Trajectory tracking control based on linear active disturbance rejection controller for 6-DOF robot manipulator[①]

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Abstract

The trajectory tracking control for a 6-DOF robot manipulator with multiple inputs and outputs, non-linearity and strong coupling is studied. Firstly, a dynamical model for the 6-DOF robot manipulator is designed. From the view point of practical engineering, considering the model uncertainties and external disturbances, the robot manipulator is divided into 6 independent joint subsystems, and a linear active disturbance rejection controller(LADRC) is developed to track trajectory for each subsystem respectively. LADRC has few parameters that are easy to be adjusted in engineering. Linear expansion state observer(LESO) as the uncertainty observer is able to estimate the general uncertainties effectively. Eventually, the validity and robustness of the proposed method adopted in 6-DOF robot manipulator are demonstrated via numerical simulations and 6-DOF robot manipulator experiments, which is of practical value in engineering application.

Key words: 6-DOF robot manipulator, linear active disturbance rejection controller (LADRC), linear expansion state observer (LESO), trajectory tracking control

0   Introduction

Recently, the robot technology has become more and more mature along with the development of control system, electronic technology, artificial intelligence and other related technologies. It plays an increasingly important role in industrial production and manufacturing, man-machine interaction service, space on-orbit maintenance fields[1-3]. The robot manipulator in series structure has been widely used in the above fields due to its simple structure, flexible control, large working space and low cost[4,5].

In the practical engineering task, the precise tracking of the spatial motion trajectory for the robot manipulator is particularly important. Due to the fact that the robot manipulator as a complex system with multiple inputs and outputs, non-linearity and strong coupling has inevitable uncertainties such as parameter perturbation, unmodeled dynamics and external disturbances, hence there are many difficult problems in precise trajectory tracking control for the robot manipulator[6]. Thus, the robust control problem has got increasing attention with model uncertainties and external disturbances. Sun et al.[7] proposed a neural network sliding mode adaptive controller aiming at the internal uncertainties and external disturbance. The neural network and adaptive control can estimate uncertainties. Finally, simulations were performed to validate the effectiveness of the proposed method. A neural adaptive robust control strategy[8] was provided to realize uncertainties estimation of the robot manipulator system. The neural network approximated the gravity term in the manipulator model on line, and the error compensation was made by adaptive robust control using linear matrix inequalities to synthesize controller gains. The control strategy combining sliding mode control and fuzzy control was applied to the robot manipulator in Ref. [9] in order to track precise trajectory. The simulation results showed that the system had strong robustness in the case of dramatic parameter perturbation and external disturbances. The non-singular terminal sliding mode control with phased control law was proposed to help to improve the convergence speed of sliding mode and reduce the buffeting phenomenon. Numerical simulation results verified the effectiveness of the algorithm applied in robot manipulator[10].

Most of above robust control methods have been validated only in numerical simulations. However, for practical robot manipulator, above intelligent algo-

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rithms are more complex and difficult to implement in engineering. At present, the proportional-integral-differential (PID) controller is generally used to control the trajectory of the practical robot manipulator. However, PID is seriously disturbed by environmental factors and has weak robustness. Therefore, from the perspective of practical engineering application, linear active disturbance rejection controller (LADRC) is proposed to control the trajectory of 6-DOF robot manipulator in this paper. The design idea of LADRC is derived from active disturbance rejection controller (ADRC)[11]. Thereinto, extended state observer is linearized so as to simplify control parameters and is able to estimate disturbances exactly, which is suitable for practical engineering application. The 6-DOF robot manipulator is decoupled into 6 joints and controlled by LADRC respectively. Then, coupling terms between each joint seen as internal disturbances of the system are estimated by linear extended state observer (LESO). Finally, both numerical simulation experiments and robot manipulator experiments have proved that the 6-DOF robot manipulator based on LADRC has great tracking control performance and favorable practical application value.

1 Dynamic model of 6-DOF robot manipulator

The scheme of the robot manipulator in series is shown in Fig. 1, where $\sum_I$, $\sum_B$, $\sum_E$ express the inertial coordinate, the base coordinate and the end coordinate of a robot manipulator respectively. O_I is origin of inertial coordinate. O_g is the center of the whole system. B_0 is the base of the manipulator. B_i is the ith link and $i = 1, \cdots, n$. J_i is the joint connecting B_{i-1} with B_i. C_i is the center of mass of B_i. $r_g \in R^3$ is position vector of the center of mass of the system. $r_i \in R^3$ is position vector of B_i. $p_i \in R^3$ is the position vector of J_i. $p_e \in R^3$ is position vector at the end of the manipulator.

For the robot manipulator system in series depicted in Fig. 1, it can be assumed to be a fully driven rigid body system, which means that there is a separate control input for each degree of freedom. Considering that the base is fixed, the dynamic model of the 6-DOF robot manipulator can be expressed by Lagrange equation as follows:

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where $q = [q_1, q_2, \cdots, q_6]^T \in R^3$ represents the angular displacement vector of each joint. $D(q) \in R^{6\times6}$ expresses the inertial matrix of the system. $B(q,\dot{q}) \in$

$R^{6\times6}$ is coriolis force and centrifugal force matrix. the detailed expressions of $D(q)$ and $B(q,\dot{q})$ are described in Ref. [12]. $G(q)$ represents the vector matrix of gravity. $\tau = [\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6]^T$ is the input drive torque to be designed.
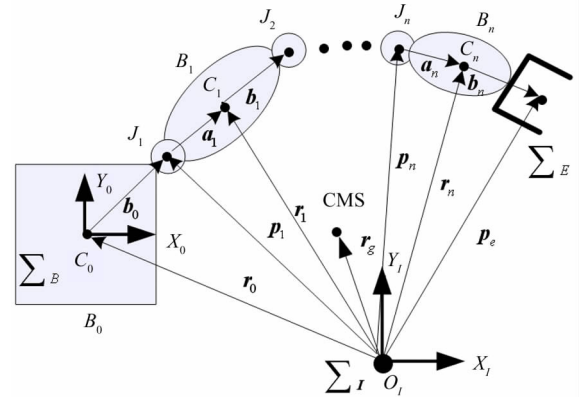

Fig. 1   The scheme of the robot manipulator in series

For rigid robot manipulator motion control, the complexity is mainly reflected in dynamics and uncertainties[13]. The dynamics are caused by the nonlinearity and coupling of the robot manipulator, and the uncertainties are caused by structural and unstructured factors[14]. Structural uncertainties refer to the accuracy of dynamic parameters. Unstructured uncertainties are caused by the flexibility of joints and linkages, actuation dynamics, friction, sensor noise, and unknown environments. Thus, in this work, all the internal and external uncertainties of the robot manipulator system are considered as a generalized disturbance, and then Eq. (1) is written as

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) + \omega = \tau \tag{2}$$

where $\omega \in R^6$ denotes the generalized disturbance of the robot manipulator system.

2 LADRC design of robot manipulator

According to the dynamic model of the robot manipulator, it is a complex system with multiple inputs and outputs, strong coupling and nonlinearity. Conventional control methods are difficult to reach high precise control requirements. However, LADRC does not depend on the mathematical model of the controlled object and has the ability to observe and compensate generalized disturbance by real-time. In order to facilitate the engineering application, LADRC is proposed as a trajectory tracking controller for 6-DOF robot manipulator, which reduces parameters setting and simplifies controller design.

Firstly, the dynamic model of the 6-DOF robot

manipulator is transformed as follows:

$$\boldsymbol{\psi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, t) = -\boldsymbol{D}^{-1}(\boldsymbol{q})[\boldsymbol{B}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\omega}] \tag{3}$$

$$\widetilde{\boldsymbol{B}}(\boldsymbol{q}) = \boldsymbol{D}^{-1}(\boldsymbol{q}) \tag{4}$$

It can be obtained by substituting it into Eq. (2):

$$\ddot{\boldsymbol{q}} = \boldsymbol{\psi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, t) + \widetilde{\boldsymbol{B}}(\boldsymbol{q})\boldsymbol{\tau} \tag{5}$$

where $\boldsymbol{\psi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, t) = [\psi_1, \psi_2, \cdots, \psi_3] \in R^6, \widetilde{\boldsymbol{B}}(\boldsymbol{q})$

$$= \begin{pmatrix} b_{11}(\boldsymbol{q}) & b_{12}(\boldsymbol{q}) & \cdots & b_{16}(\boldsymbol{q}) \\ b_{21}(\boldsymbol{q}) & b_{22}(\boldsymbol{q}) & \cdots & b_{26}(\boldsymbol{q}) \\ \vdots & \vdots & & \vdots \\ b_{61}(\boldsymbol{q}) & b_{62}(\boldsymbol{q}) & \cdots & b_{66}(\boldsymbol{q}) \end{pmatrix},$$

The equation of the ith joint ($i = 1, 2, \cdots, 6$) can be described as

$$\ddot{q}_i = \psi_i + \sum_{\substack{j=1 \\ j \neq i}}^{6} b_{ij}\tau_j + (b_{ii} - b)\tau_i + b\tau_i \tag{6}$$

For the current ith joint, torques of other joints are considered to be external disturbances, at the same time, the unknown part of the torque in this joint is also seen as the disturbance, thus, the generalized disturbance of the ith joint is denoted as

$$h_i = \psi_i + \sum_{\substack{j=1 \\ j \neq i}}^{6} b_{ij}\tau_j + (b_{ii} - b)\tau_i \tag{7}$$

Substituting it into Eq. (6), it can be expressed as

$$\ddot{q}_i = h_i + b\boldsymbol{\tau} = f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, t) + b\boldsymbol{\tau} \tag{8}$$

where $f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, t)$ is the generalized disturbance.

Therefore, LADRC is designed for the system of Eq. (8). The tracking differentiator is omitted and the nonlinear function $fal(\cdot)$ is replaced by the error term in LADRC, LESO has the ability to observe and compensate the generalized disturbance. Taking joint 1 of 6-DOF robot manipulator as an example, the control scheme based on LADRC is described in Fig. 2. q_{d1} expresses the desired joint angle, q_1 denotes the actual joint angle, τ_1 represents control torque, ω is the external disturbance acted on the joint 1. LESO is designed as follows:

$$\begin{cases} e_1 = \tilde{q}_1 - q_1 \\ \dot{\tilde{q}}_1 = \tilde{q}_2 - \beta_{01}e_1 \\ \dot{\tilde{q}}_2 = \tilde{q}_3 - \beta_{02}e_1 + b\tau_1 \\ \dot{\tilde{q}}_3 = -\beta_{03}e_1 \end{cases} \tag{9}$$

where $\tilde{q}_1$ is estimate of the joint angle and $\tilde{q}_2$ is the estimate of joint angle velocity. The expanded state $\tilde{q}_3$ is the real-time action of the generalized disturbance $f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, t)$, and the estimate $\tilde{f}$ of $f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, t)$ is compensated in the control quantity. Then the control moment of joint 1 is obtained as

$$\tau_1 = \tau_0 - \tilde{f}/b \tag{10}$$

Due to the fact that LESO can estimate the general disturbance exactly, the joint 1 subsystem is transformed into integral series and can be taken as

$$\ddot{q}_1 = b\tau_0 \tag{11}$$

In order to obtain the state error feedback control quantity τ_0, PD controller is designed as follows:

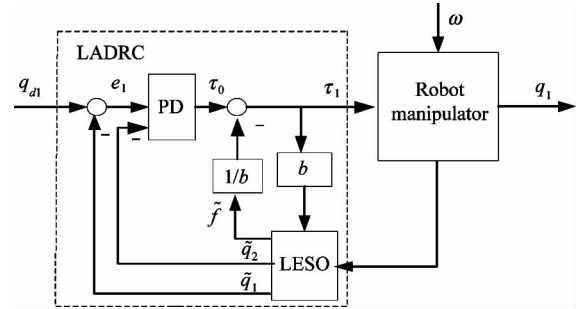$$\tau_0 = K_{p1}(q_{d1} - \tilde{q}_1) - K_{d1}\tilde{q}_2 \tag{12}$$

Fig. 2 The control scheme based on LADRC for the joint 1 of robot manipulator

with $K_{p1} = \omega_{c1}^2$, $K_{d1} = 2\xi\omega_{c1}$, where ξ, ω_{c_1} are the closed loop damping ratio and natural oscillation frequency respectively. q_{d1} denotes the desired joint angle of joint 1. To avoid differentiating the desired joint angle, the item $-K_{d1}\tilde{q}_2(k + 1)$ is introduced[15]. In addition, the control gain of LESO is defined as $L_q = [\beta_{01}, \beta_{02}, \beta_{03}]^T$, then, the characteristic equation of Eq. (9) is expressed as $s^3 + \beta_{01}s^2 + \beta_{02}s + \beta_{03}$. For the sake of simplifying control parameters of LESO, the control gain of joint 1 subsystem is set as $L_q = [3\omega_{q1}, 3\omega_{q1}^2, \omega_{q1}^3]^T$, where ω_{q1} represents the bandwidth and is equal to $(5 - 10)\omega_{c1}$. Therefore, the characteristic equation is rewritten as $(s + \omega_{q1})^3$. Then, only one parameter ω_{q1} needs to be set in LESO, which ensures much faster and more stable system.

Eventually, the precise tracking control of joint 1 subsystem can be realized by reasonable configuration of parameters $\xi, \omega_{c_1}, \omega_{q1}$. Furthermore, other 5 joint subsystems based on LADRC have the same design procedure, which is no longer the description for the sake of simplicity.

3 Numerical simulation results

The trajectory tracking compared simulations between LADRC method and conventional PID method as well as disturbance simulation of the 6-DOF robot manipulator are carried out to demonstrate the validity of LADRC. The parameters of dynamic model in simulations are taken from the 6-DOF robot manipulator, as listed in Table 1.

Table 1　The parameters of 6-DOF robot manipulator

Joint	Club length (m)	Frictional coefficient	Mass (kg)	Moment of inertia (kg · m²)
1	0.1	0.3	2.01	0.045
2	0.39	0.4	4.3	0.43
3	0.53	0.5	7.73	0.007
4	0.53	0.5	6.64	0.007
5	0.39	0.4	4.3	0.3
6	0.1	0.3	2.01	0.045

Firstly, parameters of 6 joints in LADRC are set as follows by trial and error $\omega_{qi} = 5\omega_{ci}$, $i = 1, \cdots, 6$, $\xi = 1$, $\omega_{c1} = 70$, $\omega_{c2} = 15$, $\omega_{c3} = 10$, $\omega_{c4} = 10$, $\omega_{c5} = 10$, $\omega_{c6} = 10$. In addition, parameters of PID are chosen as $K_{p1} = 20$, $K_{i1} = 1$, $K_{d1} = 8$, $K_{p2} = 15$, $K_{i2} = 0.7$, $K_{d2} = 6$, $K_{p3} = 20$, $K_{i3} = 1$, $K_{d3} = 10$, $K_{p4} = 18$, $K_{i4} = 0.9$, $K_{d4} = 6$, $K_{p5} = 14$, $K_{i5} = 0.6$, $K_{d5} = 5$, $K_{p6} = 13$, $K_{i6} = 0.4$, $K_{d6} = 6$。Assume the initial joint angles as $\boldsymbol{q}_0 = [0.6, 0.5, 0.8, 0.7, 0, 0]$ rad, the initial joint angle velocities $\boldsymbol{w}_0 = [0.3, 0.5, 0.4, 0.5, 0, 0]$ rad/s, and the desired joint angles as $q_{d1} = \sin(0.5\pi t)$, $q_{d2} = \cos(0.5\pi t)$, $q_{d3} = 1 + 0.2\sin(0.5\pi t)$, $q_{d4} = 1 - 0.2\cos(0.5\pi t)$, $q_{d5} = 1 - 0.3\cos(0.5\pi t)$, $q_{d6} = 0.2\sin(0.5\pi t)$.

The trajectory tracking compared simulations of 6 joints between LADRC method and PID method are operated, as described from Fig. 3 to Fig. 8. It can be noted that LADRC provides much greater trajectory tracking control performance than PID method. It has
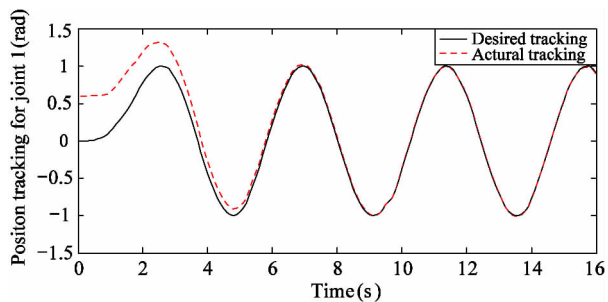
(a) LADRC method
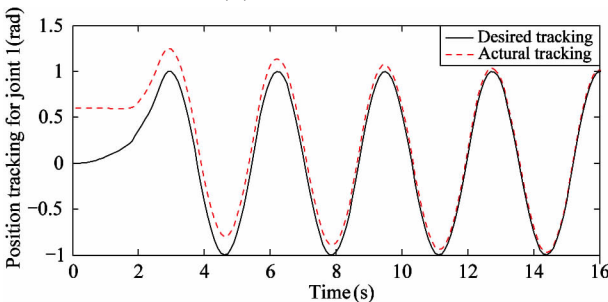
(b) PID method

Fig. 3　Trajectory tracking control for joint 1
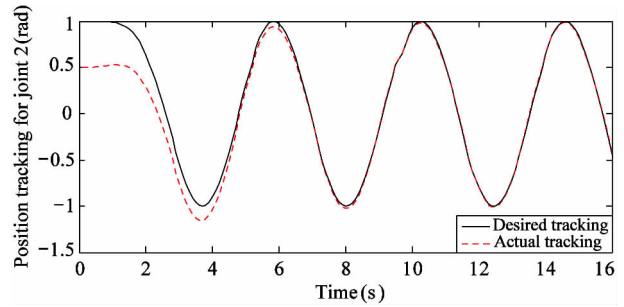
(a) LADRC method
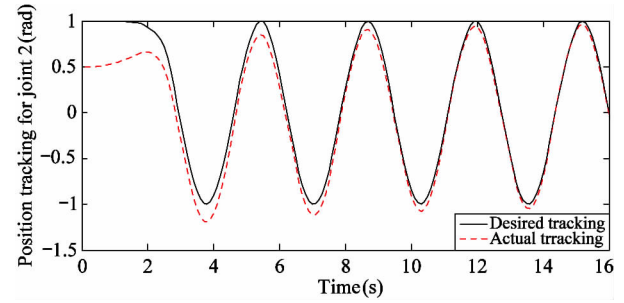
(b) PID method

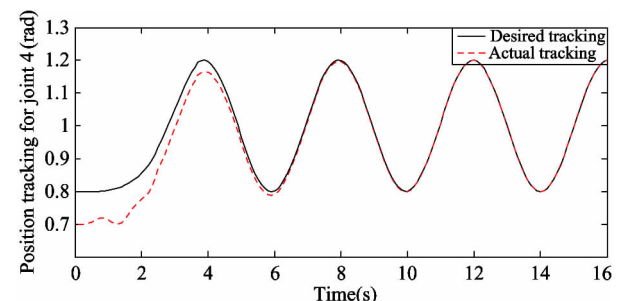Fig. 4　Trajectory tracking control for joint 2
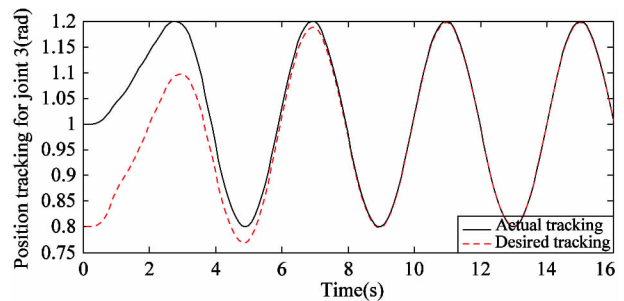
(a) LADRC method
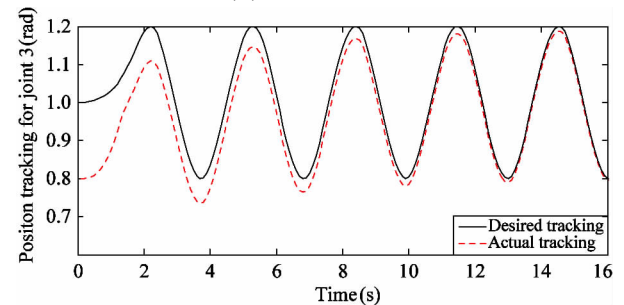
(b) PID method

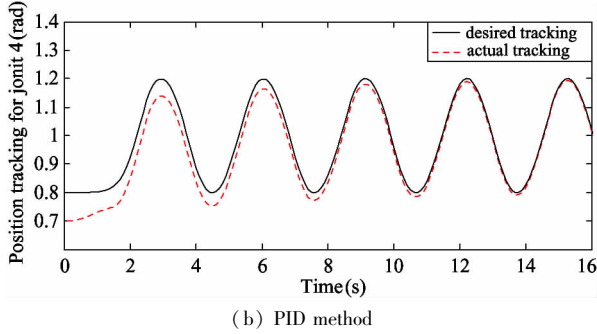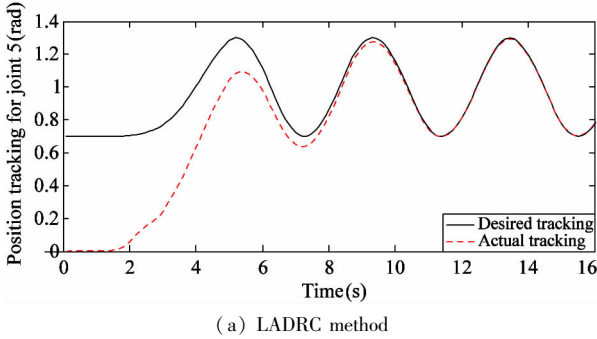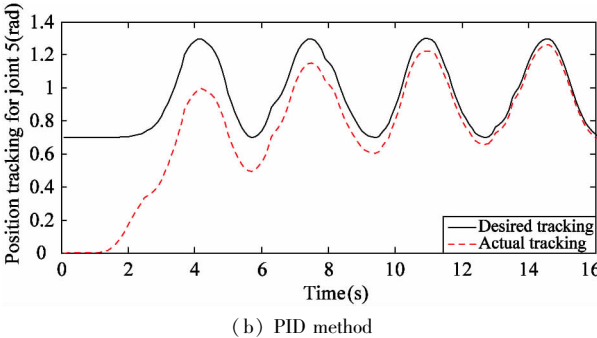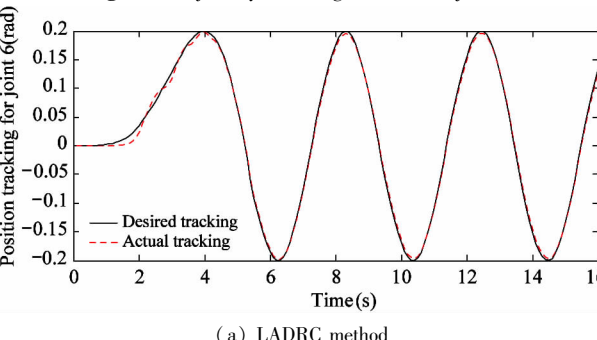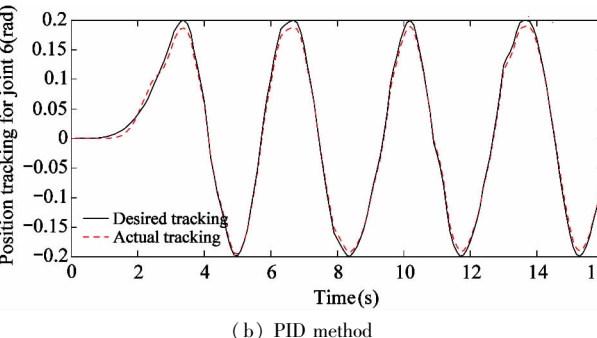Fig. 5　Trajectory tracking control for joint 3

(a) LADRC method

(b) PID method

Fig. 6　Trajectory tracking control for joint 4

(a) LADRC method

(b) PID method

Fig. 7　Trajectory tracking control for joint 5

(a) LADRC method

(b) PID method

Fig. 8　Trajectory tracking control for joint 6

long settling time, large overshoot and obvious static error in PID. However, LADRC method possesses short settling time and no static error. It is clear that LADRC method is better suited in dealing with trajectory tracking control problem about the 6-DOF robot manipulator.

Next, to further assess the robustness of the proposed algorithm, time-varying external disturbance τ_d = $0.2\sin(0.5t)$ is introduced in the 6 joints respectively. The control parameters of LADRC remain the same as the first simulation. The initial state of the robot manipulator is zero, and the desired joint angles are assumed as $q_d = [1.2, 1.8, 0.9, 1, 1.5, 1.2]$.

The simulation results are shown from Fig. 9 to Fig. 14, where it can be clearly obtained that the joint angles subjected to the time-varying disturbance have a certain overshoot, while it is thoroughly eliminated due to the fact that LESO estimates and compensates the time-varying disturbance successfully. Then, the manipulator based on LADRC can quickly track the desired joint angle accurately with zero steady state error and strong robustness. Moreover, as depicted from Fig. 15 to Fig. 20, it is evident that LESO has the satisfied uncertainties estimation performance. As a result, simulation results highlight the claim that LADRC algorithm can offer the favorable trajectory tracking control performance and strong robustness in the presence of general disturbances, which is very suitable for practical engineering application.
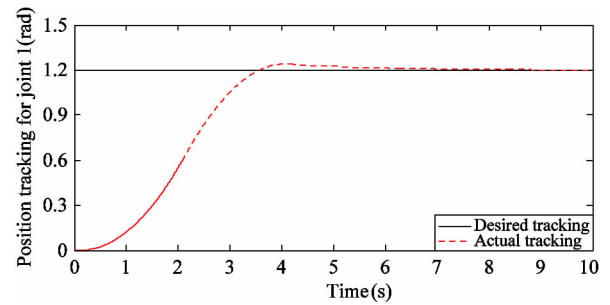

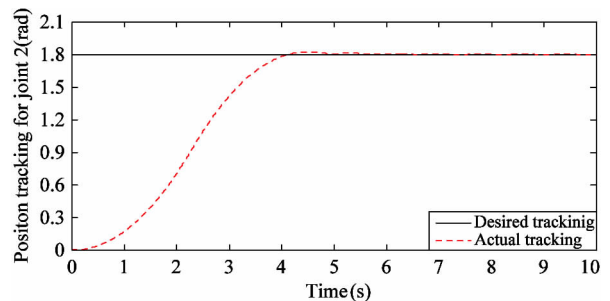**Fig. 9**　Trajectory tracking control for joint 1 based on LADRC

Fig. 10　Trajectory tracking control for joint 2 based on LADRC
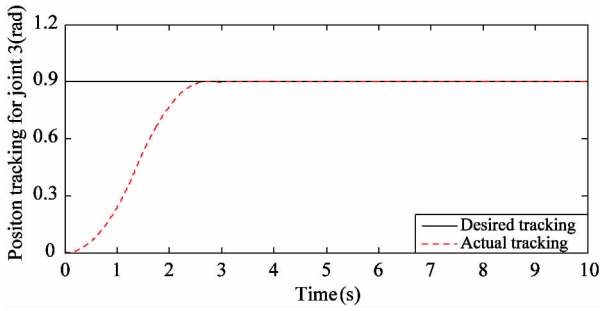
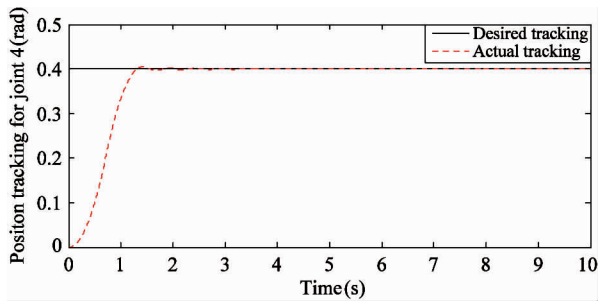**Fig. 11**   Trajectory tracking control for joint 3 based on LADRC

Fig. 12   Trajectory tracking control for joint 4 based on LADRC
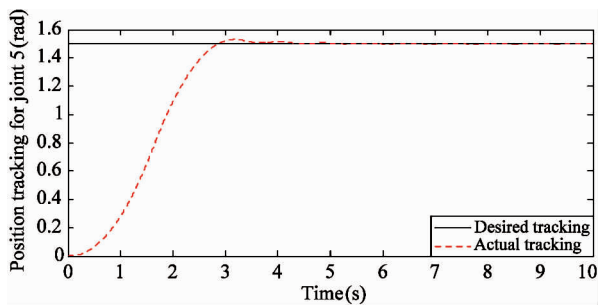

Fig. 13   Trajectory tracking control for joint 5 based on LADRC
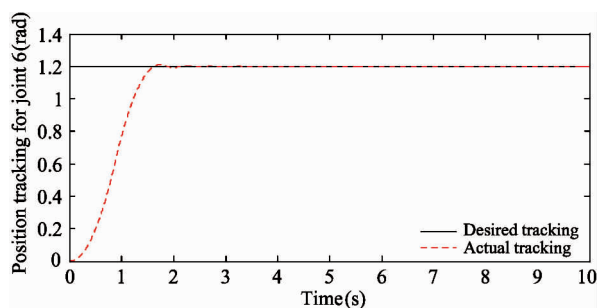

Fig. 14   Trajectory tracking control for joint 6 based on LADRC
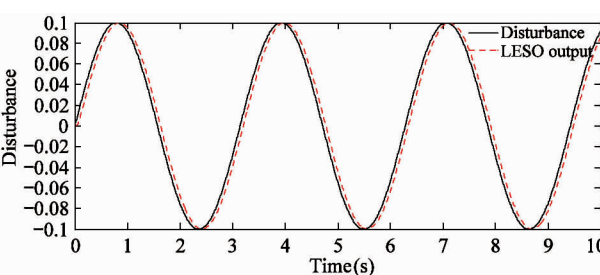

Fig. 15   Disturbance estimation result of LESO for joint 1
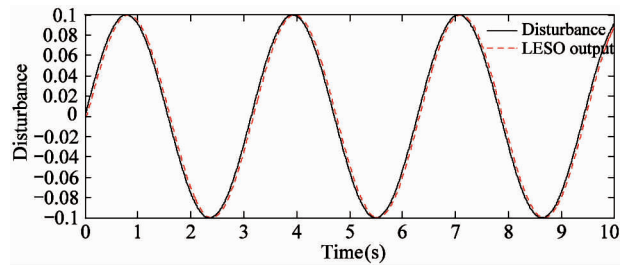

Fig. 16   Disturbance estimation result of LESO for joint 2
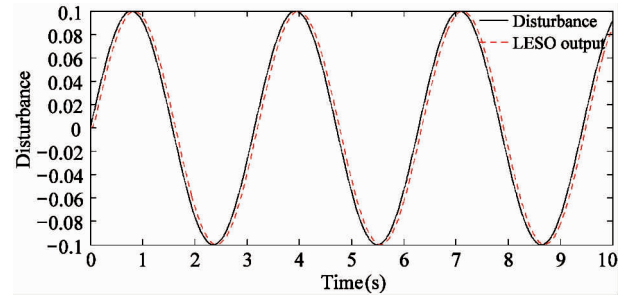

Fig. 17   Disturbance estimation result of LESO for joint 3
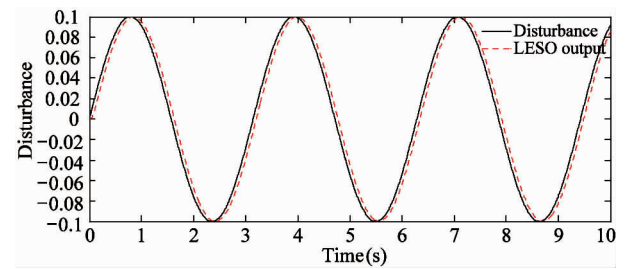

Fig. 18   Disturbance estimation result of LESO for joint 4
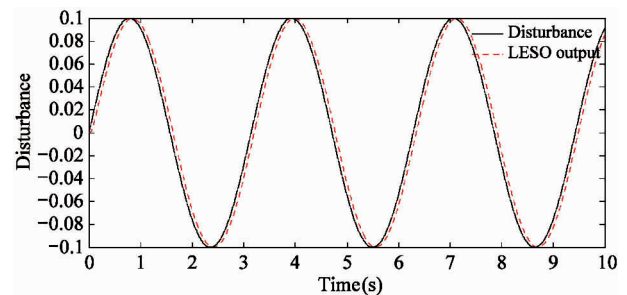

Fig. 19   Disturbance estimation result of LESO for joint 5
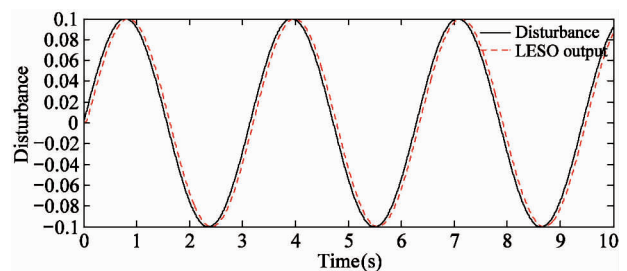

Fig. 20   Disturbance estimation result of LESO for joint 6

4 6-DOF robot manipulator experiment results

4.1 Experimental setup

The 6-DOF robot manipulator developed in the research group consists of 6 modular joints and 2 connecting rods, as shown in Fig. 21. The drive system of each joint is composed of double-winding motor and harmonic reducer. Joint sensors include position sensors, torque sensors and motor position sensors. The electrical system adopts FPGA + DSP technique to provide hardware support for the control algorithm and guarantee the real-time performance of the algorithm. Due to the fact that harmonic reducer and torque sensor as the two flexible parts introduced greatly increase the difficulty of trajectory tracking control, the joint flexibility and unmodeled dynamics regarded as generalized disturbances are estimated by LESO in this paper so as to realize the trajectory tracking control for 6-DOF robot manipulator.

Fig. 21 6-DOF robot manipulator

4.2 Experimental results

In order to verify the effectiveness of the proposed algorithm in practical engineering applications, the experiment is carried out on the 6-DOF robot manipulator developed in the research group. The parameters of LADRC are the same as those in simulations. The initial state of the robot manipulator is zero. The desired joint angles are given as $q_d = [1.6, 0.9, -1.5, 1, 0.6, -1.2]$ rad.

The trajectory tracking results are shown from Fig. 22 to Fig. 27, where it can be clearly seen that LADRC method can offer precise trajectory tracking control performance. Besides, to make a quantitative analysis, some control performance indices are elaborated in Table 2, in which it is obtained that the LADRC method is well suited in dealing with the trajectory tracking control problem with short settling time, small overshoot and no static error, which can meet the actual engineering requirements.
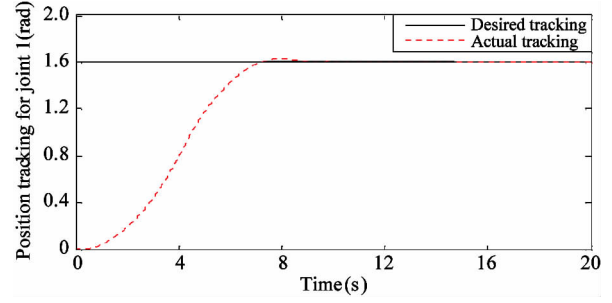

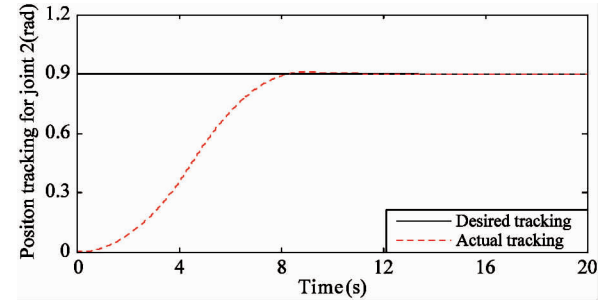**Fig. 22** Trajectory tracking result for joint 1
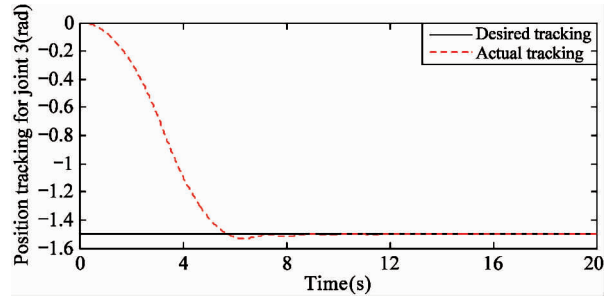

Fig. 23 Trajectory tracking result for joint 2

Fig. 24 Trajectory tracking result for joint 3
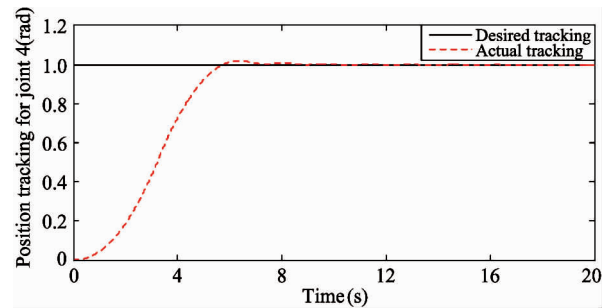

Fig. 25 Trajectory tracking result for joint 4
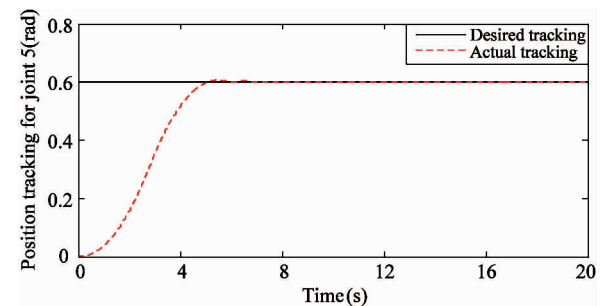

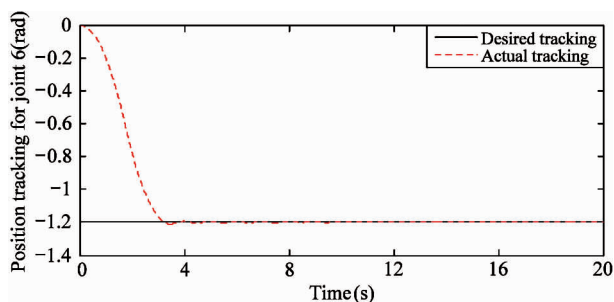**Fig. 26** Trajectory tracking result for joint 5

Fig. 27 Trajectory tracking result for joint 6

Table 2 Control performance indices

Joint	Settling time (s)	Overshoot (%)	Static error (rad)
1	9.8	1.69	0
2	8.9	1.32	0
3	7.3	2.86	0
4	8.1	1.93	0
5	5.5	1.78	0
6	5.8	1.16	0

5 Conclusion

On account of precise trajectory tracking problem for the 6-DOF robot manipulator, the LADRC strategy is proposed in the case of model uncertainties and external disturbances. Firstly, the dynamical model of the 6-DOF robot manipulator is devised. Then, unmodeled dynamics, joint coupling terms as well as external disturbances are unified as generalized disturbance of the robot manipulator system. LESO is designed to estimate and compensate the generalized disturbance effectively. The compared simulations between LADRC and conventional PID demonstrate that the 6-DOF robot manipulator based on LADRC method can offer the favorable trajectory tracking control performance along with strong robustness in the case of time-varying external disturbance. In the meantime, the feasibility of the proposed method is verified again via 6-DOF robot manipulator experiments, which has satisfactory practical engineering application value.

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