The redundant and complementary information of the known observation can be used more rationally. The RMSE of FEKF-ROIO is significantly lower than the RMSE of FEKF-IO under incomplete observations.



(a) Horizontal direction



(b) Vertically direction

Fig. 2 Average RMSE contrast between FEKF-IO and FEKF-ROIO with incomplete observations

In order to further verify the effect of the reconstructed observation method on the final estimation results under the number of different sensors.

Table 1 Horizontal direction comparison of average RMSE between FEKF-IO and FEKF-ROIO of the different sensors with incomplete observations

Number of sensors	2	5	7	10
FEKF-IO	2.4631	2.1374	1.7694	1.5267
FEKF-ROIO	1.9347	1.6473	1.3792	1.1347

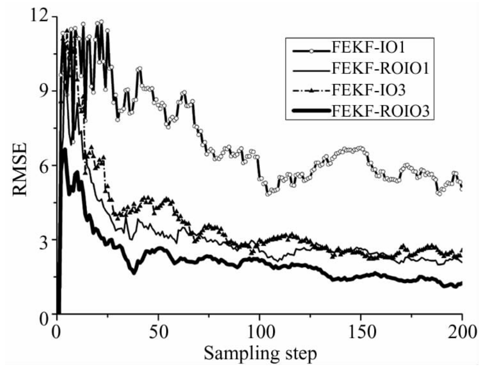
Table 2 Vertically direction comparison of average RMSE between FEKF-IO and FEKF-ROIO of the different sensors with incomplete observations

Number of sensors	2	5	7	10
FEKF-IO	2.1369	1.8743	1.5479	1.3746
FEKF-ROIO	1.8437	1.64723	1.3749	1.2479

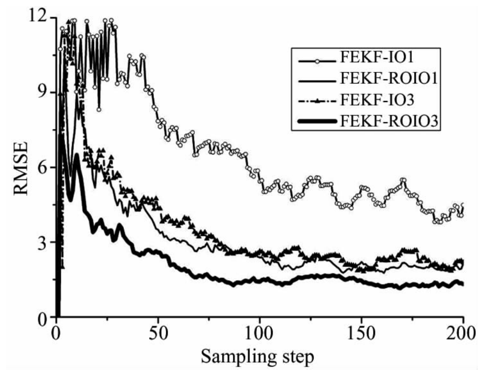
When the probability of sensor detection is 0.8, Table 1 and Table 2 show the average RMSE of FEKF-IO and FEKF-ROIO in the horizontal and vertical directions with the different sensors, respectively. It can be seen from the table that the average RMSE of FEKF-ROIO in both directions is lower than the average RMSE of FEKF-IO.

Contrasting RMSE of FEKF-ROIO with RMSE of FEKF-IO based on different probing probabilities of sensors, three sensors are taken as an example, when probability of detection is 0.6, FEKF-IO1 indicates the federal extended Kalman filter based on incomplete observations, FEKF-ROIO1 indicates the reconstructed observation federal extended Kalman filtering based on incomplete observations. When probability of detection is 0.8, FEKF-IO3 indicates the federal extended Kalman filter based on incomplete observations, FEKF-ROIO3 indicates the reconstructed observation federal extended Kalman filter based on incomplete observation.

As can be seen from Fig. 3, the RMSE of FEKF-IO3 is lower than the RMSE of FEKF-IO1, the RMSE of FEKF-ROIO3 is lower than the RMSE of FEKF-ROIO1. FEKF-IO1 and FEKF-ROIO1 have low probability of detection, lack of observation information for system and observation information has a direct effect on system estimation, resulting in a larger deviation from the true value. The RMSE of FEKF-ROIO3 is lower than the RMSE of FEKF-IO3, and the RMSE of FEKF-ROIO1 is lower than the RMSE of FEKF-IO1. Throughout the reconstructed information of observation, the redundancy and complementary information of observation are fully used to improve the utilization of observation with smaller observation noise variance, reduce the utilization of observation with larger observation noise variance and make the estimate closer to the true value.



(a) Horizontal direction



(b) Vertically direction

Fig. 3 Comparison of average RMSE between FEKF-IO and FEKF-ROIO

In order to verify the filter result of FEKF-ROIO and FEKF-IO under different probing probabilities, the number of sensors is set 3, the probabilities are 0.6, 0.7, 0.8 and 0.9, Table 3 and Table 4 give data analysis of FEKF-IO and FEKF-ROIO, respectively.

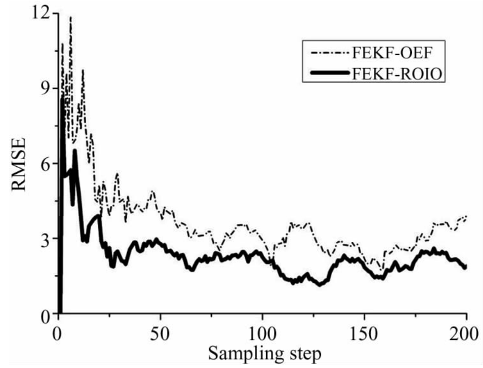
Table 3 Horizontal direction comparison of average RMSE between FEKF-IO and FEKF-ROIO under different probing probabilities

Probing probabilities	0.6	0.7	0.8	0.9
FEKF-IO	2.4976	2.4831	2.2372	2.1367
FEKF-ROIO	1.9375	1.7964	1.7234	1.5134

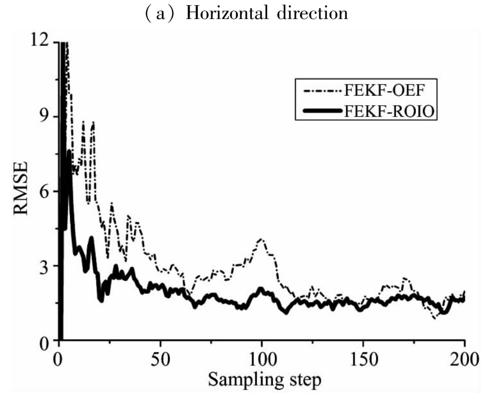
Table 4 Vertical direction comparison of average RMSE between FEKF-IO and FEKF-ROIO under different probing probabilities

Probing probabilities	0.6	0.7	0.8	0.9
FEKF-IO	2.5439	2.4571	2.2436	2.1487
FEKF-ROIO	1.9418	1.8347	1.7424	1.5472

Fig. 4 shows the RMSE comparison of FEKF-ROIO and the federal extended Kalman filtering algorithm with the outlier elimination fusion (FEKF-OEF). Fig. 5 shows the RMSE comparison between FEKF-ROIO and FEKF without feedback fusion (FEKF-WFF). In the horizontal and vertical directions, the RMSE of this algorithm is lower than the other two algorithms.

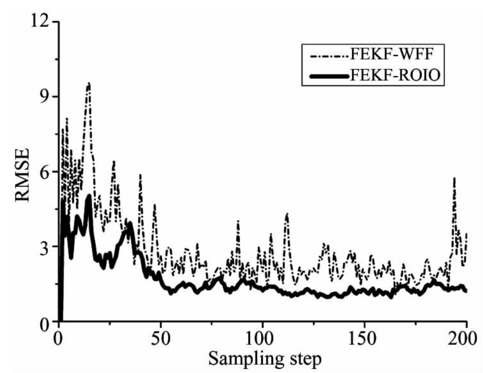


(a) Horizontal direction

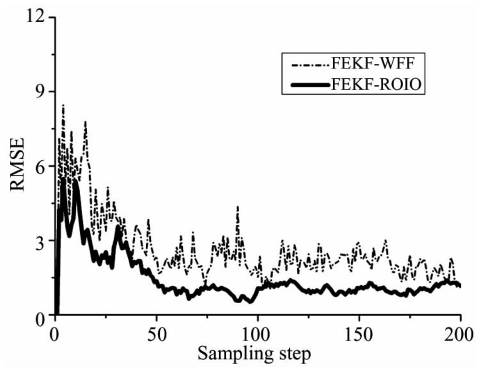


(b) Vertical direction

Fig. 4 RMSE comparison of FEKF-OEF and FEKF-ROIO



(a) Horizontal direction



(b) Vertical direction

Fig. 5 RMSE comparison of FEKF-WFF and FEKF-ROIO

4 Conclusions

Aiming at the problems that the detection probability of the sensor is less than 1 with obstacle occlusion, sensor failure and other factors in complex environment, this paper proposes a noval federal extended Kalman filtering based on reconstructed observation in incomplete observations which can be applied to nonlinear system. FEKF-ROIO is based on the multi-sensor system observation sets, the parity time observation interchange and builds a new observation set. The observation sets are processed by the extended Kalman filter algorithm, respectively. All local state estimates are fused to obtain an optimal state estimation by weight.

Compared with the existing filtering algorithm, the proposed algorithm has the following advantages. Change the detection probability of the sensor or change the number of sensors, compared with the incomplete observation of the federated extended Kalman filter algorithm, FEKF-ROIO improves observation use of the smaller observation noise variance and reduce observation use of the larger observation noise variance by reconstructed observation. The estimation of this algorithm is closer to the true value by the utilization of observation redundancy and complementary information.

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