

A decoupled multi-objective optimization algorithm for cut order planning of multi-color garment^①

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Abstract

This work addresses the cut order planning (COP) problem for multi-color garment production, which is the first step in the clothing industry. First, a multi-objective optimization model of multi-color COP (MCOP) is established with production error and production cost as optimization objectives, combined with constraints such as the number of equipment and the number of layers. Second, a decoupled multi-objective optimization algorithm (DMOA) is proposed based on the linear programming decoupling strategy and non-dominated sorting in genetic algorithms II (NSGAI). The size-combination matrix and the fabric-layer matrix are decoupled to improve the accuracy of the algorithm. Meanwhile, an improved NSGAI algorithm is designed to obtain the optimal Pareto solution to the MCOP problem, thereby constructing a practical intelligent production optimization algorithm. Finally, the effectiveness and superiority of the proposed DMOA are verified through practical cases and comparative experiments, which can effectively optimize the production process for garment enterprises.

Key words: multi-objective optimization, non-dominated sorting in genetic algorithms II (NSGAI), cut order planning (COP), multi-color garment, linear programming decoupling strategy

0 Introduction

The apparel industry is gradually shifting from mass production to customized manufacturing driven by consumers' growing demand for personalization^[1]. In particular, the 'quick-response production of small-batch orders' mode becomes a new trend in apparel industry under the e-commerce economy. Therefore, it is important to enhance order responsiveness and optimize the production process. The apparel production process has several steps: cut order planning (COP), marker planning, laying, cutting and sewing operations, and packaging^[2]. As the first step, optimizing the COP can effectively reduce production costs and improve production efficiency.

COP is to plan the number of sections, the number of fabric layers for each color, and the number of the placement of sizes based on order information such as quantity, size, and color, in order to meet production demands with minimizing production costs^[3]. Then, it is a non-linear integer programming problem as well as a non-deterministic polynomial-complete

problem constrained by production conditions^[4]. Currently, it is mainly solved through artificial expertise and optimization software in actual production, but it is difficult to rationally solve the complex COP problems with multiple colors and irregular quantities.

COP problem is solved based on expertise in early days, and four main methods were summarized in Ref. [5], namely proportional planning, grouping planning, combination planning, and increment and decrement planning. In Ref. [6], the influencing factors of the cutting bed were calculated according to the analytic hierarchy process, and the key factors influencing COP were determined. In Ref. [7], a two-stage enumeration method was proposed. However, these methods apply only to the regular COP problem. In Ref. [8], a nonlinear mixed integer programming model of COP problem was constructed with the aim of minimizing fabric cost, and was solved by using LINGO optimization software. In Ref. [9], an extended integer programming model was established to minimize the overall cutting costs, which is solved by using soft computing techniques.

With the development of intelligent production op-

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timization algorithms, taboo search (TS), genetic algorithm (GA) and simulated annealing (SA) are gradually applied to solve large-scale and irregular COP problems. In Ref. [10], two encoding schemes for GA were designed to solve the COP: GA1 and GA2. GA1 used a repair strategy to handle the constraint of the cutting bed length, while GA2 employed an enhanced pattern-based one-point and two-point crossover operator. In addition, SA algorithm^[11], two-stage optimization algorithm based on probabilistic search and GA^[12], taboo search-based genetic algorithm (TS-GA), and simulated annealing-based genetic algorithm (SA-GA)^[13] were also proposed to solve the COP problem. In Ref. [14], the COP and the two-dimensional layout (TDL) processes were integrated into a CT problem (CT = COP + TDL), and a hybrid heuristic algorithm was designed to solve the CT problem. It is worth noting that, the above studies mainly deal with the single-color COP (SCOP) problem. In contrast, the multi-color COP (MCOP) has a higher complexity and is more difficult to solve due to more types of fabric colors. In Ref. [15], an integer programming method was proposed to solve the SCOP, and it was extended to the MCOP problem, but performed worse. In Ref. [16], an improved two-population particle swarm-genetic hybrid algorithm was designed for the MCOP. In Ref. [17], aiming at the MCOP problem of fabrics of two colors, the advisory committee on immunization practices (ACIP) algorithm was proposed to solve the problem with the lowest cost as the optimization objective. However, these methods have limited performance for MCOP. In actual production, the optimization objectives such as production errors, number of sections or production costs are often considered. To this end, multi-objective optimization algorithms such as NSGAI^[18] and multiple objective particle swarm optimization (MOPSO)^[19-20] are potential methods to deal with the MCOP. In Ref. [21], a hybrid optimization algorithm, namely NSGAI-TR, was designed to solve the MCOP based on NSGAI. Currently, there are a few studies on the MCOP problem, and how to improve the solution's accuracy and efficiency remains a key issue in the apparel industry. Motivated by this challenge, a decoupled multi-objective optimization algorithm (DMOA) is proposed for the large-scale and irregular MCOP problem in this paper. The main contributions are as follows.

(1) A multi-objective optimization model is established for MCOP, taking the production errors and production costs as the optimization objectives. Meanwhile, constraint conditions such as the number of colors, fabric layers, size ranges, and cutting bed quantities are taken into consideration.

(2) DMOA is designed based on the linear programming (LP) decoupling strategy and NSGAI. First, LP is used to linearly decouple the complex nonlinear relationships between the size combination matrix and the fabric layer matrix. It reduces the dimensionality of encoding and improves both solution efficiency and accuracy. Then, an improved NSGAI algorithm is applied to obtain the size combination matrix and the fabric layer matrix. An adaptive mutation operator and an elite archive storing non-dominated solutions are designed to enhance the algorithm's solution accuracy and convergence rate.

(3) Comparative experiments are carried out using DMOA, NSGAI, NSGAI-LP, SA-GA, NSGAI-TR, and MOPSO-LP algorithms based on practical apparel orders. The experiment results validate the effectiveness and superiority of the proposed algorithm, which can provide excellent MCOP solutions for garment enterprises.

1 MCOP problem and mathematical modelling

1.1 Problem definition

The idea of MCOP (as shown in Fig. 1) is to distribute the clothing order (\mathbf{O}) with multiple sizes and colors to several sections, and then plan the size number in each section (i. e., the size combination matrix \mathbf{P}) and the layers number of each color fabrics in each section (i. e., the fabric layer matrix \mathbf{L}). Meanwhile, one cutting bed can be used to produce one or several sections. For example, if an enterprise receives a production order (\mathbf{O}) as shown in the upper left corner of Fig. 1, the feasible solutions of the size combination matrix (\mathbf{P}) and the fabric layer matrix (\mathbf{L}) should be solved. Then, the element $\mathbf{P}(1, 1) = 1$ represents the number of XS sizes placed on the first section is one. The element $\mathbf{L}(1, 1) = 5$ indicates that five layers of yellow fabric are laid on the first section. Thereafter, different sizes of mark and different colors of fabric are placed in several sections as planned for production. In general, the main goal of a COP solution is to minimize production errors and costs. In actual production, production cost usually composes of fabric cost and cutting bed use cost, and the former accounts for 50% to 60% and the latter is related to the number of sections^[22].

The MCOP problem addressed in this paper is for multiple sizes and colors of one style and fabric, which is common for clothing orders. Then, clothing orders can be essentially described by a two-dimensional ma-

trix, where rows represent colors, columns represent sizes, and elements represent production quantities.

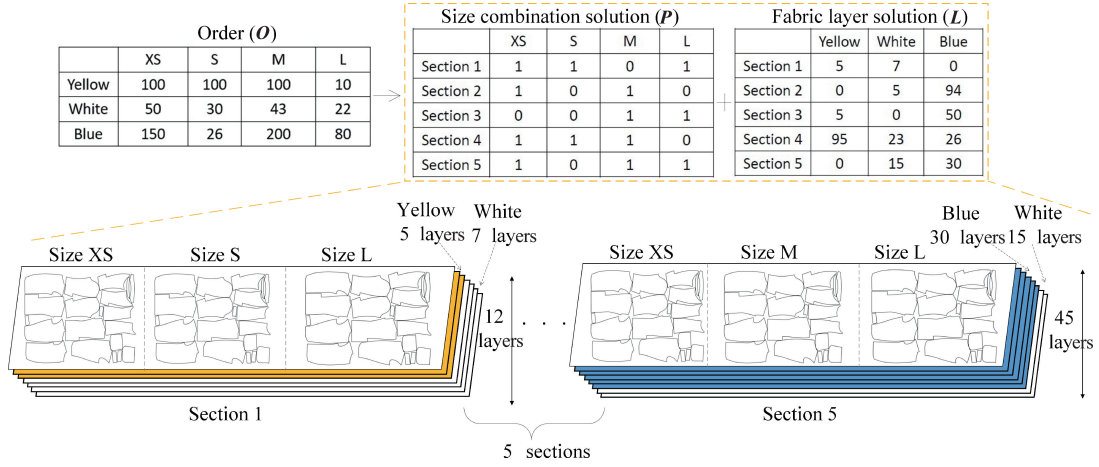


Fig. 1 Schematic diagram for MCOP

Therefore, the relevant parameters for defining MCOP are as follows.

- O : production order matrix;
- O_r : actual production order matrix;
- P : size combination matrix;
- L : fabric layer matrix;
- N : number of sections;
- S : number of size types in the order;
- C : number of color types in the order;
- b : sections ($b = 1, 2 \dots N$);
- o_{cs} : number of garments in color c and size s in the order;
- p_{bs} : number of template pattern of size s placed on section b ;
- l_{bc} : number of fabric layers of color c laid out on section b ;
- N_{max} : maximum number of sections;
- p_{max}^{bs} : maximum number of template pattern of size s placed on section b ;
- l_{max} : maximum number of fabric layers that can be laid out on the section;
- CT_f : cost of fabric consumption;
- CT_b : cost of setting up a new section;
- I : the positive integer.

Notice that the order matrix O , size combination matrix P , and fabric layer matrix L can be represented as

$$O = \begin{bmatrix} o_{11} & \dots & o_{1s} \\ \vdots & \ddots & \vdots \\ o_{c1} & \dots & o_{cs} \end{bmatrix}, P = \begin{bmatrix} p_{11} & \dots & p_{1s} \\ \vdots & \ddots & \vdots \\ p_{b1} & \dots & p_{bs} \end{bmatrix},$$

$$L = \begin{bmatrix} l_{11} & \dots & l_{1c} \\ \vdots & \ddots & \vdots \\ l_{b1} & \dots & l_{bc} \end{bmatrix}$$

The relationship between the actual production or-

der matrix and the size combination matrix and the fabric layer matrix is as follows.

$$O_r = L_T \cdot P$$

1.2 Mathematical modelling

The objective of MCOP is minimizing the production errors, while simultaneously to minimize the overall production costs. Herein, production error E is defined as the sum of the absolute differences between corresponding elements of the production order matrix O and the actual production order matrix O_r , that is

$$E = \text{sum}(|O - O_r|) = \sum_c \sum_s |o_{cs} - \sum_b l_{bc} \cdot p_{bs}| \quad (1)$$

The comprehensive production costs, denoted by C_p , is composed of the costs incurred from section and fabric materials, and can be expressed by

$$C_p = CT_b \cdot N + \sum_b \sum_c CT_f \cdot l_{bc} \quad (2)$$

where the first term represents the cost of setting up all sections, and the second term represents the cost of total fabric consumptions.

Based on the above analysis, the MCOP problem can be formulated as following multi-objective optimization model:

$$\min F = (F_1, F_2) \quad (3)$$

$$F_1 = E \quad (4)$$

$$F_2 = C_p \quad (5)$$

where, F represents the objective function, F_1 represents the production error, and F_2 represents the comprehensive production cost.

In practice, production condition constraints can be described as follows.

(1) The number of fabric layers is constrained by

$$0 \leq \sum_c l_{bc} \leq l_{max}, \forall b \in \{1, 2 \dots N\} \quad (6)$$

(2) The number of sections is constrained by

$$0 \leq N \leq N_{\max} \quad (7)$$

(3) The constraint on the number of size pieces placement is

$$0 \leq p_{bs} \leq p_{\max}^{bs}, \forall b \in \{1, 2, \dots, N\} \quad (8)$$

(4) The constraint of positive integers determined by actual production is

$$b, s, c, p_{bs}, l_{bc} \in I \quad (9)$$

2 Design of the DMOA

In this section, the design of DMOA based on LP and NSGAI is explained in detail. The algorithm flow chart is shown in Fig. 2. Firstly, the size combination matrix is defined as the particle, and the population particles are randomly initialized. In each iteration, the population particles are subjected to crossover and adaptive mutation. Secondly, the fabric layer matrix is decoupled by the LP model, and the optimal fabric layer matrix is calculated. Then, some viable solutions are saved as the elite archive, which is updated iteratively. Finally, the population particles are calculated to pursue the optimal solutions. Once the maximum iteration count is reached, the elite archive is output. The LP decoupling strategy effectively improves the algorithm's solution accuracy. Overall, the DMOA algorithm can effectively solve irregular MCOP problems.

2.1 Improved NSGAI algorithm

In order to improve the solution accuracy of the algorithm, a normal distribution crossover operator is adopted in the crossover operation of the NSGAI algorithm. Additionally, in order to improve the algorithm's ability to explore unknown space in the early stage and to accelerate the convergence speed in the later stage, an adaptive polynomial mutation operator is proposed. Finally, to obtain the Pareto solutions of the MCOP problem, an elite archive mechanism is designed, where the archive contains elite particles, i. e., non-dominant solutions after each iteration, and an updating strategy is used to update the elite archive.

2.1.1 Population encoding

Real-number encoding is adopted for MCOP, and the encoding information includes the number of sections and the size combination solution. Specifically, the encoding rules are as follows:

- (1) Encoding length: $length = 1 + N_{\max} \times S$;
- (2) The first digit in the encoding represents the required number of sections, which should satisfy Eq. (7);
- (3) Starting from the second digit, the encoding represents the size combination matrix, which should satisfy Eq. (8).

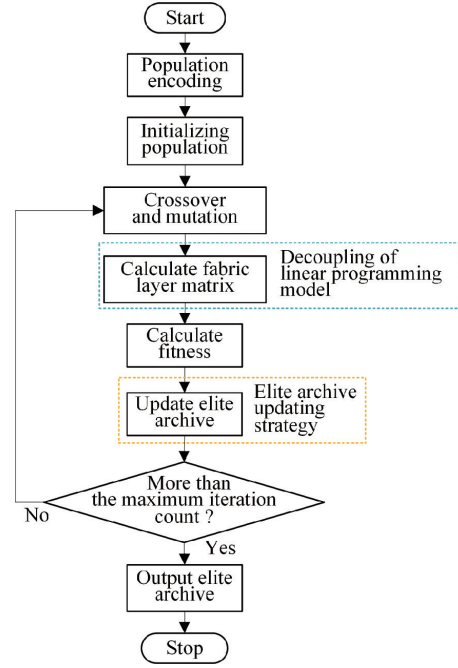


Fig. 2 Algorithm flowchart

2.1.2 Crossover operation

The normal distribution crossover operator is used to enhance the spatial search capability during the crossover operation, and enable the population particles to more easily escape from local optima and obtain a more complete Pareto optimal solution. Assuming the parents are p_1 and p_2 , and the offspring are x_1 and x_2 , the normal distribution crossover operators are

$$x_{1,i} = \frac{p_{1,i} + p_{2,i}}{2} + \frac{1.481(p_{1,i} - p_{2,i})N(0,1)}{2} \quad (10)$$

$$x_{2,i} = \frac{p_{1,i} + p_{2,i}}{2} - \frac{1.481(p_{1,i} - p_{2,i})N(0,1)}{2} \quad (11)$$

where, i represents the i -th variable, and $N(0,1)$ represents a random variable submitted to a normal distribution that mean $\mu = 0$ and standard deviation $\sigma = 1$.

The specific crossover operation process can be described as follows.

- (1) Use the binary tournament selection operation to form a mating pool, and randomly select a particle from the mating pool as the parent p_1 ;
- (2) Select the particles in the top 50% of the production error from the elite archive, and then randomly select one of these particles as the parent p_2 ;
- (3) Use the normal distribution crossover operator as described above to perform the crossover operation and produce offspring.

2.1.3 Adaptive polynomial mutation operator

In the polynomial mutation operator, the smaller the value of the mutation distribution index, the higher the probability that the offspring will deviate from the

parent. Then, an adaptive polynomial mutation operator is proposed that generates a smaller mutation distribution index in the early stages of the algorithm to enhance the global search ability, and generates a larger mutation distribution index in the later stages to improve the local search ability and accelerate the convergence speed. The adaptive polynomial mutation operator is

$$x_{t+1} = x_t + \delta(x_{\max} - x_{\min}) \quad (12)$$

$$\begin{cases} \delta = [2u + (1 - 2u)(1 - \delta_1)^{1+d_m}]^{\frac{1}{1+d_m}} - 1 & u \leq 0.5 \\ \delta = 1 - [2 - 2u + (2u - 1)(1 - \delta_2)^{1+d_m}]^{\frac{1}{1+d_m}} & u > 0.5 \end{cases} \quad (13)$$

$$\begin{cases} \delta_1 = \frac{x_t - x_{\min}}{x_{\max} - x_{\min}} \\ \delta_2 = \frac{x_{\max} - x_t}{x_{\max} - x_{\min}} \end{cases} \quad (14)$$

$$d_m = d_{m,\min} + (d_{m,\max} - d_{m,\min}) \sin\left(\frac{t\pi}{2T}\right) \quad (15)$$

where, x_t represents the particle in the t -th generation of the population that needs to be mutated; x_{\max} represents the upper bound of the population particles; x_{\min} represents the lower bound of the population particles; u is a random variable submitted to a uniform distribution between 0 and 1; d_m represents the mutation distribution index; $d_{m,\min}$ represents the minimum value of the mutation distribution index; $d_{m,\max}$ represents the maximum value of the mutation distribution index; t represents the current iteration of the population; and T represents the maximum number of iterations.

2.1.4 Elite archive updating strategy

The elite archive stores the non-dominated solutions obtained in each iteration, and the output of the algorithm is the particle in the elite archive. In order to make the particles in the elite archive approximate the Pareto front of the MCOP problem, it is necessary to maintain and update the elite archive in real time. The elite archive updating strategy is described as follows.

(1) Perform non-dominated sorting on the particles in the population and the particles in the elite archive, and select the non-dominated particles to be included in the elite archive.

(2) When the number of particles in the archive exceeds the maximum capacity, calculate the crowding distance of each particle, and remove the smallest crowding distance ones. Check whether the number of particles exceeds the maximum capacity. If so, repeat the previous step until the number of particles is exactly equal to its maximum capacity.

(3) If the number of particles in the archive does not exceed the maximum capacity, all particles will be

retained.

2.2 Linear programming model

In the MCOP problem, there is a serious nonlinear coupling relationship between the size combination matrix and the fabric layer matrix. In this paper, the LP decoupling strategy is proposed to decouple them and calculate the fabric layer matrix.

In the DMOA algorithm, the population particle is the encoding of the size combination matrix. Therefore, when the size combination matrix is known, the optimal fabric layer matrix can be obtained through the following model, so that the production error and the production cost is minimal.

$$\min F = (F_1, F_2) \quad (16)$$

$$F_1 = \text{sum}(|\mathbf{O}^T - \mathbf{P}^T \mathbf{L}|) \quad (17)$$

$$F_2 = \mathbf{C}T_b \cdot N + \sum_b \sum_c \mathbf{C}T_f \cdot l_{bc} \quad (18)$$

subject to:

$$0 \leq N \leq N_{\max} \quad (19)$$

$$0 \leq \sum_c l_{bc} \leq l_{\max}, \forall b \in \{1, 2, \dots, N\} \quad (20)$$

$$b, s, c, p_{bs}, l_{bc} \in I \quad (21)$$

Eq. (16) represents the objective function; Eq. (17) represents production errors, where T denotes transpose; Eq. (18) represents production costs; Eq. (19) represents the constraint on the number of sections; Eq. (20) represents the constraint on the number of fabric layers; and Eq. (21) represents constraints on the integer values of the variables.

As can be seen from Eq. (17), the above model is a nonlinear programming model, that is, there is a complex nonlinear relationship between the size combination matrix and the fabric layer matrix. In order to reduce the solving complexity, the above model can be transformed into a linear programming model through mathematical transformation, and the two are linearly decoupled. The specific methods are as follows.

Firstly, a new variable y is introduced to replace the absolute value variable of Eq. (17). Then, y is constrained by the constraints $y \geq \mathbf{P}^T \mathbf{L} - \mathbf{O}^T$ and $y \geq -(\mathbf{P}^T \mathbf{L} - \mathbf{O}^T)$. Finally, the linear programming model for the MCOP problem is

$$\min F = (F_1, F_2) \quad (22)$$

$$F_1 = \text{sum}(y) \quad (23)$$

$$F_2 = \mathbf{C}T_b \cdot N + \sum_b \sum_c \mathbf{C}T_f \cdot l_{bc} \quad (24)$$

subject to:

$$y \geq \mathbf{P}^T \mathbf{L} - \mathbf{O}^T \quad (25)$$

$$y \geq -(\mathbf{P}^T \mathbf{L} - \mathbf{O}^T) \quad (26)$$

$$0 \leq N \leq N_{\max} \quad (27)$$

$$0 \leq \sum_c l_{bc} \leq C_{\max}, \forall b \in \{1, 2, \dots, N\} \quad (28)$$

$$b, s, c, p_{bs}, l_{bc} \in I \quad (29)$$

Eq. (22) represents the objective function; Eq. (23) represents the sum of variable y ; Eq. (24) represents production costs; Eqs. (25) and (26) represent constraints on variable y ; Eq. (27) represents the constraint on the number of sections; Eq. (28) represents the constraint on the number of fabric layers; and Eq. (29) represents constraints on the integer values of the variables.

2.3 DMOA algorithm

In this paper, the production errors and production costs are taken as optimization objectives of the MCOP problem. To ensure that the production error meets the production requirements of the enterprise, a normal distribution crossover operator is introduced in the crossover operation, and an adaptive polynomial mutation operator is designed to enhance the global search ability and accelerate the convergence speed. To improve the solution accuracy and efficiency, a linear programming model is designed to decouple the size combination matrix and the fabric layer matrix, and the fitness of the population particles is calculated from them. Finally, the Pareto optimal solution of the MCOP problem is output from the elite archive. In summary, in order to search for the optimal solution of the MCOP, the linear programming model is embedded into the improved NSGAI algorithm, thus promoting the performance of the proposed DMOA algorithm. The pseudocode of the DMOA algorithm is illustrated in Algorithm 1.

3 Comparison of algorithms

In this section, comparative experiments are carried out to verify the effectiveness and superiority of the proposed DMOA algorithm for the MCOP problem of garment enterprises. Specifically, DMOA, MOPSO-LP, NSGAI, NSGAI-LP, SA-GA proposed in Ref. [13], and NSGAI-TR proposed in Ref. [20] are compared. The experiment was conducted on a hardware with an Intel(R) Core (TM) i7-11800H processor whose CPU frequency is 2.30 GHz and 16 GB memory, with Windows 11 operating system and PyCharm 2021.3 as the algorithm running platform, and Python version 3.9 as the software environment. Set the population size of the above algorithms to $M = 100$, the maximum number of iterations to $T = 200$, and terminate the iteration when the minimum production error remains unchanged for 20 consecutive times. Other algorithm parameter settings are described as follows: in the NSGAI-TR, NSGAI, and NSGAI-LP algorithms, the crossover factor and variation factor of the algo-

rithms are set in accordance with Ref. [20], that is, the crossover factor $d_c = 0.9$ and the variation factor $d_m = 0.1$. In the SA-GA algorithm, the initial annealing temperature $t_i = 300$ is set, and the attenuation coefficient $\alpha = 0.9$ is set after several experiments. In the MOPSO-LP algorithm, set the inertia factor $w = 0.9$, the individual learning factor $c_1 = 1.8$, and the social learning factor $c_2 = 1.5$ by the experimental method. In the DMOA algorithm, since the adaptive variation distribution index is adopted, set the mutation distribution index $d_{m, \min} = 0.1$ and the maximum value $d_{m, \max} = 1.0$.

Algorithm 1 Pseudocode for the DMOA algorithm

Input:

m : current population particle;
 M : number of particles in the population;
 t : current iteration number;
 T : maximum iteration number;

Output:

e : elite archive particle;

Initialization:

$t = 0$;
 $m = 1$;

Iteration step:

while $t < T$
 for $m = 1, \dots, M$
 Randomly select parent particles and perform a crossover operation to generate offspring;
 Perform mutation operations using an adaptive polynomial mutation operator;
 Use the linear programming model to calculate the fabric layer matrix;
 Calculate the fitness based on Eqs. (4) and (5);
 end for
 Update elite archive particles according to the elite archive updating strategy;
 Generate a new population based on the elite preservation strategy of the NSGA-II algorithm;
 $t = t + 1$;
end while

The multi-color garment orders in this section were obtained from the actual production of an apparel enterprise. Table 1 shows the details of four different orders, including one regular order and three irregular orders. Order 2 has only 4 size categories, which is relatively simple, while Order 4 has 6 size categories and a larger total quantity. The constraint parameters are set based on the actual production conditions, including the maximum number of fabric layers $l_{\max} = 100$, the maximum number of sections $N_{\max} = 15$, the maximum number of template patterns $p_{\max}^{bs} = 4$, the maximum production error rate $e_{\text{rate}} = 2\%$, the fabric consumption cost $CT_f = 10$ and setup cost $CT_b = 500$.

Table 1 Multi-color clothing orders

Order number	Color	Size					
		XS	S	M	L	XL	XXL
1	Yellow	88	250	154	148	213	/
	White	88	250	154	148	213	/
	Black	88	250	154	148	213	/
	Green	88	250	154	148	213	/
	Blue	88	250	154	148	213	/
2	Yellow	140	178	110	28	/	/
	White	179	247	141	27	/	/
	Black	323	175	173	205	/	/
	Green	412	168	250	219	/	/
	Blue	179	262	154	47	/	/
3	Yellow	323	275	66	405	173	/
	White	240	70	315	76	220	/
	Black	188	227	224	268	130	/
	Green	76	125	156	104	240	/
	Blue	54	186	248	189	116	/
4	Yellow	413	112	85	265	206	242
	White	156	351	228	78	550	369
	Black	110	168	109	115	150	64
	Green	370	165	253	216	146	108
	Blue	90	245	333	155	120	186

3.1 Application cases

In this section, the DMOA algorithm is used to optimize and solve the four different orders presented in Table 1. Table 2 provides the results of the top four optimal solutions obtained after 10 times algorithm runs. It can be observed that the error rates are far lower than the production requirements set by the enterprise. For Order 1, Solution 1 – 1 achieves zero error. Although Solution 1 – 4 has the highest error rate, it still satisfies the production error requirement, and it has the lowest cost among the four solutions. It is noteworthy that the DMOA algorithm can achieve zero error for regular orders. For Order 2, Solution 2 – 1 also achieves zero error. For Order 3, although there is no zero-error solution among the provided options, the error rate is relatively small, with a production error of only 1, fully meeting the desired error rate requirement of the enterprise. Order 4 is the most complex of these orders, yet the final error rate obtained is still very small. Generally, as the scale and irregularity of orders increase, the error rate of the MCOP solution tends to be higher. In practice application, the enterprise can tradeoff between the smaller error and lower costs of the MCOP solutions to make an appropriate decision.

Table 2 Solutions for multi-color garment orders

Order number	Solution number	Error	Cost	Error rate/%
1	1 – 1	0	4 720	0.000
	1 – 2	4	4 560	0.117
	1 – 3	8	4 520	0.234
	1 – 4	12	4 480	0.352
2	2 – 1	0	10 380	0.000
	2 – 2	1	9 070	0.027
	2 – 3	2	8 800	0.055
3	3 – 1	1	9 780	0.021
	3 – 2	2	9 430	0.043
	3 – 3	8	9 350	0.170
	3 – 4	10	9 270	0.213
4	4 – 1	1	12 630	0.016
	4 – 2	5	11 610	0.081
	4 – 3	8	11 340	0.130
	4 – 4	9	11 290	0.146

Table 3 presents size combination solutions \mathbf{P} and fabric layer solutions \mathbf{L} for the first two solutions of Order 3. The results indicate that, for example, in Solution 3 – 1, 12 sections should be setup. On Section 1, 4 XS sizes, 2 S sizes, 4 L sizes, and 2 XL sizes need to be placed; 63 layers of yellow fabric and 4 layers of black fabric are required. On Section 2, 1 S size, 4 M sizes, and 4 L sizes need to be placed; 4 layers of yellow fabric, 12 layers of black fabric, 1 layer of green fabric, and 22 layers of blue fabric are required. The same solution applies to Section 3 – 12.

3.2 Algorithm comparison

In this section, the proposed DMOA algorithm is compared with the traditional NSGAI, NSGAI-LP, MOPSO-LP, SA-GA, and NSGAI-TR. The NSGAI algorithm does not decouple the size combination matrix and the fabric layer matrix, while the MOPSO-LP algorithm is a hybrid optimization algorithm that combines linear programming and MOPSO. The SA-GA algorithm decouples the matrices based on expertise, and the NSGAI-TR algorithm uses ridge regression decoupling. Each algorithm is run ten times, and the top four optimal solutions are recorded. Table 4 and Fig. 3 present the comparative results and running time of these five algorithms for Order 4.

Table 4 shows that the DMOA algorithm can achieve solutions with smaller errors and lower costs compared with the other algorithms. In addition, its error is very close to zero, which is approximately one 25th of NSGAI-LP, one-twentieth of the MOPSO-LP,

Table 3 Size combination solutions and fabric layer solutions for Order 3

Solution number	Section number	Size					Color				
		XS	S	M	L	XL	Yellow	White	Black	Green	Blue
3-1	1	4	2	0	4	2	63	0	4	0	0
	2	0	1	4	4	0	4	0	12	1	22
	3	2	4	2	4	0	21	1	20	0	0
	4	0	4	4	3	4	0	0	0	2	26
	5	4	4	4	4	4	2	1	28	7	0
	6	4	4	4	1	0	0	8	2	0	12
	7	3	1	4	2	2	0	30	3	0	0
	8	0	0	4	0	4	0	11	0	17	0
	9	4	0	4	0	4	0	28	0	12	0
	10	0	4	0	3	4	8	0	0	22	1
	11	3	3	0	3	1	7	0	0	0	0
	12	3	4	4	4	4	0	0	1	0	2
3-2	1	4	2	0	4	2	64	0	8	0	0
	2	0	1	4	3	1	0	12	0	0	26
	3	2	4	2	4	0	23	0	24	0	14
	4	0	4	4	2	4	1	3	9	10	20
	5	1	4	0	4	3	9	0	2	0	0
	6	4	3	4	2	0	0	12	1	0	1
	7	3	1	4	3	2	3	2	28	0	1
	8	0	0	4	0	4	0	3	0	10	1
	9	3	4	4	2	0	1	2	6	0	5
	10	4	0	4	0	4	0	45	0	19	1
	11	0	4	0	4	4	2	0	4	21	0

one-fiftieth of the NSGAI-TR, and one-hundredth of the SA-GA. Compared with the NSGAI algorithm, NSGAI-LP algorithm can solve the solutions with less error and less cost. Therefore, the experimental results of DMOA, NSGAI-LP and NSGAI verify the effectiveness of the improved strategy. The MOPSO-LP algorithm, due to its utilization of LP decoupling strategy, yields significantly lower error and cost values compared with NSGAI, SA-GA, and NSGAI-TR algorithms. Obviously, the results of MOPSO-LP and DMOA show that the introduction of LP decoupling strategy can greatly reduce production errors and production costs.

Fig. 3 indicates that the DMOA algorithm requires least iterations and has the shortest running time, while the NSGAI algorithm takes the most iterations and has the longest running time. Since there exists a non-linear coupling relationship between the size combination matrix and the fabric layer matrix, linear decoupling of both is necessary. Among these decoupling algorithms, the linear programming model-based decoupling algorithm demonstrates the best performance. Therefore,

the DMOA algorithm exhibits the highest solution accuracy and efficiency.

Fig. 4 and Fig. 5 are the box plot for Order 3 and Order 4, respectively. They illustrate that the DMOA algorithm is the most effective, followed by the MOPSO-LP algorithm. Both of them exhibit significantly superior performance in various metrics compared with the other algorithms, primarily due to their utilization of linear programming model decoupling. Moreover, the NSGAI-TR algorithm outperforms the SA-GA algorithm, while the NSGAI algorithm exhibits the poorest performance.

One-way ANOVA is used to detect significance and analyze whether all algorithms have the same performance when obtaining the final result. In One-way ANOVA, the key index is the statistic F -value and its corresponding p value. Table 5 shows statistic F -values and the p -values of One-way ANOVA for Order 3 and Order 4. The results indicate that for Order 3, the F -value is 93.775 and the corresponding p -value is less than the pre-specified threshold (0.05), suggesting that all the algorithms compared are significantly differ-

ent. Similarly, for Order 4, the p -value is also less than the pre-specified threshold, indicating significant differences among all compared algorithms. Therefore, all compared algorithms exhibit statistically significant differences. In addition, according to the results of the box plot in Fig. 4 and Fig. 5, DMOA algorithm can obtain better solution results than other algorithms. In summary, all comparison algorithms have significant differences, and DMOA algorithm has the optimal performance.

Table 4 The results of the algorithms for Order 4

Optimization algorithm	Solution number	Error	Cost	Error rate/%
NSGAI	1	213	12 250	3.459
	2	224	12 170	3.638
	3	249	12 050	4.044
	4	283	12 010	4.596
NSGAI-LP	1	25	11 980	0.406
	2	26	11 310	0.422
	3	28	11 280	0.455
	4	29	10 510	0.471
NSGAI-TR	1	56	12 390	0.909
	2	102	11 830	1.656
	3	106	11 810	1.721
	4	112	11 550	1.819
SA-GA	1	109	11 930	1.770
	2	113	11 790	1.835
	3	114	11 690	1.851
	4	124	11 650	2.014
MOPSO-LP	1	19	12 940	0.309
	2	22	11 240	0.357
	3	24	11 220	0.390
	4	27	10 870	0.438
DMOA	1	1	12 630	0.016
	2	5	11 610	0.081
	3	8	11 340	0.130
	4	10	10 860	0.162

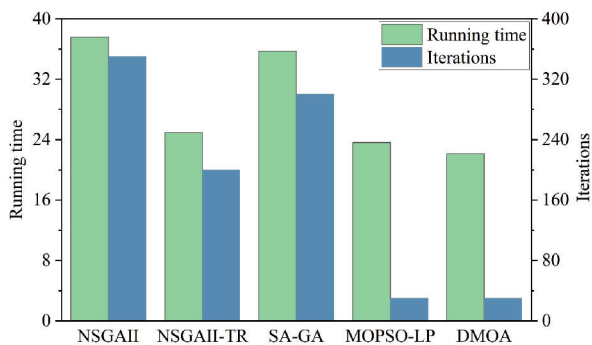


Fig. 3 The running time and iterations of algorithms

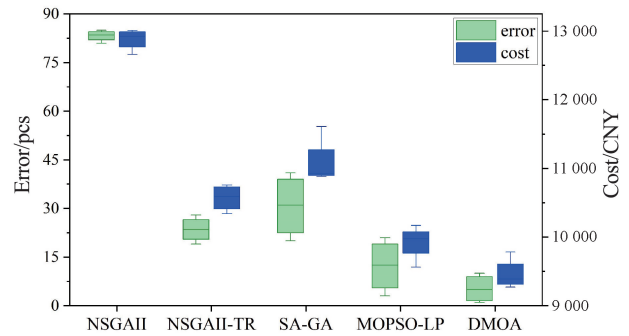


Fig. 4 Box plot of error and cost for Order 3

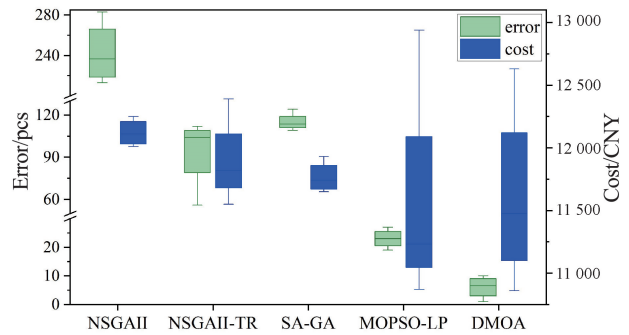


Fig. 5 Box plot of error and cost for Order 4

Table 5 p -value of One way ANOVA for Order 3 and Order 4

	F statistic	p -value
Order 3	93.775	2.001×10^{-10}
Order 4	104.035	9.482×10^{-11}

Fig. 6 presents the Pareto front curve for Order 4 using three multi-objective algorithms (DMOA, MOPSO-LP, and NSGAI-TR). It can be observed from the figure that the DMOA algorithm dominates the other two algorithms in terms of Pareto dominance, making it the preferred choice. Furthermore, it is obvious that there is a significant difference in performance between linear programming decoupling and ridge regression decoupling strategies. This discrepancy primarily stems from the fact that when the size combination matrix and the order matrix are fixed, there may be a variety of fabric layer matrices that satisfy the constraints. The LP

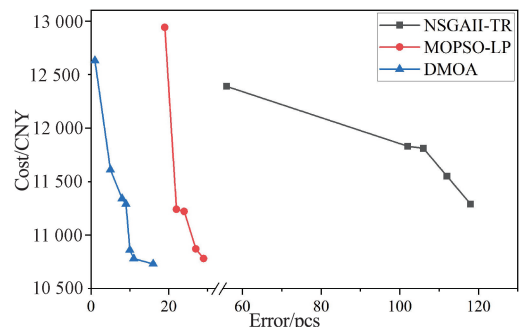


Fig. 6 Graph of the Pareto optimal front for Order 4

decoupling strategy can solve the optimal fabric layer matrix, whereas the ridge regression decoupling strategy only provides one of the fabric layer matrices that meet the constraints, which is generally not the optimal one.

4 Conclusion

A DMOA is proposed to address irregular MCOP problem in this paper. The optimization objectives focus on production errors and production costs, which are of great concern to clothing companies. LP model is designed for the first time to decouple the size combination matrix and the fabric layer matrix. Through this LP model, the optimal fabric layer matrix can be obtained by given a known-size combination matrix. This greatly enhances the algorithm's solution accuracy, and effectively resolves the coupling issue between the size combination matrix and the fabric layer matrix in the MCOP problem. In this paper, the flow of the algorithm is described comprehensively, and the superior performance of the algorithm is verified by application cases and comparative experiments, which brings remarkable economic benefits to garment enterprises.

References

- [1] XU Y, THOMASSEY S, ZENG X. Garment mass customization methods for the cutting-related processes [J]. *Textile Research Journal*, 2021, 91(7-8): 802-819.
- [2] M'HALLAH R, BOUZIRI A. Heuristics for the combined cut order planning two-dimensional layout problem in the apparel industry[J]. *International Transactions in Operational Research*, 2016, 23(1-2): 321-353.
- [3] DE S P, LANEL G H J, PERERA M T M. Integer quadratic programming (IQP) model for cut order plan [J]. *IOSR Journal of Mathematics*, 2017, 13(2):76-80.
- [4] NASCIMENTO D B, DE F J N, MAYERLE S F, et al. A state-space solution search method for apparel industry spreading and cutting[J]. *International Journal of Production Economics*, 2010, 128(1): 379-392.
- [5] ZHENG R P. Investigation of the production of a small batch of clothing in many styles[J]. *Journal of Tian Jin Institute of Textile Science and Technology*, 1988, 17(2): 97-103. (In Chinese)
- [6] TUO W, ZHENG P, CHANG T T, et al. Analysis of factors affecting cutting scheme based on analytical hierarchy process[J]. *Journal of Textile Research*, 2013, 34(4): 148-152.
- [7] ROSE D M, SHIER D R. Cut scheduling in the apparel industry[J]. *Computers and Operations Research*, 2007, 34(11): 3209-3228.
- [8] ÜNAL C, YÜKSEL A D. Cut order planning optimization in the apparel industry[J]. *Fibers and Textiles in Eastern Europe*, 2020, 28(139): 8-13.
- [9] XU Y, THOMASSEY S, ZENG X. Optimization of garment sizing and cutting order planning in the context of mass customization[J]. *The International Journal of Advanced Manufacturing Technology*, 2020, 106(7): 3485-3503.
- [10] MARTENS J. Two genetic algorithms to solve a layout problem in the fashion industry[J]. *European Journal of Operational Research*, 2004, 154(1): 304-322.
- [11] ABEYSOORIYA R P, FERNANDO T G I. Hybrid approach to optimize cut order plan solutions in apparel manufacturing [J]. *International Journal of Information and Communication Technology*, 2012, 2(4), 348-353.
- [12] LIU Y M, YAN S C, et al. Two stage optimization method of cut order planning for apparel mass customization[J]. *Computer Integrated Manufacturing Systems*, 2012, 18(3): 479-485.
- [13] TSAO Y C, VU T L, LIAO L W. Hybrid heuristics for the cut ordering planning problem in apparel industry[J]. *Computers and Industrial Engineering*, 2020, 144: 1-12.
- [14] BOUZIRI A, M'HALLAH R. A hybrid genetic algorithm for the cut order planning problem[C]//*New Trends in Applied Artificial Intelligence: 20th International Conference on Industrial, Engineering and Other Applications of Applied Intelligent Systems*. Kyoto, Japan: IEA/AIE, 2007: 454-463.
- [15] DEGRAEVE Z, GOCHET W, JANS R. Alternative formulations for a layout problem in the fashion industry[J]. *European Journal of Operational Research*, 2002, 143(1): 80-93.
- [16] DU S X, WU T. Research on the application of clipping bed sharing algorithm of double-population mixed genetic algorithm[J]. *Computer Engineering and Application*, 2021, 57(22): 182-189.
- [17] YANG C L, HUANG R H, HUANG H L. Elucidating a layout problem in the fashion industry by using an ant optimization approach[J]. *Production Planning and Control*, 2011, 22(3): 248-256.
- [18] ZHOU C, ZHANG J J, SUN X, et al. Research on optimization design method of actuator parameters with stepless capacity control system for reciprocating compressor[J]. *High Technology Letters*, 2020, 26(2): 168-177.
- [19] HAN H, ZHANG L, YINGA A, et al. Adaptive multiple selection strategy for multi-objective particle swarm optimization [J]. *Information Sciences*, 2023, 624: 235-251.
- [20] AGAJIE T F, GEBRU F M, SALAU A O, et al. Investigation of distributed generation penetration limits in distribution networks using multi-objective particle swarm optimization technique[J]. *Journal of Electrical Engineering and Technology*, 2023, 18(6): 4025-4038.
- [21] WU X, DONG H, YU L, et al. Hybrid optimization algorithm for cut order planning of multicolor garment[J]. *Control and Decision*, 2022, 37(6): 1531-1540.
- [22] WONG W K, LEUNG S. Genetic optimization of fabric utilization in apparel manufacturing[J]. *International Journal of Production Economics*, 2008, 114(1): 376-387.

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