HIGH TECHNOLOGY LETTERS | Vol. 20 No. 4 | Dec. 2014 | pp. 355 ~ 362

doi:10.3772/j.issn.1006-6748.2014.04.003

A novel multi-sensor multiple model particle filter with correlated noises for maneuvering target tracking①

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Abstract

Aiming at the effective realization of particle filter for maneuvering target tracking in multi-sensor measurements, a novel multi-sensor multiple model particle filtering algorithm with correlated noises is proposed. Combined with the kinetic evolution equation of target state, a multi-sensor multiple model particle filter is firstly constructed, which is also used as the basic framework of a new algorithm. In the new algorithm, in order to weaken the adverse influence from random measurement noises in the measuring process of particle weight, a weight optimization strategy is introduced to improve the reliability and stability of particle weight. In addition, considering the correlated noise existing in the practical engineering, a decoupling method of correlated noise is given by the rearrangement and transformation of the state transition equation and measurement equation. Since the weight optimization strategy and noise decoupling method adopt respectively the center fusion structure and the off-line way, it improves the adverse effect effectively on computational complexity for increasing state dimension and sensor number. Finally, the theoretical analysis and experimental results show the feasibility and efficiency of the proposed algorithm.

Key words: multi-sensor information fusion, weight optimization, correlated noises, maneuvering target tracking

0 Introduction

Maneuvering target tracking could be formulated as a multiple model nonlinear filtering problem. The existing solutions include an interacting multiple model (IMM) approach[1] with suboptimal filters such as extended Kalman filter (EKF) or unscented Kalman filter (UKF) and so on[2,3]. However, the inherent mixing operation in IMM yields a density mixture that is non-Gaussian. IMM deals with this non-Gaussian problem by a single Gaussian approximation that introduces errors. It is well known that particle filter (PF) is ideal for nonlinear and/or non-Gaussian problems[4,5], which provides approximate solutions to finding filtering, predictive, and smoothing densities of interest. The approximation is based on discrete representation of these densities by samples from the space of unknowns and weights associated to the samples. The method is composed of three steps: (1) the generation of particles (samples from the space of unknowns), (2) the computation of the particle weights, and (3) re-sampling. In order to improve the filtering precision of nonlinear system state estimated, some scholars attempt to replace directly a suboptimal filter by PF in IMM. And each model has a fixed number of particles. However, PF has a major disadvantage in that provided with the extremely computationally expensive, because hundreds (even thousands) of particles are needed to maintain certain accuracy in tracking applications, which makes existing and improved PF impossible for real-time applications[6]. Motivated by further reducing the computational expense, a multiple model PF (MMPF) is presented by introducing model information in sampling process of particles, and then effectively weakens the adverse effect on computational complexity because of the increase of dimension or model[7]. However, MMPF inevitably leads into the decrease of particles number which are allotted in different system models[8]. As we all know, the effective

① Supported by the National Natural Science Foundation of China (No. 61300214), the National Natural Science Foundation of Henan Province (No. 132300410148), the Post-doctoral Science Foundation of China (No. 2014M551999) and the Funding Scheme of Young Key Teacher of Henan Province Universities (No. 2013GGJS-026).

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Received on Apr. 29, 2013

sampling of particles state and the reasonable measuring of particles weight are considered as two important aspects to obtain better estimation precision in PF[9,10]. The first is to optimize the sampling particle state by the introduction of current measurement information, and the proposal distribution optimization is common solution[11]. The second is to reduce the adverse influence of random measurements noise in measuring process of particles weight as much as possible. Considering the effective utilization of redundancy and complementary information from multi-sensor measurements, combined with the information fusion technology, a kind of optimization strategy of particle weight is developed. The objective is to improve the reliability and stability of particle weight.

In addition, it is known that the implementation of PF is subject to the basic assumption with independent noise processes, namely, it is independent between process noise and measurement noise every time. Although this assumption is made very often, it is not fulfilled in many practical applications. For example, the coordinate conversion between the target motion modeling and the measurement modeling, or the space transformation and registration of distributed measurement information leads to a correlation problem of noise measurement and process noise in target tracking system inevitably. On this condition, PF and some improving algorithms degenerate in filtering precision. Aiming at the decoupling of correlated noise, Jin and others give an optimal state estimation method for data fusion with correlated measurement noise, but its drawback is not applied to noise variance matrixes with same eigenvalues[12]. Ge and others give a recursive fusion algorithm with correlated noises and one-step out-of-sequence measurements, and the algorithm well solves the measurement correlated noise problem in sensor network[13]. However, it takes no account of the influence of system nonlinear. Considering the correlations between process noise and measurement noise in the non-linear system, Chen and others design a kind of new decouple method by the rearrange of the state transition equation to a new one, remove the correlation, and apply the decouple method to the framework of PF and improve the better filtering precision[14]. In order to deal with the adverse effect from correlated noise, we develop a kind of decoupling method. The objective is to improve the filtering precision to some extend.

The rest of the paper is structured as follows. Section 1 presents the realization principle of multiple model particle filter (MMPF) in multi-sensor measurement, which is the basis of the proposed algorithm. Section 2 gives a weight optimization strategy of particle weight and a decoupling method between process noise and measurement noise, in addition, it dynamically applies them into the framework of MMPF and gives a concrete flow of multi-sensor multiple model particle filter with correlated noises (MMMPF-CN) for maneuvering target tracking. Section 3 shows the results and some discussions of applying MMMPF-CN to a maneuvering tracking task. Finally, Section 4 concludes the paper.

1 Multiple model particle filter in multi-sensor measurement

The maneuvering target tracking can be described as follows.

$$r_k \sim p(r_k \mid r_{k-1}) \quad (1)$$

$$\boldsymbol{x}_k = f_{r_k}(\boldsymbol{x}_{k-1}) + \boldsymbol{u}_{k-1,r_{k-1}} \quad (2)$$

$$z_{k,m} = h(\boldsymbol{x}_k) + \boldsymbol{v}_{k,m} \quad m = 1,2\cdots,M \quad (3)$$

where $\boldsymbol{x}_k \in \boldsymbol{R}^{n_x}$ denotes the unknown system state at time k, $z_{k,m} \in \boldsymbol{R}^{n_z}$ denotes the measurement from m homogeneous sensors which are nonlinear mapping of current state. $\boldsymbol{u}_{k,r_k}$ and $\boldsymbol{v}_{k,m}$ denote the system noise and the measurement noise sequences, respectively, of which variance is $\boldsymbol{\sigma}^2_{\boldsymbol{u}_{k,r_k}}$ and $\boldsymbol{\sigma}^2_{\boldsymbol{v}_{k,m}}$. r_k denotes the unknown discrete model state, which is subject to the discrete-time, homogeneous and finite state one-order Markov chain. $D = \{1,2,\cdots,d\}$ and $\pi_{ab} = P_r\{r_{k+1} = b \mid r_k = a\}$ denotes the state space and the transition probability, respectively, here $a,b \in D$. $\boldsymbol{\Pi} = [\boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \cdots, \boldsymbol{\pi}_d]^{\mathrm{T}}$ denotes the transition probability matrix, $\boldsymbol{\pi}_a = [\pi_{a1}, \pi_{a2}, \cdots, \pi_{ad}]$ and $\sum_{b=1}^{d} \pi_{ab} = 1$. The state estimation problem can be solved by calculating the posterior probability density function $p(\boldsymbol{x}_k \mid z_{1:k,1:M})$ of $\boldsymbol{x}_k$ on the basis of all the available data of measurement sequence. Because the complete information of sequential estimation is in $p(\boldsymbol{x}_k \mid z_{1:k,1:M})$, some parameters, which system state estimation needs to know, can be obtained, such as mean and variance, etc. For the stochastic sampling characteristics of PF, model information can be introduced in particle sampling process to realize the joint estimation of system state and model adopted at current time. Then, the arithmetic mechanism of the multiple model particle filter in the following sections will be analyzed emphatically. The key idea of MMPF is to approximate $p(\boldsymbol{x}_k \mid z_{1:k,1:M})$ by particles with model information, and can be written as

$$p(\boldsymbol{x}_k \mid z_{1:k,1:M}) \approx \sum_{i=1}^{N} \delta(\boldsymbol{x}_k - \langle \boldsymbol{x}_k^i, r_k^i \rangle)/N \quad (4)$$

where $\delta(\cdot)$ is Dirac's delta and function. $\langle \boldsymbol{x}_k^i, r_k^i \rangle$

denotes sampling particles with model information at time k, which are sampled directly from $p(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k,1:M})$, here $i = 1,2,\cdots,N$ and $N \rightarrow \infty$. Unfortunately, $p(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k,1:M})$ is unknown generally and the above process is often impossible to implement. The difficulty can be circumvented by sampling particles $\{\langle \boldsymbol{x}_k^i, r_k^i \rangle, \omega_k^i\}_{i=1}^N$ with associated importance weights from a known and easy-to-sample proposal distribution $q(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k})$. The process is described as the importance sampling. And the associated importance weights of particle is defined as

$$\omega_k^i \propto p(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \boldsymbol{z}_{1:k,1:M}) / q(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \boldsymbol{z}_{1:k,1:M}) \tag{5}$$

In order to further depict the generation of $\{\langle \boldsymbol{x}_k^i, r_k^i \rangle\}_{i=1}^N$, $q(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \boldsymbol{z}_{1:k,1:M})$ is factorized as follows

$$q(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \boldsymbol{z}_{1:k,1:M}) = q(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle, \boldsymbol{z}_{1:k,1:M}) q(\langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle \mid \boldsymbol{z}_{1:k-1,1:M}) \tag{6}$$

It is known that $\langle \boldsymbol{x}_k^i, r_k^i \rangle$ is sampled by augmenting each $\langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle$ sampled from $q(\langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle \mid \boldsymbol{z}_{1:k-1,1:M})$ with the new state sampled from $q(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle, \boldsymbol{z}_{1:k,1:M})$. Considering the multi-sensor character of the measurement system, it is needed to calculate the weight $\omega_{k,m}^i$ of every particle in line with single sensor measurement. In order to obtain the recursive equation of $\omega_{k,m}^i$, $p(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \boldsymbol{z}_{1:k,1:M})$ is expressed in terms of $p(\boldsymbol{z}_{k,m} \mid \langle \boldsymbol{x}_k^i, r_k^i \rangle)$, $p(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle)$ and $p(\langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle \mid \boldsymbol{z}_{1:k-1,1:M})$. And

$$p(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \boldsymbol{z}_{1:k,1:M}) \propto p(\boldsymbol{z}_{k,m} \mid \langle \boldsymbol{x}_k^i, r_k^i \rangle) p(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle) p(\langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle \mid \boldsymbol{z}_{1:k-1,1:M}) \tag{7}$$

Under assumptions that the characteristic of system state translation is subject to the Markov process and the measurement sequence meet the conditionally independent. Meanwhile, according to Eq. (5), Eq. (6) and Eq. (7), the particle weights $\omega_{k,m}^i$ is given by

$$\omega_{k,m}^i = \omega_{k-1,m}^i p(\boldsymbol{z}_{k,m} \mid \langle \boldsymbol{x}_k^i, r_k^i \rangle) p(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle) / q(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle, \boldsymbol{z}_{1:k,1:M}) \tag{8}$$

In the practical application, the proposal distribution is commonly selected as

$$q(\langle \boldsymbol{x}_{\mathrm{k}}^{\mathrm{i}}, \mathrm{r}_{\mathrm{k}}^{\mathrm{i}} \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_{k-1}^i \rangle, \boldsymbol{z}_{1:k,1:M}) = p(\langle \boldsymbol{x}_k^i, r_k^i \rangle \mid \langle \boldsymbol{x}_{k-1}^i, r_k^i \rangle) p(r_k^i \mid r_{k-1}^i) \tag{9}$$

Substituting Eq. (9) into Eq. (8), the weights update equation can then be shown to be

$$\omega_{k,m}^i = \omega_{k-1,m}^i p(\boldsymbol{z}_{k,m} \mid \langle \boldsymbol{x}_k^i, r_k^i \rangle) \tag{10}$$

Next, $\omega_{k,m}^i$ is normalized, and let $\varpi_{k,m}^i$ denote the normalized weights. Based on $\varpi_{k,m}^i$ and the number N of sampling particle, the re-sampling is introduced to improve the particle degenerate problem. The underlying principle of re-sampling is to eliminate particles with small weights and to duplicate particles with large weights under the conditions of the total particles number invariant[15]. A set of new particles $\{\langle \boldsymbol{x}_k^j, r_k^j \rangle\}_{j=1}^N$ are sampled after re-sampling. According to the Monte Carlo simulation technology, state estimation can be ultimately achieved by the arithmetic means of $\{\langle \boldsymbol{x}_k^j, r_k^j \rangle\}_{j=1}^N$.

2 Multi-sensor multiple model particle filter with correlated noise for maneuvering target tracking

2.1 The weight optimization strategy

In view of the richly redundant and complementary information existing in the multi-sensor measurement system, it provides the necessary condition objectively to improve the influence of random measurement noise. According to the realization principle of particle filter and the characteristic of sensor accuracy, meanwhile, combined with the construction of multi-senor likelihood function and the weight fusion ideology, the particle weight optimization strategy is designed to improve the variance of particle weight[16]. And then it gives the principle and process of particle weight optimization strategy in detail. Firstly, suppose that measurement noise is subject to the hypothesis of Gaussian distribution, and Eq. (10) is written as

$$\begin{aligned}\omega_{k,m}^i &= \omega_{k-1,m}^i \exp(-(\boldsymbol{z}_{k,m} - h(\langle(\boldsymbol{x}_k^i, r_k^i\rangle))^2 / 2\boldsymbol{\sigma}_{v_{k,m}^2}) / \sqrt{2\pi}\boldsymbol{\sigma}_{v_{k,m}} \\ &= \omega_{k-1,m}^i \exp(-(\boldsymbol{v}_{k,m} - (h(\langle \boldsymbol{x}_k^i, r_k^i \rangle) - h(\boldsymbol{x}_k)))^2 / 2\boldsymbol{\sigma}_{v_{k,m}^2}) / \sqrt{2\pi}\boldsymbol{\sigma}_{v_{k,m}}\end{aligned} \tag{11}$$

It is known that $\omega_{k,m}^i$ is subject to the Gaussian distribution with mean $h(\langle \boldsymbol{x}_k^i, r_k^i \rangle) - h(\boldsymbol{x}_k)$ and variance $\boldsymbol{\sigma}_{v_{k,m}}^2$. Secondly, it is needed to calculate which denotes weight of particle i after fusion at time k, and let $\boldsymbol{\lambda}_{k,m}$ be defined as the weight coefficient at the fusion process.

$$\omega_k^i = \omega_{k-1,m}^i \sum_{m=1}^M \boldsymbol{\lambda}_{k,m} (\exp(-(\boldsymbol{v}_{k,m} - (h(\langle \boldsymbol{x}_k^i, r_k^i \rangle) - h(\boldsymbol{x}_k)))^2 / 2\boldsymbol{\sigma}_{v_{k,m}^2}) / \sqrt{2\pi}\boldsymbol{\sigma}_{v_{k,m}}) \tag{12}$$

In accordance with its characteristics of Gaussian distribution, the following is given

$$\hat{\omega}_k^i \sim \boldsymbol{N}\left(\sum_{m=1}^M \boldsymbol{\lambda}_{k,m} (h(\langle \boldsymbol{x}_k^i, r_k^i \rangle) - h(\boldsymbol{x}_k)), \sum_{m=1}^M \boldsymbol{\lambda}_{k,m}^2 \boldsymbol{\sigma}_{v_{k,m}}^2\right) \tag{13}$$

And the standard deviation of $\hat{\omega}_k^i$ can be written as

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} = \sqrt{\sum_{m=1}^M \boldsymbol{\lambda}_{k,m}^2 \boldsymbol{\sigma}_{v_{k,m}}^2} \tag{14}$$

where the smaller $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ is, the higher the accuracy of fusion output is. Obviously, when $\boldsymbol{\sigma}_{v_{k,m}}$ is set, $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ is closely related to the distribution of $\lambda_{k,m}$. In order to obtain the highest fusion accuracy, $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ should be minimized. According to the information conservation principle, the calculation of $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ can be further attributed to the solving problem of conditional extreme value. That is when $\boldsymbol{\sigma}_{k,j}$ and $\sum_{m=1}^{M}\lambda_{k,m} = 1$ ($\lambda_{k,m} \geqslant 0$) are known, and how to find the conditions that the value of $\boldsymbol{\Lambda}(\lambda_{k,1},\lambda_{k,2},\cdots,\lambda_{k,M}) = \sum_{m=1}^{M}\lambda_{k,m}^2\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2$ meets the minimum. Considering that the above is a typical constraint condition equation of multivariable conditions extremum problems, the solution can be refer to the Lagrange multiplier method. So the modified function $\vartheta(\sum_{m=1}^{M}\lambda_{k,m} - 1)$ is introduced, and the function expression of $\boldsymbol{\Lambda}$ is given by

$$\boldsymbol{\Lambda} = \sum_{m=1}^{M}\lambda_{k,m}^2\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2 + \vartheta(\sum_{m=1}^{M}\lambda_{k,m} - 1) \tag{15}$$

The partial derivative of $\lambda_{k,m}$ is calculated on both sides of this function respectively. If and only if $\partial\boldsymbol{\Lambda}/\partial\lambda_{k,m}$ is equal to zero, and $\boldsymbol{\Lambda}$ can be taken as the minimum. The expression of $\lambda_{k,m}$ is written as

$$\lambda_{k,m} = -\vartheta/(2\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2) \tag{16}$$

Since $\sum_{m=1}^{M}\lambda_{k,m}$ is equal to 1, and

$$\vartheta = -2/(\sum_{m=1}^{M}1/\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2) \tag{17}$$

Then Eq. (17) is substituted into Eq. (16), and

$$\lambda_{k,m} = 1/(\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2\sum_{m=1}^{M}1/\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2) \tag{18}$$

After $\lambda_{k,m}$ is solved, the fusion precision $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ can be calculated by Eq. (19).

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} = 1/\sqrt{\sum_{m=1}^{M}1/\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2} \tag{19}$$

According to Eq. (19), when the measurement accuracies of sensors are the same and their values are all $\boldsymbol{\sigma}_{\boldsymbol{v}_k}$, we obtain

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} = \boldsymbol{\sigma}_{\boldsymbol{v}_k}^2/\sqrt{M} \tag{20}$$

The above equation also shows that the precision of particle weight can be improved $\sqrt{M}$ times than single sensor, while M sensors with same measurement accuracy are used. When the measurement accuracy of each sensor is different, the highest and worst accuracies are $\boldsymbol{\sigma}_{k,\max}^2$ and $\boldsymbol{\sigma}_{k,\min}^2$, respectively, and then

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} \leqslant 1/\Big(\boldsymbol{\sigma}_{\boldsymbol{v}_{k,\min}}^2 + \sqrt{\sum_{m=1}^{M-2}1/\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2}\Big) \tag{21}$$

Based on Eq. (21), the measurement accuracy of sensor will also help to improve the variance of particle weight no matter how bad it is.

2.2 The decoupling method of correlated noise

If there are dependences between process noise and measurement noise in the system described by Eq. (1) and Eq. (3), that is to say, the process noise and the measurement noise are subject to

$$E[\boldsymbol{u}_{k,r_k}(\boldsymbol{v}_{k,m})^{\mathrm{T}}] = \boldsymbol{S}_{\boldsymbol{u}_{k,r_k},\mathrm{v}_{k,m}} \tag{22}$$

where $\boldsymbol{S}_{\boldsymbol{u}_{k,r_k},\mathrm{v}_{k,m}}$ denotes the covariance between process noise and measurement noise. Obviously, it destroys the basic assumptions that the process noise and the measurement noise are independent in PF and improved PF, and then the performance of MMMPF will inevitably degrade to some extent. Aiming at the above problem, the decoupling method of correlated noise is given by learning from Ref. [14]. Its concrete procedures are as follows. Firstly, Eq. (3) is rewritten as

$$\boldsymbol{z}_{k-1,m} - h(\boldsymbol{x}_{k-1}) - \boldsymbol{v}_{k-1,m} = 0 \tag{23}$$

And then the following equation can be obtained

$$\begin{aligned}\boldsymbol{x}_k = {} & f_{r_k}(\boldsymbol{x}_{k-1}) - \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}h(\boldsymbol{x}_{k-1}) \\ & + \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}\boldsymbol{z}_{k-1,m} + \boldsymbol{u}_{k-1,r_{k-1}} \\ & - \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}\boldsymbol{v}_{k-1,m}\end{aligned} \tag{24}$$

where $\boldsymbol{\eta}_k$ is an additional parameter and it is with a proper dimensionality. In addition, the new state transition function $f_{r_k}^*(\boldsymbol{x}_{k-1})$ and new process noise sequence $\boldsymbol{u}_{k-1,r_{k-1}}^*$ are defined as

$$f_{r_k}^*(\boldsymbol{x}_{k-1}) = f_{r_k}(\boldsymbol{x}_{k-1}) - \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}h(\boldsymbol{x}_{k-1}) \tag{25}$$

$$\boldsymbol{u}_{k-1,r_{k-1}}^* = \boldsymbol{u}_{k-1,r_{k-1}} - \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}\boldsymbol{v}_{k-1,m} \tag{26}$$

$$\boldsymbol{\Theta}_{k-1} = \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}\boldsymbol{z}_{k-1,m} \tag{27}$$

Here $\boldsymbol{\Theta}_{k-1}$ is considered as the control item in the new system. The state transition equation can be simplified as

$$\boldsymbol{x}_k = f_{r_k}^*(\boldsymbol{x}_{k-1}) + \boldsymbol{\Theta}_{k-1} + \boldsymbol{u}_{k-1,r_{k-1}}^* \tag{28}$$

And the covariance of process noise and measurement noise in the new system can be obtained

$$E[\boldsymbol{u}_{k,r_k}^*(\boldsymbol{v}_{k,m})^{\mathrm{T}}] = \boldsymbol{S}_{\boldsymbol{u}_{k,r_k},\boldsymbol{v}_{k,m}} - \boldsymbol{\eta}_{k,m}\boldsymbol{\sigma}_{v_{k,m}}^2 \tag{29}$$

when $\boldsymbol{\eta}_k$ is selected as the following value.

$$\boldsymbol{\eta}_{k,m} = \boldsymbol{S}_{\boldsymbol{u}_{k,r_k},\boldsymbol{v}_{k,m}}(\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2)^{-1} \tag{30}$$

And the covariance is equal to zero in Eq. (30), that is to say, the process noise in Eq. (28) and the measurement noise in Eq. (3) are independent[14]. In addition, the mean $\mu_{\boldsymbol{u}_{k,r_k}^*}$ and the variance $\boldsymbol{\sigma}_{\boldsymbol{u}_{k,m_k}^*}^2$ of $\boldsymbol{u}_{k,r_k}^*$ can be calculated by the following two equations.

$$\mu_{\boldsymbol{u}_{k,r_k}^*} = E[\boldsymbol{u}_{k,r_k}] - \boldsymbol{\eta}_{k,m}E[\sum_{m=1}^{M}\boldsymbol{v}_{k,m}] = 0 \tag{31}$$

$$\boldsymbol{\sigma}_{\boldsymbol{u}_{k,r_k}^*}^2 = \boldsymbol{\sigma}_{\boldsymbol{u}_{k,r_k}}^2 - \sum_{m=1}^{M}\boldsymbol{S}_{\boldsymbol{u}_{k,r_k},\boldsymbol{v}_{k,m}}(\boldsymbol{\sigma}_{\boldsymbol{v}_{k,m}}^2)^{-1}(\boldsymbol{S}_{\boldsymbol{u}_{k,r_k},\boldsymbol{v}_{k,m}})^{\mathrm{T}} \tag{32}$$

According to Eq. (32), when process noise and measurement noise sequences are subject to independently identical distribution in original system, namely,

$\boldsymbol{S}_{\boldsymbol{u}_{k,r_k},\mathbf{v}_{k,m}}$ is equal to zero and

$$\boldsymbol{\sigma}^2_{\boldsymbol{u}^*_{k,r_k}} = \boldsymbol{\sigma}^2_{\boldsymbol{u}_{k,r_k}} \tag{33}$$

From the result, the new system modeled by Eq. (28) and Eq. (3) is equivalent to the original system, but also it is provided with the characteristic of uncorrelated noise, so it is of more universal.

2.3 The flow of MMMPF-CN

According to the above analysis, the realization of MMMPF-CN is similar to MMMPF in the algorithm framework, and the differences are the following two points. On one hand, particle weight is calculated by Eq. (16), the measured process of which synthesizes the redundancy and complementary information from the multi-sensor measurements. On the other hand, the estimation object of MMMPF is for the new nonlinear system decoupled by Eq. (28) and Eq. (3), not for original nonlinear system modeled by Eq. (2) and Eq. (3). The concrete steps are as follows.

- At time step $k-1$

Suppose we have $\{\langle \mathbf{x}^i_{k-1}, r^i_{k-1}\rangle, \hat{\omega}^i_{k-1}\}^N_{i=1}$ and $\hat{\mathbf{x}}_{k-1/k-1}$

- At time step k

With new measurements, $\mathbf{z}_{k,m}$, $m = 1,2,\cdots,M$

Firstly, the model states of every particle are sampled by $r^i_k \sim p(r^i_k \mid r^i_{k-1})$ at time k. The concrete steps are as follows. Supposing that the model state $r^i_{k-1} = a$ of particle i is known at time $k-1$, and ε is a uniformly distribution number in the range $(0,1]$. When ε meets $\sum_{b=1}^{c-1}\pi_{a,b} < \varepsilon \leqslant \sum_{b=1}^{c}\pi_{a,b}$, the model state of particle i is selected as $r^i_k = c$ at time k.

Secondly, according to Eq. (30) and Eq. (32), $\boldsymbol{\eta}_{k,m}$ and $\boldsymbol{\sigma}^2_{\boldsymbol{u}^*_{k,r_k}}$ are calculated.

Thirdly, combined with Eq. (28), $\langle \mathbf{x}^i_k, r^i_k\rangle$ is sampled by the following equation.

$$\langle \boldsymbol{x}^i_k, r^i_k\rangle = f_{r^i_k}(\langle \boldsymbol{x}^i_{k-1}, r^i_k\rangle) - \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}h(\langle \boldsymbol{x}^i_{k-1}, r^i_k\rangle) + \sum_{m=1}^{M}\boldsymbol{\eta}_{k-1,m}\boldsymbol{z}_{k-1,m} + \boldsymbol{u}^{i*}_{k,r_k} \tag{34}$$

Fourthly, the weight $\hat{\omega}^i_k$ of particle i is calculated by Eq. (12)

Fifthly, the re-sampling is introduced and a set of new particles set $\{\langle \mathbf{x}^j_k, r^j_k\rangle\}^N_{j=1}$ is sampled after the re-sampling stage.

Finally, the estimation $\hat{\mathbf{x}}_{k/k}$ is obtained by the method of arithmetic mean.

$$\hat{\mathbf{x}}_{k/k} = \sum_{j=1}^{N}\langle \mathbf{x}^j_k, r^j_k\rangle / N \tag{35}$$

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3 Simulation result and analysis

To exemplify the applicability of MMMPF-CN, the maneuvering target tracking system of multi-sensor measurements with three two-coordinate radars are considered as the simulation scene. In this example, the PC, 3.40GHz Intel i7-2600 CPU has 3.3GBs of RAM is used as the simulation hardware environment, and the target moves within the horizontal-vertical plane.

$$\boldsymbol{x}_k = \begin{cases} \boldsymbol{F}_1 \times \boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}\boldsymbol{u}_{k-1,1} & 1 \leqslant k < 8 \\ \boldsymbol{F}_2 \times \boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}\boldsymbol{u}_{k-1,2} & 8 \leqslant k < 16 \\ \boldsymbol{F}_1 \times \boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}\boldsymbol{u}_{k-1,1} & 16 \leqslant k \leqslant 25 \end{cases}$$

$$\boldsymbol{z}_{k,m} = [r_k \quad \theta_k]^{\mathrm{T}} + \boldsymbol{v}_{k,m} \qquad m = 1,2,\cdots,M$$

$$r_k = sqrt(x_k^2 + y_k^2)$$

$$\theta_k = sqrt(x_k^2 + y_k^2)$$

Where $\boldsymbol{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^{\mathrm{T}}$ denotes system state vector. x_k, $\dot{x}_k$, y_k and $\dot{y}_k$ denote position component and velocity component of target state on the horizontal direction and the vertical direction, respectively. The values of initial position and initial velocity are as follows, $[x_k, y_k] = [5\text{km}, 8\text{km}]$ and $[\dot{x}_k, \dot{y}_k] = [0.5\text{km/s}, 0.1\text{km/s}]$. $\boldsymbol{F}_1 = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $\boldsymbol{F}_2$

$$= \begin{bmatrix} 1 & \sin(\xi\tau)/\xi & 0 & -(1-\cos(\xi\tau))/\xi \\ 0 & \cos(\xi\tau) & 0 & -\sin(\xi\tau) \\ 0 & (1-\cos(\xi\tau))/\xi & 1 & \sin(\xi\tau)/\xi \\ 0 & \sin(\xi\tau) & 0 & \cos(\xi\tau) \end{bmatrix}$$

denote the system state transition matrixes, where $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ denote the uniform linear motion and the uniform turning motion respectively. The sampling time τ and turning angular velocity ξ are 1 and 0.15rad/s, respectively. $\boldsymbol{\Gamma} = \begin{bmatrix} 0 & 0 & \tau/2 & \tau \\ \tau/2 & \tau & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ denotes the system noise matrix. $\boldsymbol{u}_{k,1}$ and $\boldsymbol{u}_{k,2}$ denote the system noise vector, and suppose they are subject to zero-mean Gaussian white noise with standard deviation $\boldsymbol{\sigma}^2_{\boldsymbol{u}_{k,1}} = 0.2\mathbf{I}$ and $\boldsymbol{\sigma}^2_{\boldsymbol{u}_{k,2}} = 0.25\mathbf{I}$. $\mathbf{I}$ denotes the identity matrix. r_k and θ_k denote the radial distance component and the azimuth angle component in measurement vector, respectively. $\boldsymbol{v}_{k,m}$ denotes the measurement noise vector and suppose they are all subject to zero-mean Gaussian white noise process with standard deviation $\begin{bmatrix} R_{r,m} & 0 \\ 0 & R_{\theta,m} \end{bmatrix}$, here the noise standard deviations of radial distance component and azimuth angle component of three sensors are $R_{r,1} = 0.12\text{km}$, $R_{\theta,1} = 0.05\pi/180\text{rad}$, $R_{r,2} = 0.1\text{km}$, rad $R_{r,3} = 0.15\text{km}$, and $R_{\theta,3} = 0.02\pi/180\text{rad}$, respectively. The correlation coefficient of system noise and measurement noise is $0.2\mathbf{I}$. In this case, the numbers of Monte Carlo run are 50 and the total simulation steps are 25.

Three algorithms including MMPF, MMMPF and

MMMPF-CN are compared. Among them, MMPF denotes the traditional multiple model particle filter and the application objects are mainly for the single measurement condition. In the example, the second sensor measurements are used into the simulation realization in simulation. Relative to MMPF, the improvement of MMMPF is that the weight optimization strategy proposed is introduced into the framework of MMPF and the application objects of MMMPF are also expanded into the actual multi-sensor measurements. Further improvement is that we consider the problem of correlated noises in the actual engineering, the decoupling method is incorporated into the construction of MMMPF-CN. According to the realization principle of algorithms, three sensor measurements are used in the simulation of MMMPF and MMMPF-CN at the same time.

The motion trajectory of an estimated object is given in Fig. 1, and the comparisons of particle weight variance before and after the re-sampling are given in Fig. 2 and Fig. 3, respectively. The results clearly show that the stability and reliability of particle weight variance in MMMPF and MMMPF-CN are superior to MMPF, the reason is that the adverse effects caused by random measurements noise for particle weight is improved by the rational utilization of multi-sensor measurements information. And we also observe that the decoupling of correlated process and measurement noises also promote the stability and reliability of particle weight to some extend. Fig. 4 and Fig. 5 show the comparison of RMSE of state estimation for three algorithms in horizontal and vertical direction over 50 independent runs. The data in Table 1 quantitatively show the mean of RMSE of state estimation. It is shown clearly that the filtering precision of MMMPF-CN is superior to other two algorithms. Moreover, the following conclusions can be drawn by the data analysis in Table 1. MMMPF has the better filtering precision relative to MMPF, but its real-time is inferior to MMPF. The reason is that we need to calculate M particle weight $\omega_{k,m}^{i}$ according to Eq. (11) in order to obtain $\hat{\omega}_{k}^{i}$, and the more number of sensors used in the measurement system, the more computational complexity will be. But we should also find that the increase of computational complexity appear on the measure process of particle weight because of the utilization of center fusion structure, which does not reflects on the re-sampling step. So the increase will not cause rapid expansion of calculation amount. In addition, compared with MMMPF, MMMPF-CN has a little increase in the computational complexity. The main reason is that the decoupling process of correlated process and measurement noises is an off-line way.

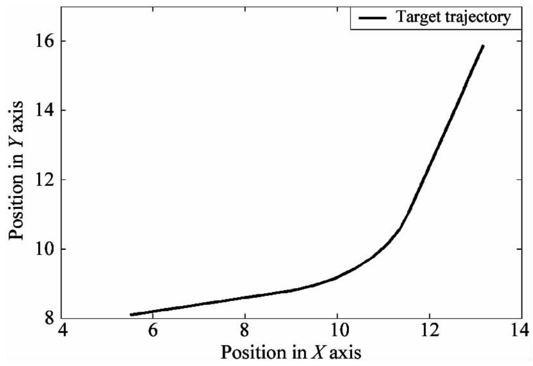

Fig. 1 Motion trajectory of estimated object

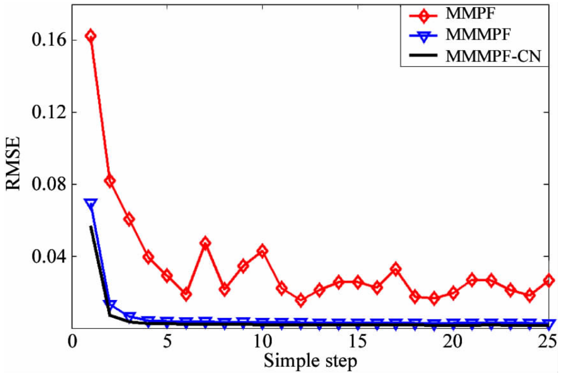

Fig. 2 Before the re-sampling

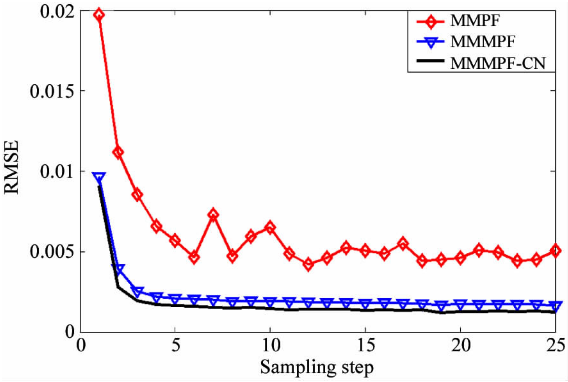

Fig. 3 After the re-sampling

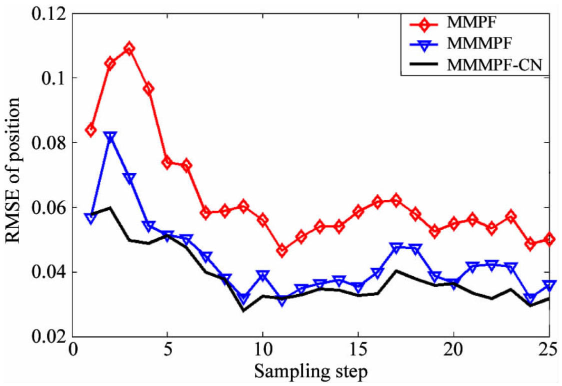

Fig. 4 Horizontal direction

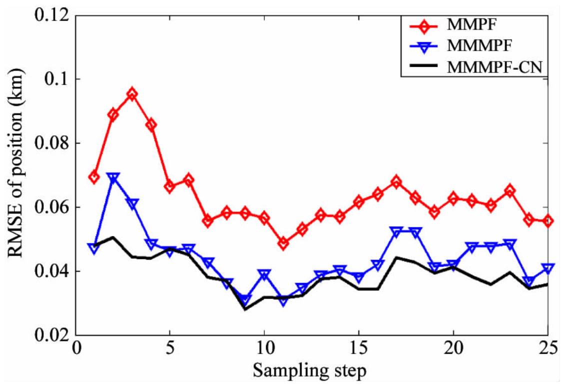

Fig. 5 Vertical direction

Table 1 The comparison for the mean of RMSE and the average time over 50 independent runs

Algorithm	MMPF	MMMPF	MMMPF-CN
Horizontal direction (km)	0.0638	0.0439	0.0386
Vertical direction (km)	0.0639	0.0442	0.0390
Cost time (s)	0.4051	0.4527	0.4553

4 Conclusions

The objective of this paper is to deal with the application of particle filter in maneuvering target tracking with the multi-sensor measurement system. Considering some actual problems occurring in maneuvering target tracking such as the effective utilization of multi-sensor measurement and correlated characteristic existing between process noise and measurement noise, meanwhile, in view of the requirement of computational complexity and real time, MMPF is used as the basic framework of the new algorithm. And then, we separately design the optimization strategy of particle weight and the decouple method of correlated noise which are dynamically combined with MMPF. The theoretical analysis and experimental results show the new algorithm has the following advantages.

1) MMPF is considered as a new solution to manage the switching problem of the multi-sensor model in accordance with the particle sampling mechanism. Compared with the direct combination of IMM and PF, MMPF solving the fast-growing problem of particle number is with the increase of the system model.

2) Aiming at the decline of filtering precision caused by the instability of particle number allotted to different models, according to the perspective to enhance reliability particle weight, a novel optimization strategy of particle weight is designed by the centralized weighted fusion mode. Besides, the optimization strategy of particle weight also gives a way of extracting and synthesizing multi-sensor information in the structure of sampling nonlinear filters, which has contributed to the application of PF in multi-sensor measurement system. In addition, the optimizing process of particle weight is only related to the measurement accuracy of sensor used at current time, not related to the number and sampling rate of sensors, so the optimization way also avoids the adverse influence from the lack and out-of-sequence of measurements.

3) Considering many actual projects with correlated measurement and process noise, we design a kind of off-line decoupling method correlated noise in order to further improve the filtering precision, meanwhile, the off-line form effectively avoids the increase of computing complexity.

4) The proposed method has two shortcomings, one is that MMMPF-CN can only treat the case of which processes noise and measurement noise must be subject to Gaussian distribution at the time. In addition, the other is that the application object of MMMPF-CN is limited to the measurement information from homogeneous sensors. So they also limit the application field of algorithm to some extent, the heterogeneous sensor system estimation with non-Gaussian correlated noise will be studied further in the future.

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