doi:10.3772/j.issn.1006-6748.2019.01.014

Study on optimal state estimation strategy with dual distributed controllers based on Kalman filtering①

Chen Yawen(陈雅雯), Wang Zhuwei, Fang Chao, Xu Guangshu, Zhang Yanhua②
(School of Information and Communications Engineering, Beijing University of Technology, Beijing 100124, P. R. China)

Abstract

Considering dual distributed controllers, a design of optimal state estimation strategy is studied for the wireless sensor and actuator network (WSAN). In particular, the optimal linear quadratic (LQ) control strategy with estimated plant state is formulated as a non-cooperative game with network-induced delays. Then, using the Kalman filter approach, an optimal estimation of the plant state is obtained based on the information fusion of the distributed controllers. Finally, an optimal state estimation strategy is derived as a linear function of the current estimated plant state and the last control strategy of multiple controllers. The effectiveness of the proposed closed-loop control strategy is verified by the simulation experiments.

Key words: optimal state estimation strategy, wireless sensor and actuator network (WSAN), distributed controllers, Kalman filter, network-induced delays

0 Introduction

Networked control systems (NCSs) in which the shared communication medium is used for the connection between the plant and the controller have attracted much attention recently due to their potential applications in various areas such as power grids, DC motors, and robotic networks[1,2]. In general, the NCS can be divided into wireline and wireless NCS based on the types of shared communication mediums. Compared to the traditional wire-based NCS, wireless NCSs, especially wireless sensor and actuator networks (WSANs), offer many advantages such as easy maintenance, low-cost, architectural flexibility, resource sharing, and additional degrees of freedom[3].

However, the wireless NCS has its own limitations, such as energy constraints, packet losses and network-induced delays, and other issues, which cause a design of optimal control strategy as a significant challenge[4]. With full plant state information, the stochastic optimal linear quadratic (LQ) was obtained for NCSs with short and long network-induced delays by Nilsson[5] and Hu[6], respectively. Over the noisy channel with the estimated plant state, Gupta et al.[7] decomposed an optimal control problem into a linear quadratic optimal regulator design and state estimation problem. Goldsmith et al.[8] addressed the LQ Gaussian optimal control problem for both finite and infinite horizon cases when the controller can exchange information with both sensor and actuator. When both stochastic delays and packet losses are considered, the optimal LQ control law with full state information is derived by Xu et al.[9].

However, the traditional work always assumes that the plant is controlled by only single controller. Challenged by the decentralized fashion of the large scale network, it is necessary to efficiently take advantage of multiple controllers to cooperatively improve the system stability[10]. Previous work[10] was the first time to investigate the information fusion of the multiple controllers and designed the optimal feedback control strategy with full state information.

However, it ignored that in a wireless network channel with noise, it was difficult to obtain the plant state[7,8]. Therefore, in this paper, the results in Ref. [10] were extended to study the optimal state estimation strategy with distributed controllers based on Kalman filtering. In particular, using the Kalman filter approach, the optimal LQ strategy is obtained as a linear function with the estimated plant state and previous control strategies through the non-cooperative theory.

The remainder of this paper is organized as follows. The system model and problem formulation are given in Section 1. Then the design of the optimal state estimation strategy for network-induced delay in Section

① Supported by the National Natural Science Foundation of China (No. 61701010, 61571021, 61601330).
② To whom correspondence should be addressed. E-mail: zhangyh@bjut.edu.cn
Received on Jan. 3, 2018

2 is proposed. Simulation experiments and conclusions are given in Sections 3 and 4, respectively.

1 System model and problem formulation

In this section, structures of wireless sensor and actuator networks (WSANs) with distributed controllers are described, and then they equivalently convert to a wireless NCS in the presence of network-induced delays. Finally, the optimal LQ control problem is formulated. This paper focuses on the case of distributed controllers with dual controllers.

1.1 System model

The structure of a wireless sensor and actuator network with dual distributed controllers is shown in Fig. 1, where the closed-loop control includes plant, actuator, and a number of sensors. The multiple controllers in a decentralized fashion are chosen to cooperatively maintain the system stability. The sampled plant state is sent to distributed controllers, and then the control strategy generated, and finally sent to the actuator from the controller nodes. As shown in Fig. 1, the relay nodes 2 and 7 are selected as the controllers to coordinately form the feed back loop to keep the system stability. As Ref. [10], the wireless sensor and actuator network with distributed controllers can be converted to an equivalent wireless NCS shown in Fig. 2. Here the network-induced delays are introduced from sensor to controllers (τ_1^{sc}, τ_2^{sc}) and from controllers to actuator (τ_1^{ca}, τ_2^{ca}).

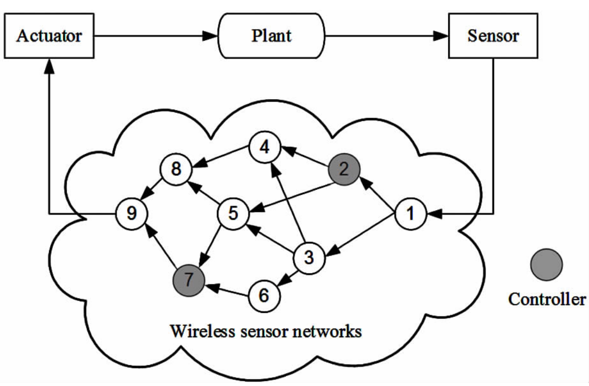

Fig. 1 The structure of the WSAN with distributed controllers

Considering the network-induced delays, a closed-loop dynamic equation for a NCS system based on dual controllers in continuous time domain is given by[5,10]

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \sum_{i=1}^{2} \boldsymbol{D}_i \boldsymbol{u}_i(t - \tau_i) + \boldsymbol{G}\boldsymbol{v}(t) \tag{1}$$

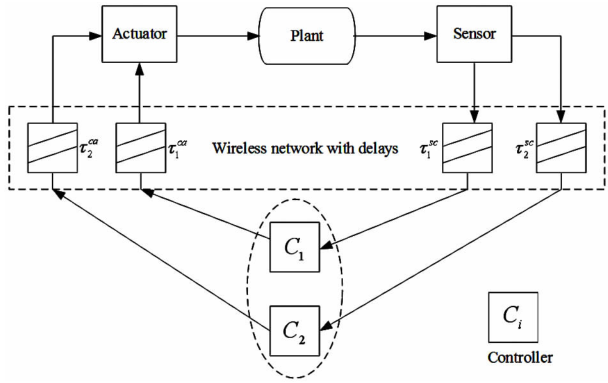

Fig. 2 Wireless NCS with dual distributed controllers

where $\boldsymbol{x}(t)$ is an M-dimensional plant state vector, $\boldsymbol{u}_i(t)$ is the K-dimensional control signal of controller i, $\boldsymbol{v}(t)$ is the noises, $\tau_i = \tau_i^{sc} + \tau_i^{ca}$ is the total network-induced delay, $\boldsymbol{A}$, $\boldsymbol{D}_i$, and $\boldsymbol{G}$ are known as matrices of appropriate sizes. Here, network-induced delay τ_i is assumed to be smaller than sensor sampling period T.

Then, the corresponding discrete-time version of Eq. (1) can be expressed as

$$\begin{aligned} \boldsymbol{x}(k+1) &= \varphi \boldsymbol{x}(k) + \sum_{i=1}^{2} (\boldsymbol{\Gamma}_{i,0}\boldsymbol{u}_i(k) + \boldsymbol{\Gamma}_{i,1}\boldsymbol{u}_i(k-1)) \\ &\quad + \boldsymbol{v}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{w}(k) \end{aligned} \tag{2}$$

where $\varphi = e^{AT}$, $\boldsymbol{x}(k) = \boldsymbol{x}(kT)$, $\boldsymbol{u}_i(k)$ is the controller signal according to $\boldsymbol{x}(k)$, $\boldsymbol{\Gamma}_{i,0} = \int_0^{T-\tau_i} e^{As}\mathrm{d}s\boldsymbol{D}_i$, $\boldsymbol{\Gamma}_{i,1} = \int_{T-\tau_i}^{T} e^{As}\mathrm{d}s\boldsymbol{D}_i$, $\boldsymbol{y}(k)$ is the observation value of the plant state , $\boldsymbol{v}(k)$ and $\boldsymbol{w}(k)$ are noises, which are set to be Gaussian distribution with zero mean and variances $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ respectively, $\boldsymbol{C}$ is the known matrix of appropriate sizes.

1.2 Problem formulation

In general, using the quadratic cost function, the design of the optimal LQ control strategy of distributed controllers is to minimize the total cost given by[11-13]

$$\min_{\{u_i(k)\}} \boldsymbol{J}_N = \boldsymbol{x}^T(N)\boldsymbol{Q}_N\boldsymbol{x}(N) + \sum_{k=0}^{N-1}\Big[\boldsymbol{x}^T(k)\boldsymbol{Q}\boldsymbol{x}(k) + \sum_{i=1}^{2}\boldsymbol{u}_i^T(k)\boldsymbol{Q}'_i\boldsymbol{u}_i(k)\Big]$$

$$\begin{aligned} \text{s.t.} \quad \boldsymbol{x}(k+1) &= \varphi\boldsymbol{x}(k) + \sum_{i=1}^{2}(\boldsymbol{\Gamma}_{i,0}\boldsymbol{u}_i(k) \\ &\quad + \boldsymbol{\Gamma}_{i,1}\boldsymbol{u}_i(k-1)) + \boldsymbol{v}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{w}(k) \end{aligned} \tag{3}$$

where N is the total number of sampling instants, $\boldsymbol{Q}_N \geq 0$ is the known symmetric positive semi-definite weight matrix, and $\boldsymbol{Q} > 0$ and $\boldsymbol{Q}'_i > 0$ are the known symmetric positive definite weight matrices.

Since the current control strategies are generated in a decentralized fashion that the above optimal LQ control strategy can be reformulated as a non-cooperative control game:

$$\min_{\{u_i(k)\}} \boldsymbol{J}_N^i = \boldsymbol{x}^T(N)\boldsymbol{Q}_N\boldsymbol{x}(N) + \sum_{k=0}^{N-1}\begin{pmatrix}\boldsymbol{x}(k)\\ \boldsymbol{u}_i(k)\end{pmatrix}^T\boldsymbol{Q}_i\begin{pmatrix}\boldsymbol{x}(k)\\ \boldsymbol{u}_i(k)\end{pmatrix}, \quad i = 1, 2$$

$$\text{s.t.} \quad \boldsymbol{x}(k+1) = \varphi\boldsymbol{x}(k) + \sum_{i=1}^{2}(\boldsymbol{\Gamma}_{i,0}\boldsymbol{u}_i(k) + \boldsymbol{\Gamma}_{i,1}\boldsymbol{u}_i(k-1)) + \boldsymbol{v}(k)$$
$$\boldsymbol{y}(k) = \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{w}(k) \tag{4}$$

where $\boldsymbol{Q}_i = \begin{pmatrix}\boldsymbol{Q} & 0\\ 0 & \boldsymbol{Q'}_i\end{pmatrix}$ is the known symmetric positive definite weight matrices.

2 Optimal state estimation strategy design

In the transmission process, due to the sensor-to-controller and controller-to-actuator network-induced delays, the current control information between the controllers cannot be shared in real-time. Therefore, for each controller, the optimal LQ control signal can be determined by the current plant state information and the last control strategies, which can be expressed as[10]

$$\boldsymbol{u}_i(k) = \boldsymbol{A}_i(k)\boldsymbol{x}(k) + \boldsymbol{B}_{i,1}(k)\boldsymbol{u}_1(k-1) + \boldsymbol{B}_{i,2}(k)\boldsymbol{u}_2(k-1), \qquad i = 1, 2 \tag{5}$$

where $\boldsymbol{A}_i(k)$, $\boldsymbol{B}_{i,1}(k)$, and $\boldsymbol{B}_{i,2}(k)$ are coefficient matrices.

Taking controller 1 as the desired controller, substituting, based on Eq. (2) and Eq. (5), the dynamic equation of controller 1 through the information fusion of the dual controllers can be expressed as

$$\begin{aligned}\boldsymbol{x}(k+1) = {} & [\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]\boldsymbol{x}(k) + \boldsymbol{\Gamma}_{1,0}\boldsymbol{u}_1(k)\\ & + [\boldsymbol{\Gamma}_{1,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,1}(k)]\boldsymbol{u}_1(k-1)\\ & + [\boldsymbol{\Gamma}_{2,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,2}(k)]\boldsymbol{u}_2(k-1) + \boldsymbol{v}(k)\end{aligned} \tag{6}$$

However, in the real NCS, the plant state information in the dynamic Eq. (6) needs to be transmitted to the controller through the wireless network. Because of the existence of the noise, it is impossible to get the state signal accurately, and the estimation state is needed. In the paper, the Kalman filter algorithm is used to estimate the plant state information, which is the optimal algorithm based on the input and output observation values.

Using the Kalman filtering algorithm, based on Eq. (2) and Eq. (6), the estimation value of the current plant state information is given by

$$\begin{aligned}\bar{\boldsymbol{x}}(k \mid k-1) = {} & [\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]\hat{\boldsymbol{x}}(k-1 \mid k-1)\\ & + \boldsymbol{\Gamma}_{1,0}u_1(k)\\ & + [\boldsymbol{\Gamma}_{1,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,1}(k)]\boldsymbol{u}_1(k-1)\\ & + [\boldsymbol{\Gamma}_{2,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,2}(k)]\boldsymbol{u}_2(k-1)\end{aligned} \tag{7}$$

where $\bar{\boldsymbol{x}}(k \mid k-1)$ is the optimal estimation of current plant state signal using the previous plant state value, $\hat{x}(k-1 \mid k-1)$ is the optimal estimation of the previous plant state value. Here, the value of $\hat{\boldsymbol{x}}(k \mid k)$ is derived by

$$\hat{\boldsymbol{x}}(k \mid k) = \bar{\boldsymbol{x}}(k \mid k-1) + \boldsymbol{K}(k)[\boldsymbol{y}(k) - \boldsymbol{C}\bar{\boldsymbol{x}}(k \mid k-1)] \tag{8}$$

where the Kalman gain is given by

$$\boldsymbol{K}(k) = \bar{\boldsymbol{p}}(k \mid k-1)C^T[\boldsymbol{C}\bar{\boldsymbol{p}}(k \mid k-1)\boldsymbol{C}^T + \boldsymbol{R}_2]^{-1} \tag{9}$$

and here $\bar{\boldsymbol{p}}(k \mid k-1)$ is calculated as

$$\bar{\boldsymbol{p}}(k \mid k-1) = [\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]\hat{\boldsymbol{p}}(k-1 \mid k-1) \times [\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]^T + \boldsymbol{R}_1 \tag{10}$$

where $\bar{\boldsymbol{p}}(k \mid k-1)$ is the estimation covariance of $\bar{\boldsymbol{x}}(k \mid k-1)$, and $\hat{\boldsymbol{p}}(k-1 \mid k-1)$ is the predict covariance of $\hat{\boldsymbol{x}}(k-1 \mid k-1)$, which is given by

$$\hat{\boldsymbol{p}}(k \mid k) = [\boldsymbol{I} - \boldsymbol{K}(k)\boldsymbol{C}]\bar{\boldsymbol{p}}(k \mid k-1) \tag{11}$$

Based on Eq. (7) and Eq. (8), the optimal plant state estimation can be iteratively calculated by Kalman filter algorithm:

$$\begin{aligned}& \hat{\boldsymbol{x}}(k+1 \mid k+1)\\ & = \bar{\boldsymbol{x}}(k+1 \mid k) + \boldsymbol{K}(k+1) \times [\boldsymbol{y}(k+1) - \boldsymbol{C}\bar{\boldsymbol{x}}(k+1 \mid k)]\\ & = [\boldsymbol{I} - \boldsymbol{K}(k+1)\boldsymbol{C}]\{[\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]\hat{\boldsymbol{x}}(k \mid k)\\ & \quad + \boldsymbol{\Gamma}_{1,0}\boldsymbol{u}_1(k) + [\boldsymbol{\Gamma}_{1,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,1}(k)]\boldsymbol{u}_1(k-1)\\ & \quad + [\boldsymbol{\Gamma}_{2,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,2}(k)]\boldsymbol{u}_2(k-1)\}\\ & \quad + \boldsymbol{K}(k+1)\{\boldsymbol{C}\{[\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]\boldsymbol{x}(k \mid k)\\ & \quad + \boldsymbol{\Gamma}_{1,0}\boldsymbol{u}_1(k) + [\boldsymbol{\Gamma}_{1,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,1}(k)]\boldsymbol{u}_1(k-1)\\ & \quad + [\boldsymbol{\Gamma}_{2,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,2}(k)]\boldsymbol{u}_2(k-1) + \boldsymbol{v}(k)\}\\ & \quad + \boldsymbol{w}(k)\}\end{aligned} \tag{12}$$

Define estimation error $\tilde{\boldsymbol{x}}_k = \boldsymbol{x}(k \mid k) - \hat{\boldsymbol{x}}(k \mid k)$, Eq. (12) can be rewritten as

$$\begin{aligned}& \hat{\boldsymbol{x}}(k+1 \mid k+1)\\ & = [\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]\hat{\boldsymbol{x}}(k \mid k) + \boldsymbol{\Gamma}_{1,0}\boldsymbol{u}_1(k)\\ & \quad + [\boldsymbol{\Gamma}_{1,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,1}(k)]\boldsymbol{u}_1(k-1)\\ & \quad + [\boldsymbol{\Gamma}_{2,1} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,2}(k)]\boldsymbol{u}_2(k-1)\\ & \quad + \boldsymbol{K}(k+1)\boldsymbol{C}[\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)]\tilde{\boldsymbol{x}}_k\\ & \quad + \boldsymbol{K}(k+1)\boldsymbol{C}\boldsymbol{v}(k) + \boldsymbol{K}(k+1)\boldsymbol{w}(k)\end{aligned} \tag{13}$$

Then, using the optimal state estimation $\hat{\boldsymbol{x}}(k \mid k)$, the objective to minimize the quadrature cost function in Eq. (4) can be converted to be

$$\min_{\{u_i(k)\}} \boldsymbol{J}_N^i = \hat{\boldsymbol{x}}^T(N \mid N)\boldsymbol{Q}_N\hat{\boldsymbol{x}}(N \mid N) + \sum_{k=0}^{N-1}\begin{pmatrix}\hat{\boldsymbol{x}}(k \mid k)\\ \boldsymbol{u}_i(k)\end{pmatrix}^T\boldsymbol{Q}_i\begin{pmatrix}\hat{\boldsymbol{x}}(k \mid k)\\ \boldsymbol{u}_i(k)\end{pmatrix} \tag{14}$$

Define the residual cost function as

$$\boldsymbol{W}_i(\hat{\boldsymbol{x}}(k \mid k), \boldsymbol{p}(k), k) = \min_{\{u_i(j)\}} \left\{ \hat{\boldsymbol{x}}^T(N|N)\boldsymbol{Q}_N\hat{\boldsymbol{x}}(N|N) + \sum_{j=k}^{N-1} \begin{pmatrix} \hat{\boldsymbol{x}}(j \mid j) \\ \boldsymbol{u}_i(j) \end{pmatrix}^T \boldsymbol{Q}_i \begin{pmatrix} \hat{\boldsymbol{x}}(j \mid j) \\ \boldsymbol{u}_i(j) \end{pmatrix} \right\} \quad (15)$$

Base on Eqs(5) and (12), it can be written as follows:

$$\begin{pmatrix} \hat{\boldsymbol{x}}(k+1 \mid k+1) \\ \boldsymbol{u}_1(k) \\ \boldsymbol{u}_2(k) \end{pmatrix} = \boldsymbol{G} \begin{pmatrix} \hat{\boldsymbol{x}}(k \mid k) \\ \boldsymbol{u}_1(k) \\ \boldsymbol{u}_1(k-1) \\ \boldsymbol{u}_2(k-1) \end{pmatrix} + \boldsymbol{H} \begin{pmatrix} \tilde{\boldsymbol{x}}_k \\ \boldsymbol{v}(k) \\ \boldsymbol{w}(k+1) \end{pmatrix} \quad (16)$$

where

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{\varphi} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k) & \boldsymbol{\Gamma}_{1,0} & \boldsymbol{\gamma}_1 & \boldsymbol{\gamma}_2 \\ 0 & \boldsymbol{I} & 0 & 0 \\ \boldsymbol{A}_2(k) & 0 & \boldsymbol{B}_{2,1}(k) & \boldsymbol{B}_{2,2}(k) \end{pmatrix}$$

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{K}(k+1)\boldsymbol{C}[\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k)] & \boldsymbol{K}(k+1)\boldsymbol{C} & \boldsymbol{K}(k+1) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{\gamma}_i = \boldsymbol{\Gamma}_{1,i} + \boldsymbol{\Gamma}_{2,0}\boldsymbol{B}_{2,i}(k), \ i = 1, 2 \quad (17)$$

Then, it can be proved that the residual cost function can be expressed as following quadrature form:

$$\boldsymbol{W}(\hat{\boldsymbol{x}}(k \mid k), \boldsymbol{p}(k), k) = \begin{pmatrix} \hat{\boldsymbol{x}}(k \mid k) \\ \boldsymbol{u}_1(k-1) \\ \boldsymbol{u}_2(k-1) \end{pmatrix}^T \boldsymbol{s}(k) \begin{pmatrix} \hat{\boldsymbol{x}}(k \mid k) \\ \boldsymbol{u}_1(k-1) \\ \boldsymbol{u}_2(k-1) \end{pmatrix} + \beta(k) \quad (18)$$

where

$$\begin{aligned} \beta(k) = {} & tr[\boldsymbol{p}(k)\boldsymbol{Q}_1^{11}] \\ & + tr\,[\boldsymbol{p}(k)(\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k))^T\boldsymbol{C}^T\boldsymbol{K}^T(k+1) \\ & \times \boldsymbol{s}^{11}(k+1)\boldsymbol{K}(k+1)\boldsymbol{C}(\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k))] \\ & + tr[\boldsymbol{R}_1\boldsymbol{C}^T\boldsymbol{K}^T(k+1)\boldsymbol{s}^{11}(k+1)\boldsymbol{K}(k+1)\boldsymbol{C}] \\ & + tr[\boldsymbol{R}_2\boldsymbol{K}^T(k+1)\boldsymbol{s}^{11}(k+1)\boldsymbol{K}(k+1)] \end{aligned}$$

and $\boldsymbol{s}(k)$ is iteratively calculated by

$$\boldsymbol{s}(k) = \begin{pmatrix} \boldsymbol{Q}_1^{11} + \tilde{\boldsymbol{s}}_{k+1}^{11} & \boldsymbol{Q}_1^{12} + \tilde{\boldsymbol{s}}_{k+1}^{12} & \tilde{\boldsymbol{s}}_{k+1}^{13} & \tilde{\boldsymbol{s}}_{k+1}^{14} \\ \boldsymbol{Q}_1^{12^T} + \tilde{\boldsymbol{s}}_{k+1}^{21} & \boldsymbol{Q}_1^{22} + \tilde{\boldsymbol{s}}_{k+1}^{22} & \tilde{\boldsymbol{s}}_{k+1}^{23} & \tilde{\boldsymbol{s}}_{k+1}^{24} \\ \tilde{\boldsymbol{s}}_{k+1}^{31} & \tilde{\boldsymbol{s}}_{k+1}^{32} & \tilde{\boldsymbol{s}}_{k+1}^{33} & \tilde{\boldsymbol{s}}_{k+1}^{34} \\ \tilde{\boldsymbol{s}}_{k+1}^{41} & \tilde{\boldsymbol{s}}_{k+1}^{42} & \tilde{\boldsymbol{s}}_{k+1}^{43} & \tilde{\boldsymbol{s}}_{k+1}^{44} \end{pmatrix}$$

$$\tilde{\boldsymbol{s}}(k+1) = \boldsymbol{G}^T\boldsymbol{s}(k+1)\boldsymbol{G}$$

$$\boldsymbol{s}(N) = \begin{pmatrix} \boldsymbol{Q}_N & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

here $\tilde{\boldsymbol{s}}^{ij}$ is the (i,j) -block of $\tilde{\boldsymbol{s}}(k+1)$, $\boldsymbol{Q}_1^{ij}$ is the (i,j) -block of $\boldsymbol{Q}_1$, $tr[\ ]$ denoting the track of the matrix.

The corresponding optimal state estimation control strategy can be expressed as the following type.

$$\boldsymbol{u}_1(k) = -\boldsymbol{L}_1(k) \begin{pmatrix} \hat{\boldsymbol{x}}(k \mid k) \\ \boldsymbol{u}_1(k-1) \\ \boldsymbol{u}_2(k-1) \end{pmatrix} \quad (19)$$

where

$$\boldsymbol{L}_1(k) = (\boldsymbol{Q}_1^{22} + \tilde{\boldsymbol{s}}_{k+1}^{22})^{-1}(\boldsymbol{Q}_1^{12} + \tilde{\boldsymbol{s}}_{k+1}^{12} \quad \tilde{\boldsymbol{s}}_{k+1}^{23} \quad \tilde{\boldsymbol{s}}_{k+1}^{24}) \quad (20)$$

Below, this paper provides the proof for Eqs(18) and (19).

Proof:

Based on Eqs(4), (15), and (18), it can be written as follows:

$$\begin{aligned} & \boldsymbol{W}[\hat{\boldsymbol{x}}(k \mid k), \boldsymbol{p}(k), k] \\ & = \min_{\{\boldsymbol{u}_1(k)\}} \left\{ \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}_1(k) \end{pmatrix}^T \boldsymbol{Q}_1 \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}_1(k) \end{pmatrix} + \boldsymbol{W}[\hat{\boldsymbol{x}}(k+1 \mid k+1), \boldsymbol{p}(k+1), k+1] \right\} \\ & = \min_{\{\boldsymbol{u}_1(k)\}} \begin{pmatrix} \hat{\boldsymbol{x}}(k \mid k) \\ \boldsymbol{u}_1(k) \\ \boldsymbol{u}_1(k-1) \\ \boldsymbol{u}_2(k-1) \end{pmatrix}^T \boldsymbol{S}(k) \begin{pmatrix} \hat{\boldsymbol{x}}(k \mid k) \\ \boldsymbol{u}_1(k) \\ \boldsymbol{u}_1(k-1) \\ \boldsymbol{u}_2(k-1) \end{pmatrix} \\ & \quad + tr[\boldsymbol{p}(k)\boldsymbol{Q}_1^{11}] \\ & \quad + tr[\boldsymbol{p}(k)(\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k))^T\boldsymbol{C}^T\boldsymbol{K}^T(k+1) \\ & \quad \times \boldsymbol{s}^{11}(k+1)\boldsymbol{K}(k+1)\boldsymbol{C}(\varphi + \boldsymbol{\Gamma}_{2,0}\boldsymbol{A}_2(k))] \\ & \quad + tr[\boldsymbol{R}_1\boldsymbol{C}^T\boldsymbol{K}^T(k+1)\boldsymbol{s}^{11}(k+1)\boldsymbol{K}(k+1)\boldsymbol{C}] \\ & \quad + tr[\boldsymbol{R}_2\boldsymbol{K}^T(k+1)\boldsymbol{s}^{11}(k+1)\boldsymbol{K}(k+1)] \end{aligned} \quad (21)$$

Eq. (21) shows that the residual cost function can be expressed as a quadratic form in $\boldsymbol{u}_1(k)$. using the iterative recursive method, minimizing it with respect to $\boldsymbol{u}_1(k)$ gives the optimal control law as in Eq. (19), and the residual cost function can be expressed as the quadrature form as in Eq. (18).

Similarly, the optimal state estimation control strategy for the other controller can be derived Eq. (19), and then the optimal control law for dual controllers can be expressed as

$$\boldsymbol{u}_i(k) = -\boldsymbol{L}_i(k) \begin{pmatrix} \hat{\boldsymbol{x}}(k \mid k) \\ \boldsymbol{u}_1(k\text{-}1) \\ \boldsymbol{u}_2(k\text{-}1) \end{pmatrix}, \ i = 1, 2 \quad (22)$$

where $\boldsymbol{L}_2(k)$ can be obtained as $\boldsymbol{L}_1(k)$ in Eq. (20) in the same derivation processing.

3 Simulation experiments

In this section, the simulation experiment of optimal state estimation control strategy stability with dual distribution controller is given.

First, a stable system as in Ref. [5] is investigated that the parameters of the control system are set as

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, \ \boldsymbol{D}_1 = \boldsymbol{D}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{Q}_N = \boldsymbol{Q} = 80 \times \begin{bmatrix} 35 & \sqrt{35} \\ \sqrt{35} & 1 \end{bmatrix}, \boldsymbol{Q}'_1 = \boldsymbol{Q}'_2 = 1 \quad (23)$$

and the sampling period $T = 0.05$, the length of sampling periods $N = 100$.

It is assumed that the sensor-to-controller and controller-to-actuator network-induced delays are the same that $\tau_1^{sc} = \tau_2^{sc} = \alpha T$. Fig. 3 shows the stability comparison of the case of proposed dual distributed controllers with the traditional single-controller case. Fig. 4 shows the stability comparison of the cases of distributed controllers with or without optimal state estimation.

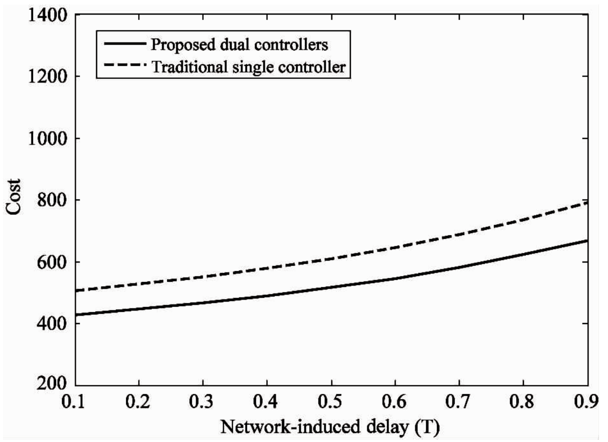

Fig. 3 Stability comparison of proposed scheme with traditional single-controller case in the stable system

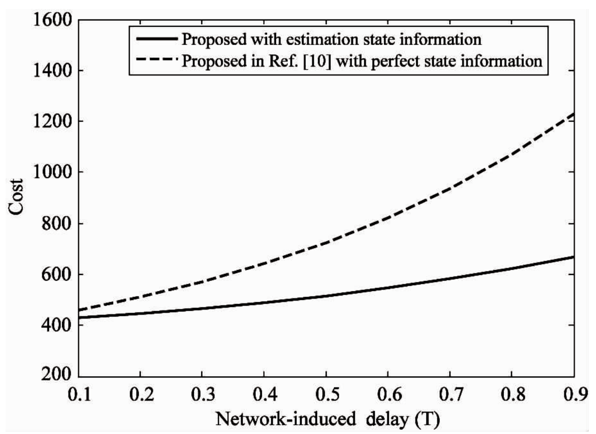

Fig. 4 Stability comparison of dual-controller cases with or without perfect state informationin the stable system

It is observed that the cost functioncan is significantly reduced by multiple controllers, which indicates that the advantage of the distributed controllers in NCS is an effective way to improve the system stability. It is also found that the imperfect information caused by the network-induced delay introduces certain stability degradation. In addition, the cost of the dual controllers with perfect state information increases significantly when the network-induced delay becomes larger because the error of the plant state becomes larger with the delay increasing, which introduces the extra system costs.

Next, this paper considers the optimal state estimation strategy with dual control to two-area LFC in the power grid systems, which is shown as Fig. 5[10,14,15].

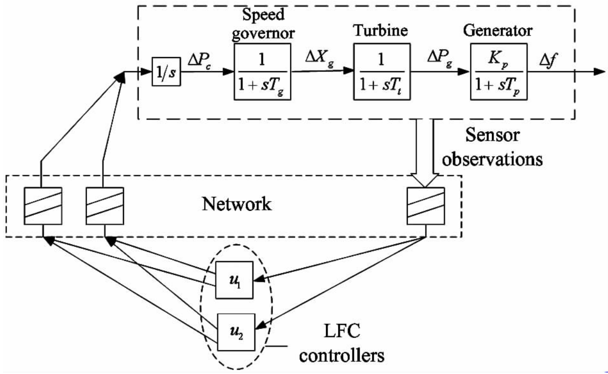

Fig. 5 Block diagram of LFC system in power grid systems

Here, the system state vector can be defined as

$$\boldsymbol{x}(t) = [\Delta P_c \quad \Delta f \quad \Delta P_g \quad \Delta X_g] \quad (24)$$

where ΔP_g, ΔX_g, ΔP and Δf are the deviations of generator mechanical output, valve position, generator output, and deviation of frequency, respectively.

Then, the linear dynamic control model can be described as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{D}_1\boldsymbol{u}_1(t - \tau_1) + \boldsymbol{D}_2\boldsymbol{u}_2(t - \tau_2) \quad (25)$$

where the system known matrices $\boldsymbol{A}$, $\boldsymbol{D}_1$, and $\boldsymbol{D}_2$ are given by

$$\boldsymbol{D}_1 = \boldsymbol{D}_2 = [1 \quad 1 \quad 0 \quad 0]$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 0 & -10/3 & 10/3 \\ 0 & 0 & 0 & -12.5 \end{bmatrix} \quad (26)$$

In the simulation, the initial value of the plant state is set to be $\boldsymbol{x}_0 = [0.25 \quad 0.15 \quad 0.2 \quad 0.1]^{\mathrm{T}}$, and

$$\boldsymbol{Q}'_1 = \boldsymbol{Q}'_2 = 1$$

$$\boldsymbol{Q}_N = \boldsymbol{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

the network-induced delays are chosen as the same as in the generic system.

In Fig. 6 and Fig. 7, it also compares the stability of the proposed dual distributed controllers with cases of traditional single controller and distributed controller with perfect state information in LFC application. The results are similar to those in the stable system, and the scheme of distributed controllers with optimal state estimation strategy is an effective way to improve the system stability.

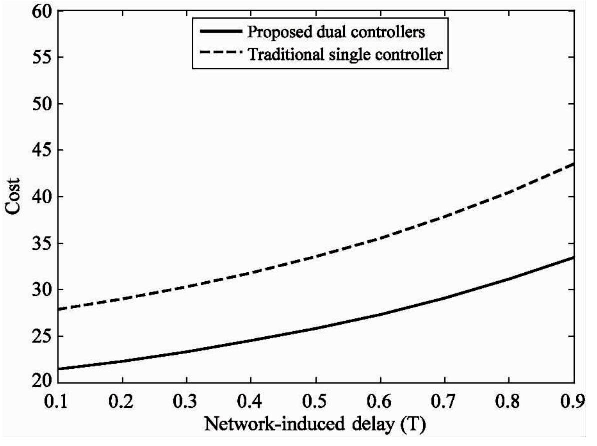

Fig. 6 Stability comparison of proposed scheme with traditional single-controller case in power grid system

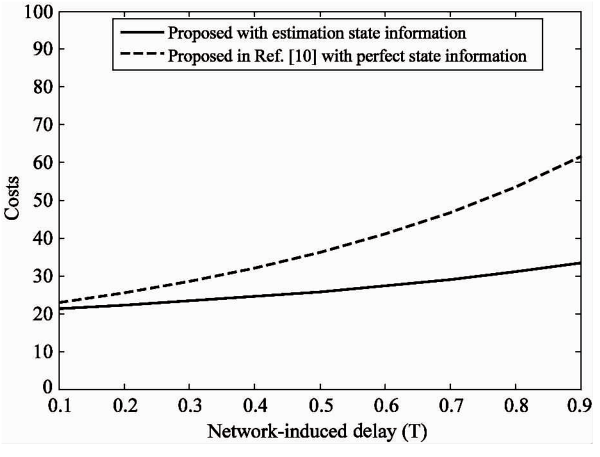

Fig. 7 Stability comparison of dual-controller cases with or without perfect state informationin power grid system

4 Conclusions

In the case of network-induced delay, this paper considers the design of the optimal state estimation control strategy for WSANs with dual distribution controller. In particular, the optimization problem is formulated as a non-cooperative LQ game, for which the optimal state estimation strategy solution is derived for distributed controllers using the Kalman filter approach. The stability of the proposed algorithm in both stable control system and application of LFC system in power grid are studied.

References

[1] Chai S, Liu G P, Rees D, et al. Design and practical-implementation of Internet-based predictive control of a servo system [J]. *IEEE Transactions on Control Systems Technology*, 2008,16(1):158-168

[2] Bullo F, Cortes J, Martinez S. Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms[M]. Princeton: Princeton University Press, 2009. 93-104

[3] Tabbara M, Nesic D, and Teel A. Stability of wireless and wireline networked control systems [J]. *IEEE Transactions on Automatic Control*, 2007, 52(9):1615-1630

[4] Hespanha J, Naghshtabrizi P, Xu Y. A survey of recent results in networked control systems [J]. *Automatica*, 2007, 95(1):138-162

[5] Nilsson J, Bernhardsson B, Wittenmark B. Stochastic analysis and control of real-time systems with random time delays [J]. *Automatica*, 1998, 34(1):57-64

[6] Hu S, Zhu W. Stochastic optimal control and analysis of stability of networked control systems with long delay [J]. *Automatica*, 2003, 39(11):1877-1884

[7] Gupta V, Dana A F, Murray R M, et al. Data transmission over networks for estimation and control [J]. *IEEETransactions on Automatic Control*, 2009, 54(8): 1807-1819

[8] Garone E, Goldsmith A, Casavola A, et al. LQG control for MIMO systems over multiple erasure channels with perfect acknowledgment [J]. *IEEE Transactions on Automatic Control*, 2012, 57(2):450-456

[9] Xu H, Jagannathan S, Lewis F L. Stochastic optimal control of unknown linear networked control system in the presence of random delays and packet losses [J]. *Automatica*, 2012, 48:1017-1030

[10] Wang Z, Wang X, Liu L, et al. Optimal state feedback control for wireless networked control systems with decentralized controllers [J]. *IET Control Theory & Applications*, 2015, 9(6): 852-862

[11] Engwerda J C. Linear Quadratic Dynamic Optimization and differential game theory [M]. West Sussex: John Wiley, 2005. 29-45

[12] Cao X, Chen J, Cheng P, et al. Optimal controller location in wireless networked control systems[J]. *International Journal of Robust and Nonlinear Control*, 2015, 25(2):301-319

[13] Wang Z, Wang X, Liu L. Stochastic optimal linear control of wireless networked control systems with delays and packet losses [J]. *IET Control Theory & Applications*, 2016,10(7):742-751

[14] Fosha C E, Elgerd O I. The megawatt frequency control problem: a new approach via optimal control theory [J]. *IEEE Transactions on Power Apparatus & Systems*, 1970, PAS-89(4): 563-577

[15] Dong L, Zhang Y. On design of a robust load frequency controller for interconnected power systems [C]. In: Proceedings of the American Control Conference, Baltimore, USA, 2010. 1731-1736

Chen Yawen, born in 1993. She is now pursuing the master degree in Information and Communication Engineering from Beijing University of Technology. She received her B. S. degree in Communication Engineering of Beijing University of Technology in 2015. Her research interests include wireless network control and optimization algorithm.